“IT’S NUMBERS AND THAT’S IT”: AN EXPLORATION OF CHILDREN’S BELIEFS ABOUT MATHEMATICS THROUGH THEIR DRAWINGS AND WORDS

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Abstract

Children's beliefs about mathematics involve epistemological beliefs about the subject, its nature and how it works, as well as beliefs about who can and cannot do mathematics. While children's beliefs about mathematics have been linked to their achievement in mathematics, there is little research that explores beliefs about mathematics in the New Zealand context. A general concern is that students do less well than they could at mathematics; hence many people give up on and disengage from mathematics.

This study explores children's and their teachers' beliefs about mathematics and is set against a backdrop of prevailing achievement discourses, both in New Zealand and abroad, that define people's perceived abilities as usually based on ethnicity and gender. It also considers the multiple worlds of the child, the worlds of mathematics beliefs and of doing school mathematics, the child's relationships with these worlds and with others who inhabit them.

The study combines complementary theories and methods to examine espoused and enacted mathematics beliefs by adopting a predominantly sociocultural perspective and including a combination of constructivist and pragmatic theories as well as multiple methods of accessing and analysing beliefs. In order to develop a picture of mathematics beliefs, I collected data from a number of sources: mathematics beliefs questionnaires from 823 children at 17 schools, drawings from 180 children at two focus schools, video recordings of multiple mathematics lessons in two focus classrooms and observations during the first year. The following year, I revisited, observed and interviewed nine focus children and their teachers. I applied multiple analysis 'frames' to the data: factor analysis, adapted visual frameworks, metaphors and themes.

By combining a variety of methods and applying a number of different analysis perspectives, this study exposed a rich and complex landscape of beliefs about mathematics. In particular, the children's drawings communicated mathematics beliefs by using metaphors such as 'maths as problem solving', 'maths as useful', 'maths as life', and 'maths as brain burn inducing'. The children and teachers exhibited a range of beliefs about the world of mathematics and who belongs to this world by positioning certain people as good at mathematics, not good at mathematics, or in certain cases, both positions depending on the context. In terms of assigned mathematics identities, both children and teachers refer to the 'Asian as good at maths' discourse but do not position Māori and Pasifika as weak; gender was not viewed as important. On the other hand, the children's responses were influenced by their ethnicities, gender, socioeconomic status and mathematics achievement levels. The implications for primary school mathematics relate to the powerful influence of how mathematics is done, taught and learnt within the dominant context of the Numeracy Projects which governs ability groupings, the dance of the mathematics class, the ascendency of strategy over algorithm, and the notion that there are multiple ways to solve problems. In particular, the implications of inequality inherent in mathematics ability grouping warrants addressing.

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Chapter 1: Setting the scene

What is mathematics?

Most people would say it has something to do with numbers, but numbers are just one type of mathematical structure. Saying "math is the study of numbers" (or something similar) is like saying that "zoology is the study of giraffes". Math may be better thought of as the study of patterns, but this too falls short...

The more I study math, the more I wonder about what exactly math is. Actually nobody knows. It seems to be a product or our minds, and yet reflects the external universe with uncanny accuracy. A mathematician develops a mathematical theory for its aesthetic unworldly beauty and its compelling evolution, with no thought of how it might be applied to the world. A century later a physicist finds this theory to be perfect to use as a framework to express his physics (this sort of thing happens frequently). Pretty weird how intimately connected our innermost "mind" and the outermost "universe" really are. This is a profound mystery!
Rafael Espericueta (2001)

"It's nummmmmbbers" and dots and x's and +'s and %'s and . and – and that's it.
Spud Murphy, age 10, Kikorangi, Alien Task, written response

Introduction

This study explores children's beliefs about mathematics, in particular beliefs about the nature of mathematics, self-beliefs about learning mathematics and beliefs about the groups of people who may or may not be good at mathematics.

Of all the curriculum subjects taught during the years of compulsory education, mathematics engenders the greatest range of emotional reactions: there are those who fear, loathe and detest it as well as many who find it fascinating, challenging and worth studying (Bartholomew, Darragh, Ell, & Saunders, 2011; Boaler, 2010; Gates, 2001). The public at large, politicians and many educators accept that mathematics is used as a form of 'border control' to keep those deemed unsatisfactory out of further study, certain qualifications and professions (Ashcraft & Ridley, 2005; Boaler, 2010; Ernest, 1995; Gates, 2001;

Leder, Pehkonen, & Törner, 2002a, 2002b; Mendick, 2006). They also view good mathematics achievement results with a sense of national pride while poor results are seen as indicative of all that may be wrong with the education system, with teachers, with the curriculum and with students' backgrounds and dispositions. Mathematics educators such as Paul Cobb have a different perspective: they value encouraging an interest in mathematics and developing "an empathy for and sense of affiliation with mathematics together with the desire and capacity to learn more about mathematics when the opportunity arises" (2007, pp. 8-9) even if students do not choose to follow a mathematics dependent career.

A rationale for undertaking research in mathematics education according to Morgens Niss is "…because there are far too many students of mathematics, from kindergarten to university, who get much less out of their mathematical education than would be desirable for them or society" (2007, p. 1293). In the context of these and other discourses about mathematics in general, and school mathematics in particular, this study of beliefs about mathematics aims to explore children's beliefs as an element that may help explain their engagement in, empathy for, and feelings about mathematics. An understanding of beliefs about mathematics may suggest ways children may get more out of mathematics education and thus improve mathematics learning.

Background to this study

In 2004, I completed a research project that examined the numeracy skills and confidence of 373 pre-service primary teachers at a New Zealand college of education. The impetus for this research was a claim by my students in an assessment course that they could not do "that divide or percent stuff" and that I over-estimated their mathematics skills. Consequently, I became interested in what the participants believed about their ability to do mathematics, and how it might affect their performance as teachers and learners, an area not analysed in the project (Solomon, 2004). I was also interested in how these teachers' beliefs may affect what the children in their classrooms believed about mathematics.

During the same period of time, I was lecturer and coordinator for a course designed by Graham Nuthall that focused on the relationship between teaching and learning. From working with some of Nuthall and his team of researchers' findings from the Understanding Learning and Teaching Project at the University of Canterbury (Alton-Lee & Nuthall, 1991, 1998; Collins & O'Toole, 2006; Nuthall, 1999; O'Toole, 2005), I became extremely interested in what happens in the classroom, what teachers and students bring into the classroom that might contribute to what is taught and what is learned, as well as what may either interfere with or augment learning; in other words, what may help explain why some students learn and remember the concepts that are taught while others do not. Although the terms *learning* and *remembering* are often used either as synonyms or as twinned processes associated with having acquired a new skill or piece of knowledge, they can also be defined as separate but related processes. If one views learning as "a long-term change in mental representations or associations as the result of experience" (Ormrod, 2008, p. 4) then both the processes of acquiring and storing are implicated. On the other hand, *remembering* includes notions of being able to locate, access and/or reconstruct what was stored, what was learnt (Nuthall, 2000; Ormrod, 2008).

For Nuthall, the traditionally accepted views about the relationship between teaching and learning could be described in terms of cultural rituals and routines surrounded by a "web of supporting beliefs or myths that explain or justify the way those routines are played out" (2005, p. 920). One of these myths is the usually accepted causal relationship between teaching and learning. Being engaged in the normal classroom routines of discussions, completing activities and/or listening to the teacher, for instance, does not necessarily equate with children learning what the teacher intended them to learn (Nuthall, 2005). In the same article, he claims that "what a student knows is a coherent body of beliefs and understandings that is for the most part logically and consistently interconnected" (Nuthall, 2005, p. 911). Both the notion of myths and beliefs associated with learning, and the network of beliefs and understandings that are part of children's knowing became central themes in my approach to understanding beliefs about mathematics. My decision to go into classrooms in order to understand how the cultural rituals associated with classroom teaching and learning played out and their relationships to children's beliefs about mathematics was based, in part, on Nuthall's findings and perspective on learning.

In addition, my experiences of teaching mathematics stimulated my interest in mathematics beliefs. Over 30 years ago in KaNgwane (now Mpumalanga, South Africa), I faced my first mathematics class full of students who believed people either could or could not do mathematics. These students were in their first year at secondary school, and this was their first algebra lesson of the year. They were so convinced that algebra was too difficult that many of them could not see the point of trying to understand and learn the material. Since then, I have worked as a teacher, tutor and lecturer with pre-service and in-service teachers, university/college students, adult literacy and numeracy students, and children. In addition, I have conducted workshops for tutors who lacked confidence in their own mathematics ability. Apart from my experiences with my pre-service education students who believed they "couldn't do that divide and percent stuff", another story stands out in the development of my interest in beliefs. My experiences with two remarkable young women at a small community college in Oregon changed my beliefs about what constitutes success in mathematics, and whom we accept as belonging to the world of mathematics. Both Cynthia and Daphne[1] were products of the special education classroom having been diagnosed as severely learning disabled and therefore excluded from all but the most basic practical mathematics courses at secondary school. Through sheer hard work and determination, they managed to pass a compulsory college algebra course[2]. They positioned themselves as capable of success in mathematics and believed in the relationship between hard work and success. As a result of their beliefs and success in mathematics, they both went on to

1 Pseudonyms

2 College algebra is a requirement for almost all degree programmes in the USA. The course at this college included algebra, trigonometry, geometry and basic calculus.

complete four-year university degrees thus managing to achieve beyond the expectations of the educational community.

Two books, both published in 2002, also contributed to the development of my interest in beliefs, in general, and beliefs about mathematics, in particular. Hofer and Pintrich's *Personal Epistemology: The psychology of beliefs about knowledge and knowing* (2002) served as an introduction to a fascinating, and, for me, unexplored, perspective on beliefs located within the fields of psychology and education. The book alerted me to issues associated with defining beliefs, as well as to different models, theories and methodologies associated with the study of personal epistemology. One of the more vexing theoretical issues associated with the study of beliefs is whether they are general, domain specific or both. *Beliefs: A hidden variable in mathematics education?* (Leder et al., 2002a) explores beliefs within the field of mathematics education by focusing on conceptualising and measuring beliefs, as well as exploring teachers' and students' beliefs from both psychological and sociocultural perspectives. For these authors, the study of mathematics beliefs is explored as a variable that may explain or suggest solutions to the decline in interest in or engagement with mathematics. Both books emphasise the lack of consensus about what is meant by beliefs and the difficulties associated with accessing and measuring beliefs.

Context

Although my interest in beliefs developed from my experiences of teaching within a variety of educational contexts, my study of beliefs about mathematics is located within a specific socio-political scene. Neo-liberalism, with its accompanying emphasis on globalisation, competition and the commodification of education (Arnot & Mac an Ghaill, 2006), has affected the way New Zealand and other western countries view educational achievement (Amit & Fried, 2008; De Lange, 2007; English, 2008; Gilbert, 2010; Grace & Thrupp, 2010; Lauder, 2010; Thrupp, 2010).

The results of the Third International Mathematics and Science Study (TIMSS), 1994-1995, under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), renewed political and educational concerns about New Zealand students' mathematics achievement and teachers' mathematics content knowledge (M. Chamberlain, 1997a; Garden, 1997). The model used by the IEA was to look at individual countries' mathematics performance within an international context by studying the "intended, implemented and attained" (M. Chamberlain, 1997b, p. 5) mathematics curricula for each of the countries taking part in the study. The only way, however, to measure the "attained" curriculum was by testing what was common across these mathematics curricula (Caygill & Kirkham, 2008; M. Chamberlain, 1997b). What was comparable was not necessarily what these individual countries viewed as important. During the process of collecting information for the TIMSS study, the IEA tested children's mathematics skills as well as administering questionnaires to the children that included questions about their backgrounds, attitudes and beliefs, collected information from teachers "about their academic and professional backgrounds, instructional practices, resources, and attitudes towards teaching mathematics …", (G. Chamberlain, 1997, p. 177) and data about the schools through their principals. Not only did children perform poorly on the tests, but teachers were also found to be ill prepared, lacking content knowledge, as well as having low confidence about their own mathematics skills (Garden, 1997; Higgins, 1999; Ministry of Education, 1992, 1997).

The response to these results was to level blame at teachers, teacher training and at the mathematics curriculum (Garden, 1997; Thomas & Tagg, 2006; Walls, 2004). A result of the Ministry of Education's concern has been the continuous implementation of initiatives addressing these problems associated with disappointing mathematics results. Some of these initiatives have looked at the curriculum and in-service teacher training and professional development, through programmes such as the Numeracy Development Projects, and Count Me in Too (Higgins, 2002, 2003; Irwin & Niederer, 2002; Thomas & Ward, 2001, 2002). Unfortunately, the 2002-2003 Trends in International Mathematics and Science Study (TIMSS) for Year 9 indicated no improvement (Ministry of

Education, 2004b, 2004c). In July 2006, the New Zealand Draft Curriculum, part of the New Zealand Marautanga Project (Ministry of Education, 2006b) was sent to concerned stakeholders for their responses. By 2007, the new curriculum had been published and is now implemented (Ministry of Education, 2007b, 2009a).

Interestingly, in the ensuing years since the almost moral panic that accompanied the 1994/1995 TIMSS results, less space in the public domain was given to the more positive results from later rounds and other measures. New Zealand 15-year olds have relatively strong mathematics results, 9th of 25 countries, according to the 2003 PISA tests of Mathematics Literacy, another international series of tests that pit one country's educational system against the rest under the auspices of the Organisation for Economic Co-operation and Development (OECD)(Ministry of Education, 2004a). However, the results from achievement measures are complex and contradictory depending both on the measures used, the combination of countries taking part in the comparisons and the way in which the results are interpreted. New Zealand, however, "slipped" to 13th of 39 countries in 2009 (Telford & May, 2010), a misreading of the results that indicate an improvement (9th out of 25 is 36th out of 100, while 13th out of 39th is 33rd out of 100 (Hannah, 2013)). The results for Year 5 students on subsequent TIMMS (1998, 2002 and 2006) show a significantly improved mean from 469 (SD 4.4) in 1994 to 496 (SD 2.1) in 2002, despite a slight dip in 2006 (Caygill, 2008; Caygill & Kirkham, 2008; Ministry of Education, 2004c). The emphasis in the public domain has still been on how we are below the 500 mean rather than on the gains made, or any discussion of the value of the specific skills, curriculum and ways of doing mathematics, and to what extent these measures may or may not be aligned with the New Zealand curriculum (Thrupp, 2010).

On the other hand, between 1995 and 2010, the Educational Assessment Research Unit at the University of Otago developed and administered the National Education Monitoring Project (NEMP). The NEMP reports not only described trends in Year 4 and 8 children's achievement, but attitudes and interests as well. The reports on each of the curriculum areas were published in a four-yearly cycle: mathematics was assessed in 1997, 2001, 2005 and 2009 (Crooks & Flockton, 2002; Crooks, Smith, & Flockton, 2010; Flockton & Crooks, 1998). All NEMP results, not just mathematics, were analysed in terms of subgroups such as school size and type, socio-economic status, student ethnicity, gender and language. They provided a picture of New Zealand children's achievement in a way that was more closely aligned to the curriculum than the OECD test results. Despite being more relevant in terms of the local curriculum, the popular press did not cover these results with the same level of angst as the TIMSS results. On the whole, the NEMP mathematics results indicate that all groups of children and schools achieve a range of results from good to bad; however, on certain tasks certain groups do better or worse. Children from lower decile schools (a socio economic indicator) do less well than those from higher decile schools, and more Māori and Pasifika children do poorly than their Pākehā[3] counterparts (Crooks & Flockton, 2002; Crooks et al., 2010; Flockton & Crooks, 1998), which could be explained by more Māori and Pasifika children attending lower decile schools than other groups. Overall, the NEMP mathematics results have not changed significantly over the years.

One common conclusion is presented in all these assessment regimes: the TIMSS, PISA and NEMP results indicate that New Zealand children's achievements range from high to low, but that Asian and Pākehā children do significantly better than Māori and Pasifika (Caygill & Kirkham, 2008; M. Chamberlain, 1997a; Crooks & Flockton, 2002; Crooks et al., 2010; Flockton & Crooks, 1998).

In New Zealand, the relationship between attitude and confidence has been seriously considered. Post TIMSS, Garden (1997) writes, "Lack of confidence in teaching mathematics… has two important side effects. First, teachers in this position are likely to have poor attitudes to the subject and to unwittingly communicate these to their students. Second, teachers may avoid teaching the subject" (p. 250). This conclusion, however, focuses on the teacher rather than student responses. Student attitudes were included in the 2006 TIMSS results,

3 Pākehā is a Māori term to describe non-indigenous New Zealanders.

which assessed Year 5 students' mathematics. Based on these, Caygill concluded that "[t]hose students who reported positive attitudes towards mathematics or were confident in their own mathematics abilities had higher achievement than those who were less positive or confident" (2008, p. 47), which perhaps implies that higher achievement supports greater confidence and positive attitude. The NEMP assessment reports analyse students' attitudes and interests in mathematics, but in terms of age rather than differences between groups or comparing patterns of responding to achievement levels. Both of these assessment and reporting regimes recognise the importance of these factors and the notion that they are, in some way, related to achievement.

That there is a relationship between attitudes, beliefs and confidence about mathematics and mathematics achievement levels is accepted by many (Begg, 1999; Carpenter, Fennema, Franke, Levi, & Empson, 2000; Crooks & Flockton, 2002; Edwards, 2000; Flockton & Crooks, 1998; Garden, 1997; Grootenboer, 2003; Moriarty, 1995; National Council of Teachers of Mathematics, 1994). For Flockton and Crooks (1998), there is a "pervasive interrelatedness that exists among mathematics understanding, skills and attitudes" (p.11). Moriarty (1995) found relationships between both confidence and attitude with children's learning. Watson (1958) and many researchers since then have found students (Hartnell, 2000; Higgins, 1999; L. Hill, 1997; Thomas, 1999; Visser, 1999), pre-service teachers (Begg, 1999; Grootenboer, 2006) as well as experienced teachers (Hill 1997; Higgins 1999; Thomas 1999; Visser 1999) who lack confidence in, have a negative attitude towards, and are both anxious and uncomfortable about mathematics. In addition, Visser (1999) notes that it is still socially acceptable to be poor at mathematics. For many of these researchers, beliefs, attitudes and feeling about mathematics are related.

Mathematics discourses or myths

Success at school mathematics works as a gatekeeping mechanism for access to prestigious, more difficult school subjects such as physics and chemistry, for entry to university, as well as to advanced training and access to professional careers (Ashcraft & Ridley, 2005; Boaler, 2010; Ernest, 1995; Gates, 2001; Mendick, 2006). Thus, if one views achievement data as indicative of a pathway towards further education, then achievement discourses, in particular, about who can or cannot do well at school, are also important. By discourses, I mean stories and conversations that have been invested with social values which regulate what is said and by whom that become associated with our belief systems (Gee, 2001). Part of the power of discourses is that they are a catalyst for positioning people, by others or themselves as participants/non-participants in a particular story (Mendick, 2006). In the context of this study, I am interested in exploring how powerful discourses may be part of students' belief systems, and how they might affect the doing and learning of mathematics in schools.

Achievement results are frequently reported in terms of ethnicity and gender, in order to compare groups or to explain patterns of performance (Caygill & Kirkham, 2008; Flockton, Crooks, Smith, & Smith, 2006). Groups are positioned as achievers and non-achievers, those who can or cannot meet a standard, show they have mastered a level, or have achieved the necessary qualifications for entry into university, for instance. These discourses privilege certain groups over others thus adding additional barriers to the *others* (Diversity in Mathematics Education Center for Learning and Teaching, 2007). One of the unintended consequences of these types of analyses is they become distilled and reinterpreted within the public domain in a way that positions members of a particular group as successful and/or unsuccessful based solely on their membership of the group (Cadinu, Maass, Lombardo, & Frigerio, 2006; Smith & Hung, 2008).

Colouring the context

In the New Zealand public press and in political circles, a discourse of the "long brown tail of underachievement" is prevalent (Collins, 2010, 2013; Fisher, 2011; Panara, 2011; Rees, 2012; Smith, 2009). Even though children from all ethnicities achieve at all levels from the highest to lowest, much emphasis is given to the underachievement of Māori and Pasifika children. Like all negative discourses, this focuses on failure rather than on students' success. However, there is an issue of underachievement for a group of Māori and Pasifika children, as illustrated in achievement reports on the National Standards (Shuttleworth, 2012, 2013), as Hekia Parata, the New Zealand Minister of Education, commented in a speech in Queenstown where she spoke about underachievement where "too many were boys, brown boys" (Fea, 2013). She also commented on the PISA results by emphasising that even though New Zealand ranked 7th overall out of the countries taking part, Pākehā 15-year olds were 2nd while Māori 34th equal and Pasifika students 44th (Fea, 2013). However, the press coverage implies that all children belonging to these groups are failing without acknowledging socio-economic factors, among others, which may explain why these children achieve less than their Pākehā and Asian counterparts (Caygill & Kirkham, 2008; Snook & O'Neill, 2010; Thrupp, 2010).

An associated discourse attributes success to Asian children. Once again an ethnic group—or rather multiple ethnicities—is positioned as always successful. In part, this discourse has been fuelled by the success of Asian countries on the TIMSS tests (Caygill & Kirkham, 2008; Mullis, Martin, & Foy, 2005). In New Zealand, despite Asian children from a variety of ethnic backgrounds doing exceptionally well on a range of academic measures, Asian children achieve on all levels of a continuum (Caygill & Kirkham, 2008; New Zealand Qualification Authority, 2010). Even though they achieve at a range of levels, proportionately more Asian students pass NCEA level 3 (New Zealand Qualification Authority, 2010) and go on to university education than their Māori and Pasifika counterparts (Cumming, 2011; Statistics New Zealand, 2006), which may contribute to the discourse of Asian success.

A variety of reasons for Asian high mathematics achievement have been postulated. At one extreme is the genetic argument that they are born better at mathematics and/or more intelligent (Lim, 2013); however, Flynn points out that this does not explain why Asian children do better than their white counterparts even when the former may have IQ scores of 20 points below the latter (Chisholm, 2010). Another suggestion is that because numbers and number concepts are more easily accessed, learned and understood in certain languages like Chinese, Japanese and Korean, it is easier for children to excel in mathematics (Berk, 2013; Gladwell, 2008; Lim, 2013). An additional discourse presents the pushy Asian parent forcing offspring to spend long hours practising (Paul, 2011), which some local Asian high achievers like Audry Tan (Claridge, 2005) and Raybon Kan (Chisholm, 2010) dispute. Kan suggests that the sort of people who leave their countries for a better life elsewhere are likely to instill the value of hard work and education in their offspring (cited by Chisholm, 2010). The value put on education and doing well incorporates practice, persistence and working hard at education (Chisholm, 2010; Claridge, 2005; Gladwell, 2008; Paul, 2011). Gladwell, who stresses the notion of persistence and hard work in his popular book *Outliers*, includes an interesting comparison of TIMSS results between countries that stress these values and other countries. He argues that answering the approximately 120 questions is a boring task and that "countries whose students are willing to concentrate and sit still long enough and focus on answering every single question in an endless questionnaire are the same countries whose students do the best job of solving math problems" (2008, pp. 247-248). Thus he argues that it is a matter of cultural values rather than innate ability that influences the achievement of certain ethnicities/populations over others.

In New Zealand, these discourses of ethnicity and mathematics performance tend to position Asians at the successful end of the spectrum and Māori and Pasifika at the unsuccessful end, without careful consideration of other complex contributing factors such as socioeconomic status and class, the range of achieving for each group, as well as the nature of the assessment tools used to judge these students. For this study, I was interested in exploring how and to what extent these discourses might colour children's beliefs about which groups of people are successful at mathematics.

Gendering the context

Discourses associated with gender and mathematics, both within New Zealand and other English-speaking countries, are both complex and contradictory.

During the early to mid twentieth century, unlike the 1800s when mathematics and science were standard subjects for those middle and upper class girls who attended secondary school (Coney, 1993; Ripley, 2005), mathematics for girls used to be viewed in terms of the skills that would be helpful for basic numerate competency, so that females could perform "women's work" as housewives, domestics, shop assistants, secretaries, perhaps even as teachers and nurses, while boys reputedly needed enough mathematics to go into the male professions (Coney, 1993; Fry, 1985). Many girls were denied access to higher school mathematics, not because they were not capable, but rather because they were not expected to be capable of achieving at this level (Coney, 1993; Mendick, 2005, 2006; Walkerdine, 1989, 1998). Many girls who had been very good at arithmetic in the primary years were actively discouraged from taking mathematics beyond the first couple of years of secondary school, and even when they did continue, it often was a programme in practical or useful mathematics rather than algebra, geometry, trigonometry and calculus, the higher status areas (Fry, 1985). Mathematics was also presented as a "remote, inaccessible" (Ernest, 1995, p. 449) and "hard" subject, and girls were not often encouraged to study hard subjects (Ben-Zeev, Duncan, & Forbes, 2005; Berk, 2013; Boaler, 2010; Fennema & Leder, 1990; Mendick, 2006; Paechter, 2001).

Today, in New Zealand, the discourse of girls as successful and girls as doing better than boys is the prevailing one. The international achievement results such as TIMSS and PISA, as well as the local measures, NCEA and NEMP, indicate that, on the whole, there is no significant difference between the results for females and males (Caygill & Kirkham, 2008; Flockton et al., 2006; New Zealand Qualification Authority, 2010; Telford & May, 2010). On the other hand, more females than males are going to university (Statistics New Zealand, 2006,) which in the public and political arenas goes to "prove" that girls have caught up with or even overtaken the boys. The discourses associated with boys doing less well than girls are covered in a variety of publications from the academic to the popular. In 1997 Fergusson and Horwood reported that "the traditional educational disadvantage shown by females has largely disappeared and has been replaced by an emerging male disadvantage" (1997, p. 83). In 2003, the

National Party called for an inquiry into the "major crisis in boys' education" claiming, "Girls were even outperforming boys in traditional male strongholds such as mathematics and science" (Ross, 2003). Recently, in the more popular print and on-line media, reports position boys' underachievement as a "timebomb" (Fox, 2006), blaming male disadvantage on the education system (Education system failing to keep boys, October 11, 2008), feminism and the feminisation of education (Laws, 2009), and on the differences in nature and acculturation of boys and girls (Edmunds, 2012; Ferguson, 2012).

There are some unintended negatives that are associated with the discourse of girls excelling, or at least doing well. For many in the public domain, females and males are constructed as binary opposites; thus if girls are excelling, naturally the boys must be failing (Arnot & Mac an Ghaill, 2006; Jackson, Paechter, & Renold, 2010; Martino, Kehler, & Weaver-Hightower, 2009; Skelton, 2006; Weaver-Hightower, 2008). This discourse ignores another possible interpretation, that of girls catching up; boys are not doing less well but rather the same, while the girls are doing better than previously. The USA, UK and Australia seem to have adopted similar discourses about failing boys, for example, labelling all males as at risk (http://www.ehow.com/info_8216179_gender-between-boys-girls-school.html; Gallagher, 2010) or those in one ethnic group or class at risk (White working class boys failing, 2008). This gendered discourse associated with pitting girls' achievement against boys' misses one of the essential variables associated with academic achievement: class and/or socioeconomic status. In other words, on the whole girls' and boys' achievement results are similar; however, certain boys and certain girls are achieving less well and are less engaged with school than others (Arnot & Mac an Ghaill, 2006; Elwood, 2010; Francis, 2010; Lingard, Martino, & Mills, 2008; Walkerdine & Ringrose, 2006).

One of my concerns with the discourse of girls as successful, particularly in mathematics, is that this notion of success is problematic and requires more unpacking (Boaler & Sengupta-Irving, 2006; Walkerdine & Ringrose, 2006). If one looks at this success, especially in terms of the level and types of mathematics that girls are studying, then a different picture emerges, a picture that seems similar across a range of English speaking countries (Boaler & Sengupta-Irving, 2006; Fennema & Leder, 1990). Mathematics in the final years at New Zealand secondary schools is divided into two strands, mathematics with calculus and mathematics with statistics. Girls are still choosing or being steered towards the easier statistics version and not towards the hard, male calculus (Forbes & Robinson, 1990; Ministry of Education, 2010a; Shannon, 2004). The route to continuing with school mathematics or doing calculus is complicated in that it is not merely previous success, ability, or teachers, counsellors, and parents that help girls make these choices, but the girls themselves who may have been affected by peer discourses of what being female means to their particular groups (you don't want to be a nerd, maths[4] is boring, you will frighten away cool guys, etc.) (Jackson, 2006; Paechter, 2000; Skelton, 2006), as well as by what they have absorbed from the public domain through text and images in print, on-line and on television. If bright, high achieving girls are still not taking calculus, they are being excluded from a number of high status educational and professional pathways. In a similar vein, although more girls than boys are continuing to university, they are still, on the whole, more likely to pursue female-friendly areas such as psychology, education, the humanities and biological science rather than the hard sciences (physics, astronomy, chemistry), engineering and mathematics (Callister, Newell, Perry, & Scott, 2006; Gibb & Fergusson, 2009; Ripley, 2005).

In 1994, twenty years since Fennema's original article on gender and mathematics in the JRME, Fennema and Hart noted that gender differences seemed to be declining; however, they still existed in three areas, "the learning of complex mathematics, personal beliefs in mathematics, and career choices that involve mathematics" (1994, p. 650). Certain elements have improved, but not as much as it first seemed. Now, in 2013, despite the improvement in scores and the number of girls pursuing higher education, some of the older discourses about gender are still present. Leder and Forgasz in an Australian study on the public's

4 In this study, I use interchangeably the formal term 'mathematics' and the more readable 'maths', the term the children and their teachers use.

perceptions on learning mathematics found that the younger respondents, those under 40, had returned to a more traditionally gendered belief that "boys are more suited to and successful in mathematics than girls" (2011, p. 453). Mathematics is still viewed as an area of study suitable for the male, naturally talented, rational and logical, and thus unsuited to the emotional female mind and women's ways of knowing and doing (Clinchy, 2002; Fennema & Hart, 1994; Mendick, 2005, 2006; Paechter, 2000; Walkerdine, 1989). Not much seems to have changed since Ernest's 1995 findings that "[a] widespread public image of mathematics is that it is difficult, cold, abstract, theoretical, ultra-rational, but important and largely masculine" (1995, p. 449). Not only is mathematics viewed as a boys' subject, but success for girls is often explained in terms of hard work and rule following while boys are still more likely to be positioned as naturally talented and brilliant (Jackson et al., 2010; Mendick, 2005, 2006; Paechter, 2000, 2007; Skelton, 2006; Walkerdine, 1998; Walkerdine & Ringrose, 2006). These discourses spill over into boys' and girls' beliefs about and explanations for why and how they are successful mathematics students.

Putting up signposts

The theoretical approaches in this study of New Zealand children's beliefs about mathematics are eclectic in that my predominately sociocultural perspective includes a combination of interpretive/constructivist and pragmatic theories. My experiences led to the topic and research questions which in turn affected the complementary methods I employed in order to access and understand these beliefs. The study is situated in primary schools and mathematics classrooms within a context of prevailing achievement discourses. Although I view the classroom as a separate, self-contained microcosm with its own values, rules and ways of doing things, it has links to the school, the curriculum (both national and localised) and society at large. It is a place familiar to children where they "do" school (Bloome, Puro, & Theodorou, 1989; Bracey, 1990), or in this case mathematics, a place where they construct and reconstruct their identities as pupils, as peers, as learners and doers of mathematics. I am particularly interested in understanding the phenomena, beliefs about mathematics, in a naturalistic setting and allowing the participants' voices and experiences to help me understand and interpret their beliefs. I use metaphors extensively as a way of making sense of what the children say about beliefs, of my own journey of understanding, as well as a tool for communication.

Metaphors

Metaphor is more than merely a literary or linguistic device, but rather a conceptual tool fundamental to human understanding and essential for abstract thought (Gauntlett, 2007; Lakoff & Johnson, 2003). Metaphors are part of the way we think and process information. When we encounter the new, or the not as yet understood, we compare it to something we already know, based on our prior experience. In other words, we link the new to our prior knowledge, and categorise the new in terms of similarities and differences with the known. Metaphor is thus a basic cognitive tool used both for making meaning and for communicating ideas to others (Gauntlett, 2007; Golden, 1968; Lakoff & Johnson, 2003).

I use multiple metaphors during the process of explaining and clarifying my understandings of children's beliefs about mathematics. Some of the dominant metaphors are those associated with *place* or geography and those associated with *seeing* and *viewing*.

The *place* metaphors include landscapes, terrain, domain and worlds. They also contain associated metaphors of navigating, travelling and exploring these landscapes by using maps, guides and signposts (the tools of both the traveller and the viewer), looking at different features as the unseen is revealed. These are peopled landscapes. My position as a researcher within this landscape is of a foreigner in a foreign land, a secondary and tertiary teacher travelling in a New Zealand primary school context, exploring what children believe, someone who had never trained as a mathematics teacher researching mathematics education. I am a traveller, an explorer, and, at times, a wanderer in this research story.

The *viewing* metaphors include a combination of nouns and verbs: lenses, colour, narrowing foci, zooming in, perspectives and looking at portraits and at drawings. However, like a braided river these metaphors overlap, separate, combine, disentangle, re-combine while some vanish into unproductive patches of gravel (Gray, 2010); for instance, landscapes are not only traveled but viewed through different lenses or perspectives, while drawings are observed, read as well as explored.

Nomenclature: the trouble with terms

The study of beliefs is complex. Part of the issue is the variety of definitions used to describe beliefs: "the personal assumptions from which individuals make decisions about actions they will undertake" (Silva & Roddick, 2001, p. 101), as "the lenses through which one looks when interpreting the world" (Philipp, 2007, pp. 257-258), and as "personal, internal knowledge"(Lester, 2002, p. 351) are a few examples. Definitions of beliefs are discussed in more detail in Chapter 4 of this study. Beliefs are discussed in the literature of fields as diverse as philosophy, psychology, sociology, anthropology and mathematics. An associated complication is the locus of beliefs: do they reside in the affective (McLeod, 1992; McLeod & McLeod, 2002) or the cognitive domain (Philipp, 2007)? Or both, or another as yet unexplained domain? Where do they reside in reference to knowledge and truth? And where in reference to affect and emotion? Goldin answers some of these questions in his definition of beliefs as "multiple-encoded cognitive/affective configurations, to which the holder attributes some kind of truth value (e.g., empirical truth, validity, or applicability)" (2002, p. 59).

Unpacking how concepts such as attitudes, feelings and beliefs are related is difficult, in part, because of the lack of consensus in the beliefs literature. For De Corte, Op 't Eynde and Verschaffel (2002) attitudes belong in the affective domain while beliefs are located in the cognitive; Pintrich, on the other hand, is not sure where attitudes belong (2002). Kloosterman views beliefs as synonymous with attitudes and dispositions (2002). Philippou and Christou position emotions, attitudes and beliefs on a "hierarchical scale" from the more affective to the more cognitive (2002, p. 213). Goldin also suggests a scale with emotions at the most affective, moving to attitudes, then beliefs and finally to values, ethics and morals which are the most cognitive of the four (2002). From my perspective, any discussion of attitudes to, feelings about, interests in and opinions about mathematics are part of beliefs and belief systems about mathematics.

Because of the complexity associated with exploring beliefs, an area defined in a variety of ways with differing definitions, locations and academic traditions, the terms used in this study needed to be identified and explained.

Epistemological beliefs are beliefs about the nature of knowledge and truth, as well as the sources of that knowledge. A sociological perspective of epistemology looks at what is socially valued knowledge (Sierpinska & Lerman, 1996). A perspective for educators is to examine beliefs about knowledge and learning, in part by distinguishing between what constitutes knowledge for the teacher as opposed to knowledge for the learner (Schommer-Aikins, Duell, & Hutter, 2005; Sierpinska & Lerman, 1996). In addition, there are some interesting distinctions between mathematics educators' epistemological beliefs and those of students, teachers and the general public. Some researchers and theorists view the former as beliefs in their own right while the latter are often referred to as myths (Ernest, 1996; Franks, 1990; Paulos, 1992; Sam & Ernest, 1998). Comments like Bertrand Russell's, "Mathematics is the subject in which no one ever knows what he is talking about nor whether what he is saying is true", (quoted in Brown, 1996), although amusing, reinforce this distinction. The public image of mathematics is that it is an exclusionary, cold world only accessible to the very bright (Cotton, 2001; Ernest, 1996; Zevenbergen, 2001). Ernest (1996) cites Erlwanger's 1973 study of successful twelve year olds who saw mathematics as "a wild goose chase searching for the many unrelated and arbitrary rules sanctioned only by the dictates of authority" (p 805). Epistemological beliefs can have a direct effect on achievement as Smith and White discovered when they encouraged their mathematics students to endorse beliefs such as "Asians are better than Whites" and "men are better than women" (2002).

Epistemological beliefs overlap with and impact on **self-belief**, which are also known as self-concept, ability beliefs or competence beliefs. These are beliefs individuals identify and use to predict their performance, competence or achievement in particular domains, for instance, mathematics (Krause, Bochner, & Duchesne, 2003; Schunk & Pajares, 2002; Wigfield & Eccles, 2002; Woolfolk, 2005). These perceptions are based on a combination of students' own experience and their comparison with others' performance, as well as their perceptions of their teachers' beliefs in their, the students', competence (Schunk & Pajares, 2002). Self-beliefs are affected by social factors such as gender, ethnicity and socio-economic status (SES) (Krause et al., 2003; Leder, Forgasz, & Solar, 1996). They also seems to be linked to the attribution theory concept of the 'loci of control' (Eggen & Kauchak, 2004; Krause et al., 2003), or the explanation of success and failure in terms of ability or effort. Often students who do not believe they are capable of success in mathematics will attribute this failure to factors which absolve them of all control or responsibility such as ability, luck, the teacher, hard assignments, etc.; conversely, those who believe themselves capable explain their performance in terms of effort and persistence, thus acknowledging control of their performance (Eggen & Kauchak, 2004; Leder et al., 1996; Schommer-Aikins et al., 2005). Self- beliefs can thus work as catalysts for, as well as barriers against, success in mathematics.

Identity beliefs is another term related to self-beliefs. Identity beliefs are associated with recognising oneself as part of a group, a particular kind of person (e.g., good at maths, clever, popular) in term of one's characteristics (e.g., scored highly on an assessment task). Identities[5] can be based on the individual's experience and self-identification or the results of positioning by others. For Grootenboer and Zevenbergen, mathematics "identities incorporate a range of dimensions including knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions related to mathematics and mathematics learning" (2008, p. 244). Identities are not static; instead they are constantly forming and reforming, being negotiated and renegotiated depending on the context (Franke, Kazemi, & Battey, 2007; Mendick, 2006).

In this study, the term **ability beliefs** is used for a group of beliefs which overlap with both self-beliefs and identity beliefs. These are beliefs about how certain groups are positioned as either successful or unsuccessful at mathematics. Individuals within these groups may be positioned in these categories based on their membership of a group rather than as a result of their real achievement levels or ability. These beliefs include beliefs about gender, ethnicity and social-economic status and are often based on experience and dominant social discourses. In Chapter 6, *ability* also refers to beliefs about what makes people good at mathematics; in this case, the 'what' refers to behaviours, dispositions and genetic qualities.

Barbara Hofer describes **personal epistemology** as "the study of individual beliefs about knowledge and knowing" (Hofer, 2002, p. xi). In this study I've borrowed the term and have used it as an umbrella term under which epistemological, self-, identity and ability beliefs fit.

I use the term **phenomenon(a)** as it has been used in English since the Sixteenth Century to mean an object, thing, notion or event that appears and is viewed (Phenomenon, 2005), or, in this case, is explored and analysed during the

5 The term *identity* has a number of conceptions ranging from the psychology emphasis on self-concept, through a socio-cultural focus on the individual interacting with social contexts to the post structural concern with the ongoing process of negotiation (Boaler, Wiliam & Zevenbergen, 2000; Grootenboer, Smith & Lowrie, 2006; Mendick, 2006; Nuthall, 2007).

process of trying to understand. Although I use this term as well as van Manen's notion of "lived experience" (1990), this is not a phenomenological study.

Complementary methods. Usually, the term *mixed methods* is used by the research community when multiple methods of data collection and analyses are combined within one study. I have, however, chosen to use the term *complementary* methods (Green, Camilli, & Elmore, 2006a; Jaeger & American Educational Research Association., 1988) in part because of the metaphors underpinning these terms. Complementary includes the connotation of working together to enhance meaning and understanding while mixed has overtones of blurring. If one mixes red and blue, one may end up with a pleasing purple; on the other hand, if one were to mix four or five colours together, the result may be a murky brown.

Style and voice

The incorporation of complementary theories and methods in this study has influenced the style in which this thesis is written. Parts of it are in the formal, impersonal, passive voice with the researcher positioning herself out of the narrative. Another formal device is the use of "one", the third person singular, in discussion. At other places, in an attempt to engage the reader, the more conversational "we" is employed. However objective a piece of academic writing may sound, it is personal in that the hand of the researcher is behind every research question and decision (Denzin & Lincoln, 2011a). For most of this piece of writing, I acknowledge my presence and choose to identify myself as part of the exploration of mathematics beliefs by using the first person singular and the active voice where appropriate.

One of the devices I use to illustrate the development of my thinking, my changing perspectives throughout the exploration, or a point with which I was grappling, is the inclusion of extracts from my research and observation journals. The boxed extracts and the autobiographical approach are stylistic tools I use both to indicate my positions towards a topic, an idea or an experience and to make the narrative more accessible to a range of readers.

Throughout this study, the narrative includes verbal and as well as visual texts. The visual texts are made up of drawings, details from drawings, diagrams and tables that I use as data, as conclusions and as part of my line of reasoning.

Research questions and focus

This study explores beliefs about the nature of mathematics, what it means to be good at mathematics, to be able/not able to do mathematics, what the children believe may cause one to be good at mathematics, and how these beliefs play out in the classroom, a combination of espoused and enacted beliefs. My decision to explore the mathematics beliefs of primary school children was influenced by my concerns about the influence of teachers who believe they themselves are weak at mathematics (Solomon, 2004). At issue is what can we do to improve mathematics engagement, enjoyment and achievement? My position is that we cannot solve this problem without understanding factors like beliefs that affect children in the learning of mathematics. I am also interested in the students' socio-economic backgrounds, cultural backgrounds, ethnicity, and gender, and the relationship these may have to their beliefs and their engagement in their mathematics classes. There exists an element of reflexivity between experiences of school mathematics, attitudes to mathematics outside school, student beliefs about mathematics and the doing and learning of school mathematics. Although the focus of this research is on the children and their beliefs, I also look at a small sample of teachers from two focus schools, at their beliefs, because these beliefs influence teaching and learning—the core business of the classroom—as well as at how teachers' beliefs may influence or be different from the children's beliefs.

The research questions are divided into three main questions, incorporating sub-questions:

What are children's epistemological beliefs about mathematics?

This includes questions about what children believe about nature of mathematics, what it is and how it works. What do they believe are the salient features of this world of mathematics?

What are children's self-beliefs about themselves as learners of mathematics?
This question focuses on how students view themselves in relation to the world or domain of mathematics. How good do they think they are at mathematics? How do they identify themselves as natural or foreign inhabitants of the world of mathematics? What are their mathematics identities?

What are children's beliefs about others and mathematics?
What sorts of people can do mathematics, are good at or bad at mathematics?

Roadmap for this thesis

Travellers can be categorised into various groups: those who research the trip before leaving home; those who consult guidebooks, guides, libraries and museums *en route* for each section, place or aspect as it unfolds; and those who use a combination of approaches. The usual map of the PhD dissertation follows a standard route with the literature researched and presented in the planning stages before commencing on the design, data collection and analysis of the research journey. Perhaps there is a return to the literature once back home, in a reflection on the journey, a discussion or conclusion. In this version of an education dissertation, I have chosen a different approach to the journey by weaving the literature through the exploration thus following Wolcott's literature "when-and-as-needed" recommendation (2009, p. 68).

In Chapter 1, I have introduced the background and context of the study. Chapter 2 (*Methodological meanderings*) introduces and examines the theoretical and methodological influences on the study, which includes a combination of interpretive/constructivist and pragmatic approaches. Chapter 3 (*Routes chosen*) introduces the multiple complementary methods used to gather the data from a combination of questionnaires, drawings, observations, recordings and interviews. It also explains the analysis processes included to make sense of these data. Chapter 4 (*The landscape of beliefs*) is based on the data collected from the Maths Beliefs Questionnaires (MBQ). It includes a combination of literature, quantitative analyses and discussion, culminating in a description of a four factor mathematics beliefs framework (MBF) derived from a factor analysis (principal component analysis). In Chapter 5 (*Reading the pictures*), the literature associated with drawings and visual methodologies is introduced, and the children's drawings are analysed and discussed. Chapter 6 (*Narrowing the focus*) brings the focus of the research into two classrooms and follows nine children and their teachers' beliefs about mathematics, about being mathematical and about doing mathematics. In Chapter 7 (*Stepping back (...and beyond)),* I review the findings and the research process, look at the implications of the research and suggest areas for further study. The roadmap for this thesis is summarised in Figure 1.1:

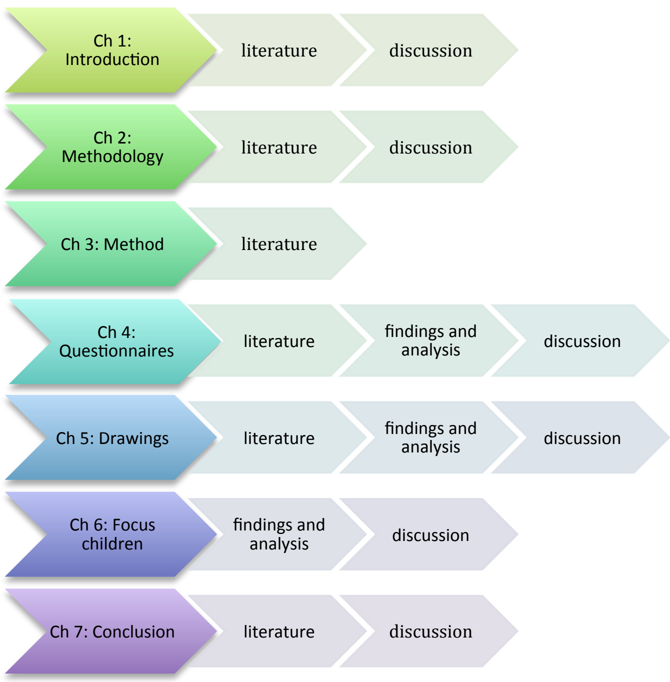

Figure 1.1: Roadmap of thesis

A note to the reader: The clarity and legibility of the drawings is much clearer in the pdf read on a computer than in a printed form where much of the colour and fine detail is lost.

Finally, a note about the use of appendices: I have included information that is supplemental to the narrative and that, though important and interesting to some readers, might have become a distraction to others were it to remain in the main body of this thesis (American Psychological Association, 2010; Wolcott, 2009). The appendices contain the expected—questionnaires, additional information about samples, interview questions and cues, an example of consent forms and analysis method—but also additional statistical analyses, comparative analyses from the focus schools MBQs, codings for the focus children and teachers, and examples of drawings from an associated study discussed in the final chapter.

Chapter 2: Methodological meanderings[6]

> "All research is interpretive: guided by a set of beliefs and feelings about the world and how it should be understood and studied" (Denzin & Lincoln, 2011a, p. 13)

The questions and the context of the research guided me towards methodologies and methods that best suited the study, which meant adopting a complementary or eclectic approach. I use the term *eclectic* because it implies the notion of borrowing the most appropriate elements from a range of sources, theories and methods (Eclecticism, 2001; Larsen, 1999). The following two chapters frame the research and discuss the ideas that influenced my approach to this research and its resulting structure, as well as elements associated with its design. The chapters are roughly divided into methodology and methods, but as these concepts are not mutually exclusive, my line of reasoning wanders between the two at times. An example of this wandering is the placement of ethical consideration in Chapter 2, from a methodological perspective, and Chapter 3, in which the consent process is described.

The following definitions of research methodology show how method is inextricably linked to methodology and both are directly affected by worldviews and their accompanying theoretical underpinnings. Research methodology can be defined as

> "… a broad approach to scientific inquiry specifying how research questions should be answered. This includes worldview considerations, general preferences for designs, sampling logic, data collection and analytic strategies, guidelines for making inferences, and the criteria for assessing and improving quality" (Teddlie & Tashakkori, 2009, p. 21).

Vann and Cole (2004) define methodology with a slightly different emphasis but also relate it to research methods: "... the logic by which theoretical principles are linked to data through combination of methods. Just as methods mediate the

6 I chose the term, *meanderings* after the river of that name to suggest winding routes (rather than the second meaning which implies aimlessness). The term also refers to an ornamental running pattern. Thus this word choice references both travelling and looking metaphors, the geographic and the visual.

relations between empirical phenomena and data, methodologies mediate the relationships between methods and theories" (p. 152). Whether methodology is viewed as an approach that influences methods and analysis used in research, or as the "logic" that links method with theory, methodology and method are in essence entangled.

The framing of this research is influenced by both pragmatic and interpretive / constructivist worldviews, by the research of Graham Nuthall and of Paul Cobb and associates, as well as by my approach to the issues associated with researching children, ethical considerations and the notions of validity/trustworthiness with complementary methods research.

Eclecticism

I am having an inordinate amount of trouble trying to unravel and/or impose a theoretical framework or multiple frameworks on my research. I had hoped to allow the data to dictate both theory and framework of the study; however, I would be naïve if I were to believe one could approach a topic without some system of personal beliefs that permeate one's thinking. Part of my dilemma is that if I articulate these beliefs adequately, they will start colouring any interpretation of my findings more than I wish them to; conversely, if I allow them to remain unidentified and amorphous, I will continue to lack the focus and discipline needed to keep on track rather than exploring every enticing trail, relevant or not.

I keep thinking of Doyle's comment as I grapple with this dilemma: "I have no data yet. It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts" (Doyle, 1891, p. 63).

Even though I claim that my research is derived from no single theoretical perspective or school of theories, it is obviously influenced by my experience as learner and teacher as well by reading in and around my topic. I could waffle on about the extent to which I've been influenced by post-structuralism, feminist notions of knowledge, post-modernism in terms of a constructivist approach to learning, or even modernism with an interest in discovery; however, exploring these and other –isms hasn't brought any additional clarity either to my questions or research methods. Conversely, the connection between my experience of both learning and 'doing' maths (Schoenfeld, 1994b) and my research is much more obvious: as a teacher, I am interested in the relationship between students' mathematical beliefs and their engagement with the subject, a relationship that seemed observable in my classes; as a learner, I can describe and analyse incidents that have partially shaped my interest. I am also interested in the match/mismatch between learners' and their teachers' beliefs about maths.

Journal entry, 3 December 2008

Eclecticism as a philosophy and research approach resonates with my equivocating stance. Collier and Elman describe *eclecticism* both as another term for qualitative methods and as a "sophisticated use of multiple, complementary methods in composite research designs, based on nesting or the iterative use of alternative qualitative and quantitative strategies" (2008, p. 780). This definition implies two different understandings of eclecticism, one as a synonym for qualitative methods, the other for combining quantitative with qualitative approaches. Eclecticism implies not being bound to a rigid position, paradigm or set of assumptions; instead, the researcher or theorist is prepared to combine methods and methodologies that may illuminate or add to the understandings of the issues or phenomena of interest.

This research fits within a constructivist, interpretive frame where multiple truths, understandings and interpretations are possible, where "[a]ny gaze is always filtered through the lenses of language, gender, social class, race and ethnicity. There are no objective observations, only observations socially situated in the worlds of—and between—the observer and the observed " (Denzin & Lincoln, 2005a, p. 12). Knowledge is socially constructed, and thus there may be many realities or perspectives. If one accepts that beliefs are socially constructed and that mathematics is a human construct (Ernest, 1998, 2008) or what Cobb refers to as "a complex human activity" (2007, p. 29), then this research resides comfortably within a constructivist, interpretive frame. Contexts of the research, the values of the researcher and the lived experience of the subjects of the research are extremely important. Constructivists also claim that conclusions are subjective, are reached inductively and are explanatory rather than generalisable (Denzin & Lincoln, 2005a, 2005b; Mertens, 2010; Teddlie & Tashakkori, 2009). The study is interpretive in that, as a researcher, I acknowledge that my experiences, social circumstances, beliefs and values influence the way I frame and interpret the questions and data (Creswell, 2007; Denzin & Lincoln, 2011a). This study takes place in schools and classrooms where real people's experiences are set within a social and cultural context. The focus of the research is on investigating the participants' beliefs about mathematics within their natural milieu. The emphasis is on the understanding

(Verstehen) rather than the causal explanation (Erklären) of these phenomena (Hatch, 2002; Keeves, 1999).

A constructivist frame, however, does not imply a single approach but includes a range of philosophical perspectives, which accept that individuals construct their own understandings and meanings of their experiences. Schunk (2008) writes about three different perspectives on a continuum: one extreme is exogenous constructivism, labelled "trivial constructivism" by Palincsar (1998, p. 347), which views knowledge as "a reconstruction of the external world" (Schunk, 2008, p. 238); at the other extreme is endogenous or radical constructivism (von Glasersfeld, 1984) where knowledge construction is internal and develops from the individual's prior knowledge; in between these extremes is dialectical constructivism which is neither all the external nor the internal world, but a world where the individuals interact with their environments physically, socially, and culturally (Cobb, 2007; Palincsar, 1998; Schunk, 2008). A Piagetian approach would favour endogenous constructivism while a Vygotskian or social cultural one is closer to dialectical constructivism. Palincsar sums up the difference between these two approaches:

> Where constructivists give priority to individual conceptual activity, sociocultural theorists tend to assume that cognitive processes are subsumed by social and cultural processes. Where social constructivists emphasize the homogeneity of thought among members of the community engaged in shared activity, cognitive constructivists stress heterogeneity of thought as individuals actively interpret social and cultural processes, highlighting the contributions that individuals make to the development of these processes. (1998, p. 371).

Palincsar thus differentiates between constructivism and social constructivism by teasing out the differences between a cognitive focus on the individual and a sociocultural focus on the community or cultural group. These constructivist perspectives relate not only to the construction of knowledge but also to the understanding of the world and what is accepted as reality.

The term paradigm (Strike, 2006), which is "[a] conceptual or methodological model underlying the theories and practices of a science or discipline at a particular time; (hence) a generally accepted world view", (Paradigm, 2005) is often used in the literature to refer to worldviews, especially as used within the research community when describing the differences between positivist/postpositivist and interpretive/constructivist worldviews (Creswell, 2007; Denzin & Lincoln, 2011a; Johnson & Onwuegbuzie, 2004; Lincoln, Lynham, & Guba, 2011; Tashakkori & Teddlie, 2008; Teddlie & Johnson, 2009). However, according to Johnson and Onwuegbuzie (2004) there is a third research paradigm, *pragmatism*.

Pragmatism accepts both objective and subjective views of knowledge[7], accepts that conclusions and interpretations cannot be value-free, and considers both the possibility of making generalisations and ideographic statements. Logic can be both inductive and deductive depending on the research method used and the stage of the enquiry (Creswell, 2007; Denzin & Lincoln, 2005a; R. B. Johnson & Onwuegbuzie, 2004; Mertens, 2010; Tashakkori & Teddlie, 2003a). Johnson and Onwuegbuzie (2004) in their extensive summary describe pragmatism as a practical paradigm, a compromise between philosophical dualisms (e.g., "Platonic appearance vs. reality, facts vs. values, subjectivism vs. objectivism and theoretical dualisms", p. 18), as well as between the theoretical dualisms associated with positivist/postpositivist and the constructivist, interpretive worldviews. They also note the importance of both the physical world and the "psychological world that includes language, culture, human institutions, and subjective thoughts" (R. B. Johnson & Onwuegbuzie, 2004, p. 18). They recognise the "entanglement" of values and beliefs with facts, or what is accepted as "knowledge" (Freeman-Moir, 2013).
In addition, Tashakkori and Teddlie define pragmatism as

> a deconstructive paradigm that debunks concepts such as 'truth' and 'reality' and focuses instead on 'what works' as the truth regarding the research questions under investigation. Pragmatism rejects the either/or choices associated with the paradigm wars, advocates for the use of mixed methods in research, and acknowledges that the values of the researcher play a large role in interpretation of the results (2003a, p. 713).

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7 Objective knowledge refers to the notion of being value-free, apart from the researcher, part of the physical world and can be tested or deduced. Subjective knowledge, on the other hand, recognises that knowledge is neither value-free nor disentangled from the researcher (Guba & Lincoln, 2005).

Denzin and Lincoln see pragmatism firmly rooted in the postrealist tradition. However, unlike the previous authors, they seem to imply criticism, claiming that pragmatism "is a theoretical position that privileges practice and method over reflection and deliberation" (Denzin & Lincoln, 2005b, p. xiv), implying a lack of serious theoretical cogitation, a point not mentioned by the others. Both Johnson and Onwuegbuzie, and Tashakkori and Teddlie emphasise pragmatism as a practical solution to the acrimony associated with the positivist versus the interpretive/constructivist stances rather than Denzin and Lincoln's concern about the lack of thoughtful rigour. Despite these differences of opinion, pragmatism allows the researcher to take an eclectic stance by selecting methodologies and methods that suit the research topic and questions, by accepting the realities of both the external and internal worlds, and by acknowledging the values and experiences of the researcher.

From worldviews to methodologies

Each of these worldviews, positivism/postpositivism, constructivism and pragmatism, has engendered its own research methodologies and methods. Traditionally, positivists and constructivists have developed and inhabited mutually exclusive worlds while the pragmatists have attempted to straddle both.

Quantitative methodologies align themselves with a positivist, rationalist approach to research, which has developed from scientific, empirical observation associated with a deductive approach to research (Guba & Lincoln, 2005; Wiersma & Jurs, 2009). Quantitative methodologies have been applied to the social sciences and education, disciplines rather removed from the study of scientific phenomena. In this approach, the subjects of the research are viewed objectively, as objects dispassionately and rationally. Value has no weight or meaning. Truth and knowledge are important with knowledge perceived as knowable, value-free, and external from the observer. A postpositivist approach is less certain: it accepts there may be more than one reality that may have to be dealt with; conclusions, though still objective, are probable rather than definite; and the researcher's values may influence the research (Lincoln et al., 2011; Wiersma & Jurs, 2009). The results from quantitative research are based on objective, value-free analyses of numerical data and are deemed generalisable (Teddlie & Tashakkori, 2009; Todd, Nerlich, & McKeown, 2004). However, if one accepts that quantification is also a cultural tool—"It is both a mode of representation and a tool, a cultural resource that enables researchers, as much as our 'subjects' to construct meaning" (Vann & Cole, 2004, pp. 151-152)—then one needs to be willing to interrogate the notion that quantification is completely objective and thus value-free. Even a quantitative approach ought to require an acknowledgement that truth, at least in the areas of education or social sciences research, is relative, in that it is coloured by the researchers' perspectives, and is context-bound.

Qualitative methodologies, on the other hand, are aligned to a humanistic worldview which recognises the subjectiveness of research in terms of data collection, observations and interpretation of perceptions. The position and values of the researcher(s) need to be acknowledged and identified. Qualitative methodologies take a holistic, deep perspective of a subject or research question/ phenomenon, and a deductive approach to logic; however, qualitative research aims to understand, and tries to make sense of the data in terms of a variety of perspectives. Knowledge is viewed as something constructed by the individual or group of individuals rather than an external, generalisable universal (Denzin & Lincoln, 2005b; Hatch, 2002; Mertens, 2010).

These two worldviews and their approaches towards research inhabit different worlds with different vocabularies, methods, standards of excellence, as well as conceptions of truth, epistemology, validity and reliability. When two worldviews inhabit different, exclusionary worlds, there is an issue of how one can criticise the other. In addition, if all theories of the world by necessity are based on the individual's personal interactions with the world, then how can a worldview be separated from experience and prior knowledge in order to argue the "Truth" of one over the other? Both of these approaches, quantitative and qualitative, are concerned with empirical research based on methodical observation, with reaching logical conclusions based on the observations, and with validity, reliability and ethics of the research (Creswell, 2003; Johnson & Onwuegbuzie, 2004; Teddlie & Tashakkori, 2009; Todd, Nerlich, McKeown, & Clarke, 2004). However, "qualitative researchers accuse quantitative ones of positivism, reductionism, determinism, and objectivism" while "quantitative researchers accuse qualitative ones of fuzziness and subjectivity" (Todd, Nerlich, & McKeown, 2004, p. 5); in other words, rather than appreciating the similarities between the worldviews and the potential for combining aspects in order to expand understandings, researchers and theorists emphasise the differences between and incompatibility of these two worldviews.

Research in education, especially classroom research, is too complex to be hampered by these exclusionary, over-codified worldviews (Keeves, 1999). To understand what happens in the classroom, one needs to be flexible enough to utilise multiple approaches in order to access the full range of lived experiences. Keeves rejects "the misleading dichotomy of research procedures into qualitative and quantitative methods, since it is argued that the choice or procedures to be employed depends on the nature of the problem under investigation" (1999, p. 3). Bergman (2008) finds the notion of a clear distinction between the two worldviews problematic as each encompasses a range of methods and approaches. For Todd, Nerlich and McKeown (2004), these paradigms are not necessarily competing but they serve different purposes. These worldviews can be viewed as complementary rather than exclusionary (Hammersley, 2008 ; Husén, 1999).

The terms mixed, multiple (Denzin & Lincoln, 2011b), complementary (Green et al., 2006a; Jaeger & American Educational Research Association., 1988) and eclectic (Collier & Elman, 2008; Teddlie & Tashakkori, 2011) have all been used in trying to describe methods and methodologies that adopt a pragmatic approach to selecting methods that suit a phenomenon, problem or a set of research questions rather than being trapped within either the quantitative or qualitative worldview. I have chosen to use the term *complementary* because it resonates with the approaches I have adopted in this study. Rather than mixing methods, which suggests a metaphor of adding to, as in adding flavour to the research, I prefer the notion of complementing, of "combining in such a way as to enhance or emphasise the qualities of each other or another" (Complementary, 2013). Green et al describe complementary methods as "exploring which approaches might be productively brought together to study a common phenomenon, and which might be juxtaposed to make visible similarities, differences and complementarities between phenomena" (2006b, pp. xvii-xviii). A complementary perspective suggests one is choosing methodologies and methods that have the potential to work together to enhance and deepen one's understanding of the phenomena under study.

Mixed methods research traditionally consists of a combination of qualitative and quantitative methods that permeate the research in terms of questions, data gathering and analysis (Creswell, Plano Clark, Gutmann & Hanson, 2003; Tashakkori & Teddlie, 2003a). A variety of process models are included in mixed methods research, often described in terms of either a temporal sequence (qualitative then quantitative, or quantitative then qualitative), or a concurrent design. With both of these processes, one paradigm (quantitative or qualitative) may be privileged over the other, or they may be given equal emphasis (Creswell et al., 2003; Johnson & Onwuegbuzie, 2004; Tashakkori & Teddlie, 2003b; Teddlie & Tashakkori, 2009).

At times, the processes described in mixed methodology research seem to have become as governed by rigidity and conventions as approaches included within the quantitative or qualitative paradigms (Todd, Nerlich, & McKeown, 2004). If one views mixed methods in terms of multiple, complementary and/or eclectic methodologies and focuses on a *perspective* towards one's research (Brannen, 2008; Green et al., 2006b), on "an attitude of inquiry, and approach to research quality and to what makes for adequate explanations of social phenomena" (Fielding, 2008, p. 51), then avoiding these conventions is possible. To do this, Maxwell and Loomis suggest an integrated, interactive model incorporating a network of components (purposes, conceptual framework, research questions, methods and validity) rather than a linear development (2003, p. 246). Teddlie and Tashkkori's definition of methodological eclecticism as including "*selecting and then synergistically integrating the most appropriate techniques from a myriad of QUAL, QUAN, and mixed methods* (authors' italics) in order to more thoroughly investigate a phenomenon of interest"(2011, p. 286) suggests a similar perspective towards mixed methods research.

Even though the framework for this research is basically interpretive, both qualitative and quantitative methods were combined in a pragmatically integrated manner, as described by Teddlie and Tashahori (2011) in order to make sense of a complicated set of phenomena and research questions within the complex settings of multiple school contexts.

Other influences

At the beginning of my career in tertiary education, I would have placed myself within the tradition of cognitive psychology because I was interested in the mental processes of individual students and teachers who were successful and/or unsuccessful learners/teachers. The more I read, taught and observed, the more influenced I became by sociocultural theories and explanations of learning. I became interested in the ways individuals interacted with their social, cultural and historical environments. Two groups of classroom researchers, Graham Nuthall and colleagues in New Zealand and Paul Cobb and colleagues' analyses of mathematics classrooms, were particularly influential in changing my focus and approach to studying within the context of the classroom. Both groups of researchers approached their extremely detailed studies of the complexities of classroom interactions by applying a combination of cognitive and sociocultural perspectives. Both groups developed frameworks for understanding and interpreting students' experiences that I found illuminating (Cobb, 2007; Nuthall, 1999, 2004, 2005, 2007; Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). Because the influences of Nuthall and Cobb are so important to this study, their research orientations are now discussed in some detail.

Nuthall

Graham Nuthall and his approach to classroom research by focusing on the learner's experiences has been an enormous influence on my practice as a teacher and as a researcher. I worked as a tutor for Graham in the early 2000s and took over teaching one of the courses he had designed once he retired. I subsequently adopted some of his course materials and research in designing and teaching an educational psychology course on cognition and learning.
I was fascinated by the research he and Adrienne Alton-Lee, and others, undertook that followed individual children within their classrooms by mapping their learning against teacher-led classroom discussions, peer discussions, conversations, self-talk and interaction with classroom resources/learning tools. My university students were particularly interested in the predictive nature of their findings, such as the 'rules for learning' (Nuthall & Alton-Lee, 1994, p. 39), which are based on a learner having at least four meaningful engagements with all the information, or the equivalent partial experiences, needed to learn a concept (Nuthall, 2005, 2007; Nuthall & Alton-Lee, 1994). I, however, was more influenced by the way cognitive and sociocultural perspectives were combined in order to gain insights into the children's experiences, their thinking, what they learnt and what they failed to remember (Nuthall, 1997).

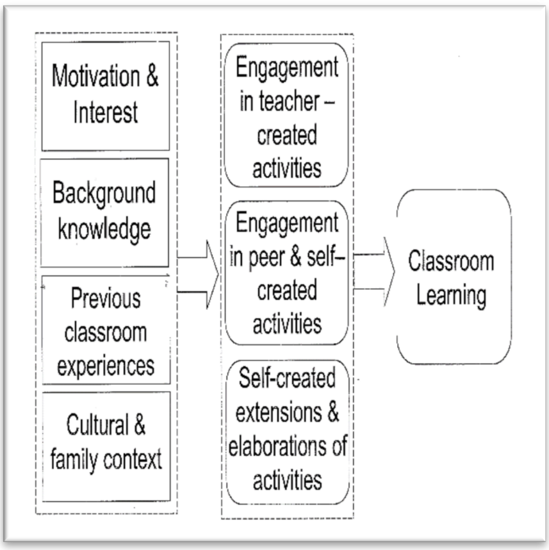

Figure 2.1: Student classroom experiences (Nuthall, n.d.)

This approach to classroom research combines a study of the individual with a specific background as part of the sociocultural world of specific peer groups and a specific classroom. Figure 2.1 summarises the idiosyncratic combination of elements–personal interest, prior knowledge, cultural and family values and past school experiences–children bring to the classroom that influences how and to what extent they engage with learning activities. In this diagram, Nuthall makes the interesting distinction between teacher, peer and self-created activities that may lead to learning a particular concept (Nuthall, 2004), a precursor to what he subsequently referred to as the three worlds of the classroom. Nuthall viewed the three intersecting worlds of the classroom as distinct cultural contexts (2005):

> First, there is the public world that the teacher sees and manages. It is the only world most of us see when we go into the classroom. In this world, the students (mostly) do what the teacher wants them to do, by following the public rules and customs of the classroom. This is the world structured by learning activities and routines the teacher designs and manages (Nuthall, 2007, p. 84).

This is the easiest of the three worlds of the classroom for the researcher to explore, the world where the teacher's beliefs and values are reflected in the classroom routines and activities (Nuthall, 2004, 2005). It is where the accepted ways of doing, responding and engaging in the work of the classroom are observable.

> Second, there is the semiprivate world of ongoing peer relationships. This is the world in which students establish and maintain their social roles and status. It has its own rules and customs, and students are acutely aware of them as they participate in the public world of the teacher. Transgressing peer customs may have worse consequences than transgressing the teacher's rules and customs. This peer-relationship world flows over into out-of-class activities, where clique formation goes on uncontrolled, and where adults do not usually see the teasing and bullying (Nuthall, 2007, p. 84).

This world is full of constant negotiations of power, roles, identities and allegiances. It is affected by individual children's status which in turn may be affected by gender, ethnicity and class, all of which influence individuals' ways of doing school, and engaging with school activities (Alton-Lee, Nuthall, & Patrick, 1987; Nuthall, 1992, 1997, 2007, 2012). Identities and power negotiations may change from day-to-day, subject-to-subject, from activity-to-activity or the group with whom the child is working or wanting to be a member. The second world is extremely difficult for an adult to access. Even the classroom teacher is often unaware of the complexities of its workings. It is only by careful observation combined with access to video and/or audio recordings of group and peer interactions that a researcher may be able to access this world.

> Finally, there is the private world of the child's own mind. This is where children's knowledge and beliefs change and grow; where self-beliefs and attitudes have their effects; where individual thinking and learning takes place. This world, continuous over all aspects of a child's life, brings home life into the school and playground, and brings school life back into the home.
> (Nuthall, 2007, p. 84).

The third world, like the second, can be difficult to access unless the researcher can observe the child working and speaking to her/himself as well as carefully asking the individual about this world. One needs to realise that the child may not wish nor be able to communicate information about this hidden world.

Accessing the third world was extremely important in this study of children's beliefs about mathematics because this is the realm of individual beliefs. The only way I could begin to understand these beliefs and how they play out in the classroom was by exploring the worlds about which the teacher knows very little. These are the worlds where individual's and groups' beliefs about the nature of mathematics, who is good at mathematics, who is allowed to explain, and whose explanations are accepted are most likely to be encountered. I also needed to be aware that these worlds, the semi private and the private, affect and are affected by the other worlds inhabited by the classmembers, worlds outside the specific classroom. Nuthall's three worlds of the classroom formed a framework for my approach to accessing, understanding and interpreting classroom data.

Cobb

Another framework I found particularly helpful is the interpretive framework that includes both social and cognitive perspectives developed from Cobb, Yackel and other colleagues' research into the complexities of the mathematics classroom (Cobb, Yackel, & Wood, 1992a, 1992b; Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). For them, mathematics is not value-free, but rather a human construct that can be viewed "as both an individual and collective human activity" that reflects cultural practices (Cobb et al., 1992a, p. 26). Cobb defines a cognitive perspective as one that emphasises "the nature of individual students' reasoning or, in other words, … their specific ways of participating in communal classroom activities" (2007, p. 29). The sociocultural aspects are described by two sets of classroom social norms which are useful for analysing what goes on in the classroom: the general classroom norms, which include the usual routines, patterns and expectations of teaching and learning in a classroom, and "sociomathematical" norms, the patterns, routines, behaviours and expectations that are specific to the mathematics classroom (Cobb, 2007; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). These norms are affected by the teachers' values and beliefs, by the students' beliefs and goals, as well as by the interactions between students and teacher. Thus there is an element of negotiation inherent in establishing both of these sets of norms (Yackel & Cobb, 1996). Cobb et al show the relationship between the social and the individual cognitive approach, which in this case they label the psychological perspectives in the following table:

Table 2.1: An interpretive framework for analyzing the classroom microculture

Social Perspective	**Psychological Perspective**
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and activity

(Cobb et al., 1997, p. 154, Figure 8.1)

In this table, they position beliefs as cognitive that interact with both the social and the sociomathematical norms of a classroom. Although the sociocultural and cognitive perspectives in this model are addressed as independent, they are intertwined:

> [T]he collective activities of the classroom community (social perspective) emerge and are continually regenerated by the teacher and students as they interpret and respond to each other's actions (cognitive perspective). Conversely, the teacher's and students' interpretations and actions in the classroom (cognitive perspective) are not seen to exist apart from their participation in communal classroom practices (social perspective) (Cobb, 2007, p. 29).

How the individual engages in doing and learning mathematics cannot be separated from the classroom ways and routines for doing and learning mathematics (McChesney, 2004).They are two interdependent ways of looking and making sense of the complexity of the mathematics classroom .

Considerations when working with children

What is it that I think is so important to cover about doing research with children? Why am I suddenly referring to my informants as children rather than students as I first described them. Initially, I thought the label 'students' was less patronising and offered them more respect, a higher status than mere 'children'. In addition, the notion of the 'student' is an attempt at removing some of the ageism that surrounds the word 'children'. Describing the informants as students indicated a sense or illusion of overlapping worlds or contexts: they were school students and the researcher, a university student. However, now that I am thinking about the challenges of using student/younger informants, often as constructed by the research community (ies) rather than the realities of working alongside younger informants, I realise that, in terms of my concerns, 'children' implies a different set of political (for want of a better word) connotations.

I want to cover the outside/other's concerns about working with the young as well as my concerns: most importantly the challenge of how to access, to listen and to hear what children say, and why this important. I also want to look at how/why/what is different or the same between listening to adults and children, and why this matters. Ideas and concerns about power, approaches and the role of the researcher need to be addressed.

Journal entry, October 2010

Researchers' attitudes and beliefs about children and childhood affect the approach they take to studying children. Traditionally, much research has been ‘done to’ children. They have been positioned as *other*; as intrinsically different from adults (Punch, 2002), viewed as incompetent, as the opposite to the mature, whole person or adult; as objects (O'Kane, 2008; Woodhead & Faulkner, 2008); and as unreliable informants (Christensen & James, 2008a; Greene & Hill, 2005; Roberts, 2008) who can be studied by allowing adults—*parents, teachers, caregivers, and researchers*—to speak for them and interpret what they say (Christensen & James, 2008b; Scott, 2008). At the other extreme are those who believe children are the same as adults (Punch, 2002), and thus what they say can be heard and interpreted in exactly the same way. In these cases, as Balen, Holroyd, Mountain and Wood (2000/2001) explain, “[c]hildren have been expected to communicate in an adult frame of reference, and instead of focusing on answering the questions posed, they have spent more effort just trying to work out what it is that is required of them” (p. 24). However, a third approach views children as similar to adults as reliable informants, but who may have different competencies, skills and interests to adults, and as such researchers need to be aware of these differences (Christensen & James, 2008a; Punch, 2002).

One of the main challenges for the adult researcher is how to access what children believe and wish to say, and how to allow what they say to help the adult understand the lived experiences of these children. In other words, how to access what they think and feel with respect, while entitling the children to retain control within the research experience. The researcher needs to be aware that children don’t just speak with a single voice. They are no more a single amorphous block than adults are: what they have to say is as varied as their backgrounds, their experiences, their cultures, their contexts and their personalities (Christensen & James, 2008a; Greene & Hill, 2005; O'Kane, 2008; Roberts, 2008). And like adults, children are social actors, who can be competent informants, collaborators and subjects in educational research (Christensen & James, 2008b; Hogan, 2005; O'Kane, 2008; Woodhead & Faulkner, 2008).

Listening to children

Children are essential players and thus experts in their own experiences and worlds (Roberts, 2008) who have the right to be listened to and to explain their own viewpoints (Scott, 2008; Woodhead & Faulkner, 2008). They are capable of communicating what they know and believe, and adult researchers cannot really access this world without listening to them (Greene & Hill, 2005; Hill, 2005). However, even when children are encouraged to give their views, adult researchers do not always hear them. What they say may be faithfully transcribed, but it is then filtered and interpreted in terms of the adult's perspectives and understandings of the child's view (O'Kane, 2008; Roberts, 2008). A question that warrants consideration is how is it possible to "[keep] faith with children's own perspectives and voice" (Greene & Hill, 2005, p. 12). Researchers such as Roberts (2008) put forward a model of collaborative research with children, of working together with and consulting children on issues of concern. Others like Mayall try "to work with generational issues, rather than to assume adult superiority or to downplay those issues" by "asking children, directly, to help me, an adult, to understand childhood" (2008, p. 109). Whether adopting a model of collaboration or using the young informants as guides through their own terrain as Mayall does, the researcher needs to be aware of the differences between listening, hearing and interpreting what children have to say.

The rights of children to be listened to and heard are inexorably linked to issues of ethics and power. The UN Convention on the Rights of the Child (UNICEF, 1989) covers the "status, role and rights of children" (Woodhead & Faulkner, 2008, p. 12), but Article 12 also includes "respect for the views of the child" (UNICEF, 1989). Hill presents a list of children's rights: to self-determination, privacy, dignity, anonymity, confidentiality, fair treatment and protection from discomfort or harm" (Hill, 2005, p. 65). Freire and Macedo put the responsibility on educators (I would add, researchers) to be aware that students have "the right to express their thoughts, the right to speak, which corresponds to the educator's duty to listen to them" (1987, p. 40). Even though the children's rights are enshrined in the UN Convention (UNICEF, 1989), the researcher is responsible for upholding these rights when listening to and researching with children.

Power

When undertaking research with children, one also has to confront the issue of power between adult and the child, between the researcher and the researched. That is not to ignore that power can also be an issue with researching other marginalised groups, but it is more obvious in this case. Adults by their age and size wield an amount of power and control over the worlds of the child that can be exacerbated “by differences in gender, ethnicity, culture and social background” (Balen, Holroyd, Mountain, & Wood, 2000/2001, p. 28). One result of this power imbalance the adult researcher needs to be aware of is “[c}hildren are not used to expressing their views freely or being-taken seriously by adults because of their position in adult-dominated society” (Punch, 2002, p. 325). By being aware of the power differential between adult and child, and acknowledging cultural, ethnic and gender differences, the researcher can work towards taking children seriously, and encouraging children to express themselves in a safe, accepting, non-intimidating environment. Also associated with this power imbalance are adult notions of knowledge, that they know more than children and thus have the right to interpret for children, judge the correctness of children’s responses, and analyse them in terms of an adult worldview (Mayall, 2008; Punch, 2002; Woodhead & Faulkner, 2008). But how can the adult researcher protect against this? One potential solution is to listen carefully to the young informants, suspend judgment, be aware of one’s own biases; another is to analyse one’s role as a researcher extremely carefully (O'Kane, 2008).

Another power issue of which the researcher needs to be aware is that adult gatekeepers control access to children (Punch, 2002). Who is allowed to talk, be listened to and heard is managed by parents, caregivers, schools, teachers and public authorities. Even though the child may have a right to be heard, any one of these entities may legally stop the child from voicing an opinion. The researcher has access only to children whose caregivers have authorised this access, or to those old enough to decide for themselves whether to take part.

An additional concern is that, although the obvious power imbalance exists between adults and children, one may also exist between children: "Factors of age, gender, birth order, educational attainment, caste/class, ethnicity, (dis)ability, as well as individual personality and physical stature all play a role in shaping power relations in childhood" (O'Kane, 2008, p. 126). Children who are perceived as having a higher status due to any of these characteristics may wield power over their peers, which in turn may affect how they engage with a researcher. Who has the authority to speak and be listened to, and who does not, affects what the researcher may hear (Nuthall, 2007, 2012). Especially in the context of the classroom and playground, these power dynamics need to be recognised by researchers.

Role of the researcher

The researcher is an integral part of the research process; as a result, the relationships between the researcher and participants/ informants influence how the data is collected, what is collected, whose voices are shared, and how the data are analysed (Connolly, 2008). These relationships are also influenced by the researcher's worldview which, in turn, influences the study design. Denzin and Lincoln (2005a) claim that "[t]he gendered, multiculturally situated researcher approaches the world with a set of ideas, a framework (theory, ontology) that specifies a set of questions (epistemology) that he or she then examines in specific ways (methodology, analysis)" (p. 21). The researcher, however, needs to recognise his/her position/role as well as the limitations of never completely being able to understand the lived experiences of others because individuals' experiences and backgrounds will always be different (Denzin & Lincoln, 2005a). This limitation is amplified when dealing with children where part of their world will always be "inaccessible to an outsider", the adult (Greene & Hill, 2005, p. 5).

Researchers seem to take a variety of positions or roles when working with children: those who do research *on* children comfortably retain a disinterested, objective adult pose; at the other extreme are those who believe they can inhabit the world of the child as a faux child. Both of these positions, especially the latter, are problematic when trying to access what children have to say. Balen et al (2000/2001) caution the researcher against going "native" (p. 27). Other researchers support this criticism: Greene and Hill argue against those who believe "...researchers can be flies on the wall or in some way neutralize themselves" (Greene & Hill, 2005, p. 11; Li, 1999); as does Mayall who explains "even when researchers try to be accepted as one of the children, children still view them as adults "because "...a central characteristic of adults is they have power over children" (Mayall, 2008, p. 109), a position echoed by O'Kane (2008). A more honest approach would be to follow Emond who explains "...the researcher, as a person, must negotiate and develop relationships with children which acknowledge that they can control the extent to which the researcher is allowed in" (2005, p. 125). Another approach that seems to work is that of the not quite competent adult; in this role the researcher can sometimes be admitted as an honorary member of the child/children's world by having "successfully negotiated a space somewhere between adult figures of authority and the children themselves" (Greene & Hill, 2005, p. 11). Corsaro and Molinari write of the "... importance of developing a particular status as an atypical, less powerful adult in research with young children" (Corsaro & Molinari, 2008, p. 240; Yin, 2006), who can be regarded as someone who needs help (Emond, 2005). From my perspective, adopting either the position of an objective, disinterested observer or of a pretend child was unacceptable. I opted for a negotiated, flexible position, a space where children could choose whether and/or how much to share, where they could advise and help me when need be, and where they could recognise me in a more traditional adult position at other times.

Methods

Apart from issues of power and how to listen to and hear children, researchers need to consider methods of collecting data from young informants/participants.

Greene and Hill (2005) claim that if one wants to learn about individual children's lived experiences, one needs to employ "methods that can capture the nature of children's lives as lived rather than those that rely on taking children out of their everyday lives into a professional's office or 'lab' " (pp. 3-4). In other words, data collection needs to take place in familiar locations and contexts such as schools and homes (Scott, 2008). In order to make the research appropriate for children, the researcher can use "practices that resonate with the children's own concerns and routines" (Christensen & James, 2008b, p. 8). These methods need to match "children's level of understanding, knowledge, interests and particular location in the social world" (Greene & Hill, 2005, p. 8). Punch (2002) suggests using a combination of visual and written task-based methods as do Balen et al (2000/2001) to which they add open-ended questions when interviewing. In addition, the research questions need to be relevant to the informants' lives, experiences and concerns (Balen et al., 2000/2001; Punch, 2002; Scott, 2008; Woodhead & Faulkner, 2008).

According to Punch (2002), Scott (2008), Woodhead and Faulkner (2008), following these approaches improves data quality and answers criticisms about the reliability of children as informants. Developing rapport between the researcher and the child (Punch, 2002; Scott, 2008) which often means spending enough time with the informants because the child needs to be able to trust the researcher (Balen et al., 2000/2001) may also assist data collection.

Finally, adult researchers need to ensure that their young participants/informants are given the same respect, ethical considerations and confidentiality as are offered to adult participants. Researchers should be willing to explain to the children what the research is about, to allow children to give informed consent whether they take part, to empower them to withhold information, to refuse to complete a task, and/or remove themselves from the research project (Cullen, 2005; Finch, 2005; Hill, 2005; Roberts, 2008; Woodhead & Faulkner, 2008).

Ethical Considerations

In this study, my approach to being ethical was influenced by my attitude towards undertaking research with children within the context of schools and by my concerns associated with affording all the research participants, children as well as adults, respect and as much self-determination as possible. My approach was also aligned with the University of Canterbury Human Ethics Committee's principles and cultural values of "justice, safety, truthfulness, confidentiality and respect" towards human participants (University of Canterbury, n.d.), particularly in terms of "the protection of human rights and cultural values of the participants, including the obtaining of informed consent, and recognition of participants' right to decline", (Item 3 of the ethics Terms of Reference, (University of Canterbury, n.d.). The notions of informed consent, voluntary participation, anonymity, confidentiality and power to withdraw are the norm in educational research (Christians, 2011; Cohen, Manion, & Morrison, 2011; Cullen, 2005; Finch, 2005; Mutch & New Zealand Council for Educational Research, 2005; Strike, 2006; Strike & Egan, 1978). However, the notions of justice and respect for the cultural values of the informants is of equal importance, especially when dealing with marginalised groups. In this study, there were two overlapping groups which could be defined as marginalised: children and within this category, Māori children (Greene & Hill, 2005; Smith, 1999, 2008). I attempted to prioritise the safety, well-being and autonomy of all the participants in this study. The proposal for this study was reviewed and accepted by the University of Canterbury Human Ethics Committee. As part of this process, a Māori expert reviewed the study for its suitability for all New Zealanders.

Although I cannot claim to be following a Māori kaupapa[8], I am influenced by some of the same considerations, in particular respect for and the importance of forming relationships with the children and adults in the schools that I visited. Cullen refers to two guiding approaches when addressing research ethics:

8 Principles or strategies that include Māori cultural values

"universal ethical principles" (Cullen, Hedges, & Bone, 2005, p. 2), such as those espoused by the University of Canterbury, quoted in the previous paragraph, and relationships (whanaungatanga), which are context-based (Cullen, 2005). Linda Tuhiwai Smith includes a list of Māori research principles and "ethical codes of conduct":

> 1) aroha ki te tangata (a respect for people), 2) kanohi kitea (the seen face, that is present yourself to people face to face), 3) titiro, whakarongo ... kōrero (look, listen ...speak), 4) manaaki ki te tangata (share and host people, be generous), 5) kia tūpato (be cautious), 6) kaua e takahia te mana o te tangata (do not trample over the *mana*[9] of people), and 7) kaua e māhaki (don't flaunt your knowledge).
>
> (L. T. Smith, 1999, p. 120).

This list emphasises the importance of the researcher honouring the dignity of those involved by listening, being cautious and respectful. In writing about research ethics with "indigenous and other marginalized groups", she stresses the importance of "establishing, maintaining, and nurturing reciprocal and respectful relationships" (Smith, 2008, pp. 128-129). My approach to visiting and gathering data in schools reflects similar values and concerns.

I view the researcher in schools and classrooms as a visitor or guest invited into the institution. As a guest in the school, one ought to respect the hosts, in this case the tangata whenua[10], by observing, listening, as well as answering questions. The guest follows the customs, protocols and routines of the school and the classroom and is expected to respect the values and efficacy of others. The researcher respects the rights of the participants – the schools, teachers, students, school leaders, etc. I spent time in all 17 schools talking and listening to teachers, administrators and children before collecting data. This time spent face to face with potential participants was particularly important for developing relationships with the two bi-lingual programmes I visited. By following this approach, I was able to comply with the spirit of principles 1,2,3, 5 and 6. I was able partially to follow principle 4 by sharing ideas, initial findings, resources and food at the two focus school sites. A researcher who follows this philosophy

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9 Authority, status, dignity

10 Literally, the people of the land; in this case the hosts, the inhabitants of the schools

is there to try to understand the phenomena at that cultural institution with that particular group rather than to criticise, judge or intervene.

For me, it was important to spend enough time at the focus schools to reach a level of acceptance, especially by the students but also by both the school administrators and the teachers. In order for me to listen to the students and teachers, they needed to feel empowered enough to decide whether to take part, how much to share, what not to share, and when to change their minds about participation. Only by prolonged engagement in these two schools, over 18 months, in the staffrooms, talking to administrators, and spending time both in the classrooms and more public school spaces was I able to earn the trust of the children. By doing this, I began to learn about the students, their teachers, and their experiences of the mathematics classroom as well as about other things that were of interest or concern to them that was beyond this particular study (Harris, 2007).

The ways I approached this study of mathematics beliefs were not only influenced by the requirements of the university ethics committee and my approach to research but also by my attitudes towards working with children. The role I played at the focus schools was one of the *not quite competent adult*, which perhaps satisfies Smith's principle 7 for ethical research conduct (Smith, 1999, 2008). My membership of the adult tribe was easily recognisable due to my age and size. The children knew I was a grown-up and treated me as such when they needed help in the playground, advice with kapa haka[11], or a question in mathematics or another school subject. However, they also knew I was a student, I was called by my first name, and that, because I was not a primary school teacher, I had no clue about how their classes or schools worked. They acted as my guides explaining how things worked in the classroom, the school, the library and the playground. They also advised me about the real work of the mathematics classroom, how lessons were structured, how groups operated, and how tasks were undertaken and completed. They allowed me glimpses into the

11 In this context, a group that performs Māori songs and dances

social workings of their school lives telling me about cliques, bullies, greyhounds, teachers (both awful and wonderful), families, that is, any topic that seemed relevant to them at the time.

Negotiating research quality

Considering the elements necessary for acceptable quality with complementary methods research is complex and confusing. Traditionally, empirical research follows a positivist approach to research design that is judged in terms of validity (Does this measure what it claims to measure? etc.), reliability (Is the measure consistent? Are those who rate or mark a survey or test consistent? etc.), and generalizability (Does it apply to other studies, situations, populations? etc.). Embedded in this tradition is the idea that all research ought to be "generalizable and replicable" (Willis, Jost, & Nilakanta, 2007, p. 218). With the proliferation of more qualitative, interpretive approaches, criteria for judging the quality of research has expanded by adopting different terms and processes associated with academic rigour (Creswell, 2007). In the following paragraphs, I explore some of the terms and processes associated with ensuring rigour in qualitative research and end with a brief description of the process I employed within this study in order to establish rigour or trustworthiness.

Criticism levelled against qualitative research's lack of rigour has been countered by writers such as Lincoln and Guba (1985) who discuss how their notions of credibility (equivalent to internal validity), transferability (to external validity), dependability (to reliability) and confirmability (to objectivity) lead to "trustworthiness" of research (Creswell, 2007, p. 256; Graneheim & Lundman, 2004; Mertens, 2010). Similarly, Tashakkori and Teddie also use the term trustworthiness along with "authenticity" and "plausibility" (2008, p.102) when discussing "inference quality" which they define as "the process of meaning making and/or its outcomes", a term they prefer to validity (p.102). In addition, others use the concept of triangulation in the place of validity and reliability (Willis et al., 2007).

Schoenfeld (2007), in his writing about research in mathematics education, rather than differentiating quality by research paradigm (quantitative and/or qualitative), claims that trustworthiness or “believability” (p. 93) can be judged by answering the following questions:

> Why should one believe what the author says? (the issue of trustworthiness)
> What situations or contexts does the research really apply to? (the issue of generality or scope)
> Why should one care?"(the issue of importance) (2007, p. 81).

He accepts that different types of research do not answer these questions in the same way or with the same emphasis. He uses terms like “descriptive and explanatory power”, “rigor and specificity”, “care and precision” and triangulation as ways of ensuring trustworthiness. His second two questions are more problematic to deal with in interpretive than in quantitative research. He describes three types of generality as a means of addressing this problem: “*potential generality* …the set of circumstances in which the results of the research might reasonably be expected to apply” (p.88), “*warranted generality* … the set of circumstances for which the authors have provided trustworthy evidence that the findings do apply” (p. 89), and “*limited warranted generality* “, cases that may “offer existent proofs, bring important issues to the attention of the field, make theoretical contributions, or have the potential to catalyse new lines of inquiry” (p. 89) as explanations of which concepts may appear in different types of research. This study falls into the category of “limited warranted generality” in that I plan to present a well-reasoned argument in order to raise issues which should engender new lines of enquiry for subsequent research and cogitation, as well as present ideas that may be of interest to the teaching profession. Schoenfield’s third question poses a dilemma: on the one hand, I would not have embarked on this journey had I not thought the study of children’s mathematics beliefs important; on the other hand, the judging of importance is an *a posteriori* rather than an *a priori* judgement in that ultimately it is the reader rather than the researcher who decides on importance in terms of a specific piece of research.

Despite the use of a variety of terms associated with ensuring quality of research, there seems some agreement around strategies or processes appropriate for reaching rigour. These include extended time involved in research sites/contexts, member checking, and peer debriefing. In addition, they include dealing with researcher bias, dealing with negative cases, triangulation, "thick descriptions" and audit trails (Creswell, 2007; Lincoln & Guba, 1985; Mertens, 2010; Willis et al., 2007). A dependability audit trail is created by including "careful documentation of how the research was conducted and the associated data analysis and interpretation process, as well as the thinking processes of the researcher " (Mertens, 2010, p. 255). These processes seem to imply a certain amount of overlap between trustworthiness, credibility and transferability in that any one of the strategies listed above could satisfy quality conditions for more than one of these categories (Graneheim & Lundman, 2004). For instance, triangulation can contribute to trustworthiness (Schoenfeld, 2007) as well as to credibility (Lincoln & Guba, 1985) or plausibility (Tashakkori & Teddlie, 2008).

Triangulation as a concept associated with the quality of research is viewed in a variety of ways, as part of the process of achieving trustworthiness (Creswell, 2007; Mertens, 2010; Schoenfeld, 2007), as synonymous with trustworthiness itself (Willis et al., 2007), and as a justification for including complementary methods research design (Bryman, 2008). Triangulation involves using multiple sources of data, contexts, methods, researchers and/or theories in order to clarify, distil or confirm an inference (Creswell, 2007; Hammersley, 2008; Soy, 1997; Willis et al., 2007). Although I use multiple sources of information, multiple perspectives, and multiple methods of analysis, I am bothered by the term triangulation. My first issue is a minor one, which perhaps reflects a personal limitation, but I am irritated by the application of *triangulation* when used to refer to numbers other than three (two, four, five, methods, examples, or types of analyses, etc.). My second problem with the term is associated with my understanding of the essence of qualitative research, which I view as essentially interpretive, situated in a socially constructed world where understandings rather than truths are the aim. Triangulation, on the other hand, as a science metaphor for finding a focal point, implies that if one were to take three readings/look at three positions, one would find an exact point or meaning, a true reality. I prefer Richardson and St Pierre's and metaphor of the crystal:

> the central imagery is the crystal, which combines symmetry and substance with the infinite variety of shapes, substances, transmutations, multidimensionalities and angles of approach. Crystals grow, change and are altered, but are not amorphous. Crystals are prisms that reflect externalities and refract within themselves, creating different colors, patterns, and arrays casting off in different directions. What we see depends on our angle of repose.
>
> (2005) p. 963

With this metaphor, meaning moves in and out of focus, clarified or distorted depending on the position and perspective of the viewer. Richardson and St Pierre include both reflection and refraction: however, prisms can also separate light into a spectrum of colours, suggesting an image of different interpretations and understandings making up the whole/the white light. During the process of establishing the trustworthiness of this study, I have tried to articulate my "angle[s] of repose" through a dependability audit trail as I explored and tried to make sense of children's mathematics beliefs by applying different theoretical lenses and different data collection methods for accessing these beliefs.

Another term that needs further discussion is *thick description*, a device used to supply the reader with enough detail about the study, the participants and context to enable her/him to decide if the findings can be generalised/transferred to another context (Creswell, 2007; Flyvbjerg, 2011). Holliday goes beyond describing the who, what, where of a study by viewing thick description as a "creative device" (p. 733) requiring transparency which "must be convincing, and it must demonstrate how the connections were made and where they came from" (A. Holliday, 2004, p. 732). In contrast, Sandelowski emphasises interpretation rather than merely presenting and describing the facts of the study when creating *thick description* (2004). In all of these explanations of *thick description,* it is presented as part of the process of establishing the trustworthiness of a piece of research.

In this study, I needed to address trustworthiness in all the aspects inherent in combining research methods. To ensure believability (Schoenfeld, 2007), authenticity and plausibility (Tashakkori & Teddlie, 2008), I included a number of strategies, all of which contribute to the dependability and thus the trustworthiness of this study. In Chapter 1, I described the story of my interest in mathematics beliefs as well as my position and biases as a researcher, something I return to at other points as well. A detailed description of the research design, the contexts of the study, the participants, as well as the data collection methods is included in Chapter 3. In terms of the quantitative elements in the study, I trialled the questionnaires, included a large number of participants, included a reliability measure (Cronbach's Alpha), wrestled with significance and effect size, and compared the extracted beliefs framework to the literature; these aspects are covered in Chapters 3 and 4. In order to achieve "inference quality" (Tashakkori & Teddlie, 2008), I report the analysis phases in detail (Chapters 3, 4 and 5). In addition, I used numerous methods of data collection—questionnaires, drawings, interviews, observations, recording —and applied multiple readings and analyses of the data, thereby altering my angles of repose (Richardson & St. Pierre, 2005) during the process of understanding what I was seeing. As part of this process, I spent an extended amount of time at the focus schools, checked initial analyses with focus children and teachers, and discussed analyses with a variety of colleagues and experts (Creswell, 2007, 2008; Lincoln & Guba, 1985; Mertens, 2010; Willis et al., 2007). Finally, I kept detailed notes and reflections documenting observations and decisions as the study progressed, some of which are presented in textboxes. Many of these elements or processes contribute to a dependability audit trail (Creswell, 2007). Perhaps in this way, I address Schoenfeld's three questions (2007).

Conclusion

This chapter has introduced some of the elements that helped frame this study of children's mathematics beliefs within the classroom context. I have described and discussed theories associated with interpretive/constructive and pragmatic worldviews that underpin an eclectic perspective on methodologies, and the choice of complementary methods. The influences of previous research by Nuthall and by Cobb and associates who combined cognitive with social perspectives in their classroom research were discussed. In addition, the chapter included an explanation of my attitudes to working with children as informants, and I addressed ethical issues such as combining relational considerations alongside the usual ethical concerns (Cullen, 2005). Finally, notions of trustworthiness were introduced and discussed in terms of how they relate to this research, and the steps taken to ensure that this piece of work meets an acceptable standard of rigour.

In the following chapter, the research design that resulted from this framing is described, in particular, sampling, data collection and data analysis.

Chapter 3: Routes chosen

Methods

In the preceding methodological meander, a metaphor suggestive of a contemplative wander through ideas and theories, I introduced the influences on this study; this chapter tells a more direct story of the research design journey. All research design requires a coherent process of sampling and data collection. Complementary methods research (Green et al., 2006a) by its nature affects the complexity of these processes and requires the application of a variety of sampling, data collection and analysis methods (Kemper, Stringfield, & Teddlie, 2003; Onwuegbuzie & Teddlie, 2003; Teddlie & Tashakkori, 2009). In this section, I describe and explain the sampling procedure, data collection methods and analysis processes used for this study.

The study was designed to include a series of different phases. The first phase involved administering a mathematics beliefs questionnaire (MBQ) to a large sample of primary school children in and around a city in New Zealand. The next phase took place at two focus schools where the children completed a drawing task. The third phase involved recording mathematics classes in a focus classroom at each of the focus schools. The final phase, involving questionnaires, observations and interviews, took place the following year with nine focus children and their teachers. Observations took place during each of these phases. See Figure 3.1 below.

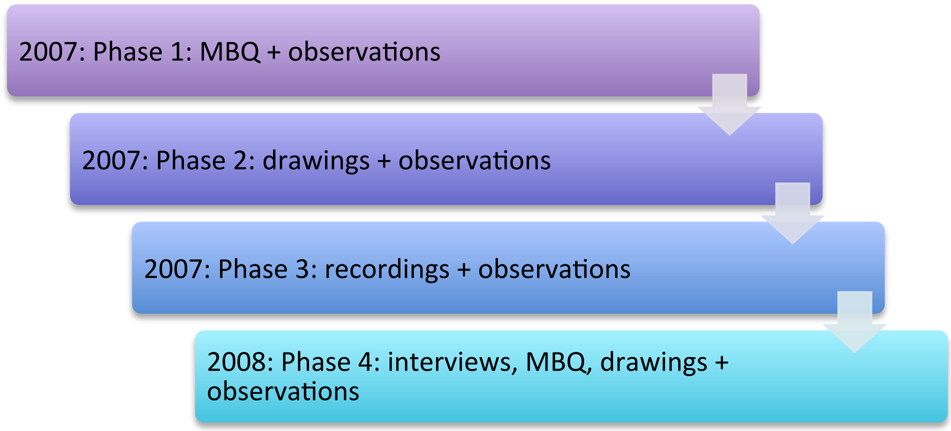


Figure 3.1 Data collection map

Rationale

In order to explore the range of mathematics beliefs held by these primary students, I decided to include a combination of data collection methods. According to Johnson & Turner (2003) there are three reasons for combining methods:

> (a) to obtain convergence or corroboration of findings,
> (b) to eliminate or minimize key plausible alternative explanations for conclusions drawn from the research data, and
> (c) to elucidate the divergent aspects of a phenomenon (p. 299).

There were three additional influences on my decision to include multiple, complementary methods of data collection: the difficulties with collecting data about beliefs, Nuthall's classroom data-collection method, and my concern, as an inexperienced researcher, of misinterpreting what I was seeing in the data.

Firstly, problems exist not only with the defining and locating, but also with the collecting of data about or accessing beliefs in order to study them (Hofer & Pintrich, 2002; Leder et al., 2002a; Lester, 2002). Hofer writes of the difficulty inherent in accessing beliefs, "…something as elusive as individual conceptions of knowledge and knowing"(Hofer, 2002, p. 9). Beliefs have been gathered by using different data collection methods, such as questionnaires (Bendixen, 2002; Moore, 2002; Perry, 1970; Presmeg, 2002; Schommer-Aikins, 2002; Schommer-Aikins, Brookhart, & Hutter, 2000; Schraw, Bendixen, & Dunkle, 2002; Yackel & Rasmussen, 2002), interviews (Baxter Magolda, 2002; Kloosterman, 2002; Perry, 1970; Presmeg, 2002), and observations (Leder & Forgasz, 2002; Lester, 2002).

There are, however, limitations associated with each of these data collection methods. Both questionnaires and interviews are forms of self-report where participants are responding to the questions at hand and are perhaps reporting what they think the researchers want to hear (Creswell, 2003; Johnson & Turner, 2003; Mertens, 2010; Wiersma & Jurs, 2009). Observations are also problematic when trying to infer beliefs from observed behaviour (Lester, 2002). In order to mitigate the shortcomings inherent in any one of these methods, I have chosen to adopt multiple methods for this study.

Beliefs, such as thinking, reside within the mind. It is impossible to look at an individual and infer that this is what she/he believes or thinks. Although I am not exploring the link between learning and what children say and do in the classroom, I have followed Nuthall's methods as a way of matching children's beliefs with what they say, especially to their peers and themselves, and what they do in the classroom. Nuthall's approach to data collection includes the use of fixed video cameras, individual microphones, careful observation, collection of classroom work and interviews (Nuthall, 2001, 2004, 2005, 2007). When I began recording the mathematics classes, I took notes on the contexts of the classes, the choreography[12], plus 15-second interval notes about the focus students (Collins & O'Toole, 2006; Nuthall, 2007). In part, this was to provide a back-up record for later analysis of the recorded footage. I also needed to make sure that I had enough supporting detail to corroborate or contradict findings from other data sources.

Finally, I chose to include a questionnaire not only because it seemed the most common form of collecting belief data, but because, as an inexperienced researcher who was able to find very little in the research literature about New Zealand children's beliefs about mathematics, I felt I needed a general picture of the range of beliefs espoused by these children on which to map the analyses of the more qualitative collection methods. I was concerned about the potential of

12 By choreography of the classroom, I mean the steps that make up the routine of a particular mathematics class, its dance of doing/learning/teaching mathematics.

misinterpreting what I was seeing and hearing without this background information. It was an attempt to validate my understandings of the beliefs by analysing expressions of these beliefs through the different collection methods (Creswell, 2007; Richardson & St. Pierre, 2005; Willis et al., 2007; Yin, 2006). I hoped that by combining methods I would expand my understanding of the range of beliefs about mathematics as well as guard against either over-generalising the findings or drawing unsubstantiated conclusions.

Sampling

Even though researchers attempt to include rigorous and valid sampling techniques, they need to acknowledge that, "because of ethical concerns, all samples are in the end volunteers. In addition, reality constraints such as access and cost must be considered in all sampling decisions" (Mertens, 2010, p. 325). At the present time in New Zealand, access to school children and to collecting data in schools presents particular issues (Snook, 2003). The only method of accessing students was through inviting schools to take part, thus it was impossible to use a random sampling technique. To mitigate bias associated with voluntary samples, I decided to include a large number of Year 5 and 6 students from a wide variety of schools including a range of differing socio-economic areas that might produce representative answers to the research questions (Kemper et al., 2003). By representative, I mean a range of typical responses. The children were between eight and eleven years old although most of them were between nine and ten. I selected this age group, the last two years of contributing primary school, the transitional stage between the primary and the intermediate school years, for the following reasons: they had studied mathematics for at least four years, long enough time to have developed beliefs about the nature of school mathematics; most had enough literacy and communication skills to describe their beliefs; they were pre-teens, an age before the increased stress and peer pressures of adolescence.

Gatekeepers

At the end of 2006, I sent 140 emails to the principals of primary schools in and around a city in New Zealand inviting them to take part in this project. Only seven principals responded. Then in early 2007, I sent a second round of email which I followed up with phone calls. Phoning the schools uncovered some of the problems with gaining access to schools. For many of the principals, this was the first time they had heard about the project because front desk gatekeepers made decisions about what was important for their principals to deal with. Some of the principals supported or were interested in the project, but their head teachers/syndicate leaders/subject leaders were not interested. In other cases, lead teachers were interested, but the principals vetoed access. To address this issue, I only included schools where both the principals and lead teachers were interested in taking part.

Schools sample

For Phase One of this study, a purposive sampling approach was applied to obtain a representative group of schools (Miles & Huberman, 1994), or what Davidson and Tolich refer to as “essential and typical units” (2003a, p. 35) based on “their characteristics and availability” (Wiersma & Jurs, 2009, p. 282). My aim was to include a range of types of schools from the whole gamut of socio-economic neighbourhoods, and a representative range of ethnicities; based on these criteria, I selected 17 of the 24 schools interested in taking part in the study. The following table summarises the information about the participating schools. The schools ranged from small, semi-rural schools to large city ones, from a student roll of 95 to over 600. Most of the schools were state schools but the sample also included three integrated and one independent school.

Table 3.1: Summary information of schools

School #	School Authority	Years	Type	Decile	Number of students	Other
1	State	1-6	Contributing	3	186	
2	State	1-6	Contributing	6	649	
3	Independent	1-8	Full	10	368	
4	State	1-8	Full	9	111	
Kikorangi	State	1-8	Full	4	357	a
6	Integrated	1-10	Composite	6	447	
7	State	1-8	Full	9	106	
8	State	1-8	Full	10	95	
9	State	1-8	Full	4	376	
10	Integrated	1-8	Full	6	239	
Whero	State	1-6	Contributing	10	601	a
12	Integrated	1-8	Full	3	151	
13	State	1-6	Contributing	6	423	
14	State	1-6	Contributing	10	441	
15	State	1-8	Full	1	139	b
16	State	1-8	Full	10	150	
17	State	1-8	Full	2	258	b

Key: a: pseudonyms for the focus schools. Kikorangi is blue and Whero red in Māori.
b: Māori bilingual programme

New Zealand schools fall under three types of authorities: most schools are state schools; a small number are completely independent or private but subsidised by the state; in addition, a small but significant group are integrated, which means they are funded by the state for salaries and operations, keep their special character and own their buildings, but must follow the national curriculum and teacher-hiring practices (Immigration New Zealand, 2006). Primary schools are either full-primary schools (Years 1-8, up to age 12/13) or contributing (Years 1-6, age 10/11). Children from contributing primary schools go on to intermediate schools (Years 7-8) or on to the few secondary schools that have intermediate programmes attached (as is the case for many of the integrated Catholic secondary schools). In addition, a few schools, like School 6, have a Years 1-10 programme.

Table 3.2: Summary of sampled schools by authority and type

School Authority	% of schools N = 17	% of students N = 823	School Type	% of schools N = 17	% of students N = 823
State	76	82	Full primary	65	44
Integrated	18	14	Contributing	29	50
Independent	6	4	Composite	6	6

There was a range in the socio economic status of the schools (SES); however, the sample was skewed towards the higher end. The socio economic status of New Zealand schools is described in terms of decile rankings[13]. For ease of analysis, the ten decile groups have been merged into three: low decile schools which include deciles 1-3, middle decile include 4-7, and high 8-10. The sample included four lower decile/SES schools, six from the middle and seven from the higher level. Figure 3.2 shows how this student population is skewed towards the middle and high decile schools with 40% attending each of these while 20% attend low decile schools, instead of the 27:30:43 for this particular city and environs (Ministry of Education, 2007a) or the expected NZ wide split of 30:40:30.

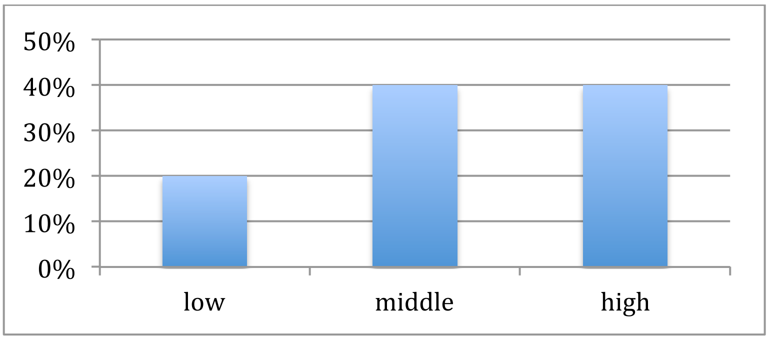


Figure 3.2: SES of students by percent

Students

All students in Years 5 and 6 were invited to participate by completing the Maths Beliefs Questionnaire (MBQ), which is discussed later in this chapter. Gender,

13 Decile, a 1-10 scale, is calculated by looking at the population from which the school draws its students. It is calculated by considering household income, parent occupation, the number of individuals living in households, parent educational qualifications, and the percentage of parents receiving benefits (Ministry of Education, 2009b).

ethnicity and achievement data were collected about these students in order to answer the research questions, especially in terms of whether certain groups would answer the survey questions in significantly different ways.

Eight hundred and twenty-three students in Years 5 and 6 (between 8 and 11 years old) from the 17 primary schools completed this questionnaire. The student sample was almost equally divided for gender with 412 students identifying themselves as female, 406 as male, while five failed to identify themselves. Most of the children identified their ethnicity: 71% as Pākehā (584), 18% as Māori (149), 6% as Asian (51) and 4% as Pasifika (36). The 20 students who identified themselves as other were combined with the Pākehā group (Flockton et al., 2006). The demographic data are presented below in Table 3.3.

Table 3.3: Summary of students by school, year level, gender and ethnicity

School	Number	Year		Gender		Ethnicity			
		5	6	f	m	A	M	Pk	Ps
1	24	1	23	13	11	2	12	9	1
2	86	47	39	49	37	4	19	54	7
3	34	34	-	15	19	4	4	26	-
4	14	5	9	5	9	-	1	13	-
Kikorangi	56	26	30	28	28	4	11	41	-
6	50	-	50	31	19	-	4	43	3
7	28	13	13	13	15	-	4	21	3
8	10	3	7	8	2	-	1	8	1
9	29	16	13	19	10	2	2	22	3
10	33	15	18	17	16	2	4	24	2
Whero	179	94	85	82	95	21	16	139	3
12	32	19	13	18	14	3	4	22	3
13	74	65	9	37	36	4	12	55	3
14	44	-	44	19	25	4	1	39	-
15	38	19	19	15	23	-	17	18	2
16	20	7	13	6	14	-	-	20	-
17	72	33	39	37	33	1	37	29	5
Total	**823**	**397**	**424**	**412**	**406**				

Key: A = Asian
M = Māori
Pk = Pākehā
Ps = Pasifika

The ethnic breakdown of students attending schools in this city during 2007 were as follows: 79% Pākehā, 11% Māori, 5 % Asian, 3% Pasifika and 25% other, which seems to indicate identification in more than one group (Ministry of

Education, 2008b). As these statistics include all registered students from New Entrants to Year 13 rather than restricted to Years 5 and 6, it is difficult to compare the percentages to decide exactly how representative this sample is. My sample may have a higher percentage of Māori children because two of the schools that took part in this study had Māori bi-lingual programmes.

Existing mathematics achievement data were collected from the schools. Achievement levels in terms of the Numeracy Project Stages were available for 86% of students based on the Numeracy Project Assessment (NumPA) Diagnostic Interview (Ministry of Education, 2006a), and Progressive Achievement Test (PAT) Mathematics stanines for 47% (Darr, 2006; Reid, 1993). NumPA scores were available for 706 students with a median score of Stage 5. The PAT Mathematics stanines for 387 students had a mean of 4.98 and standard deviation of 2.07. However, collecting achievement data was problematic as schools store this information in a variety of ways and gather the information through different means, ranging from standardised tests like the Progressive Achievement Test of Mathematics (PAT: Mathematics) to informal interviews and class work. In addition, some schools were not able to supply information on either of these two measures.

The achievement data were further examined to explore whether the differences in mean scores could be explained in terms of gender, ethnicity and the school SES. I undertook this exploration because I was concerned about drawing conclusions about the relationship between achievement and mathematics beliefs response patterns without understanding the complexity of the issues associated with achievement data and these factors (Caygill & Kirkham, 2008; Flockton et al., 2006). Before these analyses took place, the scores on both achievement measures were recoded and merged into three groups. For the NumPA stages 1, 2 and 3 were labeled as low, 4,5,6 middle, 7 and 8 as high. The PAT stanines were merged into low (1-3), middle (4-6) and high (7-9). T-tests for gender as well as ANOVAs for ethnicity and achievement were run for both achievement measures. .

Sub-sample

In order to access a more detailed picture of a range of children's beliefs, I chose two focus schools by using a purposive technique (Cohen, Manion, & Morrison, 2007; Miles & Huberman, 1994) where samples are selected "with the goal of identifying information-rich cases that allow them [researchers] to study a case in depth" (Mertens, 2010, p. 320). One of the issues confronting me was how to obtain the greatest variation when choosing two focus schools in order to access a range of beliefs that may indicate differences worth discussing. With this in mind, I identified schools with different population bases, SES, size, type and philosophies. Both focus schools were state schools, within the environs of the city, whose principals and teachers were happy to grant me access to classrooms, teachers and students.

Focus Schools

In the following subsection, I introduce the focus schools and classrooms. A more detailed description appears in Chapter 6, Narrowing the Focus. All the names of the schools, teachers and children are pseudonyms. I chose names for all the participants except those in the two focus classrooms who selected their own. (More detail about the focus schools is included in Appendix A.)

Characteristics of the focus schools:

Kikorangi School: Full-primary (Years 1-8), 357 students, Decile 4, discovery and restorative justice focus. The teachers usually wore casual clothing, jeans were not uncommon, and the students called teachers by their first names. Homework was optional.

Kikorangi had three composite Year 5/6 classes. One of them operated as a separate entity while the other two, of over 45 students, were team-taught by Ginny, Head of mathematics, and Ron, by using a large classroom/teaching space and a smaller one for break-out groups. The class was divided into two for mathematics: the higher achieving students accompanied Ron to the smaller classroom; the rest remained with Ginny.

Whero School: Contributing-primary (Years 1-6), 601 students, Decile 10. Teachers dressed conservatively and were addressed by title and surname, as in Ms Lillywhite or Mr Greensleeve. Homework was obligatory, and non-compliance resulted in consequences such as staying in during interval. Whero had six Year 5/6 classes and operated a mathematics *interchange* programme. During the same mathematics lesson time, students attended streamed mathematics classes. The school described these classes as one very accelerated class, an advanced class, two mid-level ones, a slightly lower one, and the lowest, a smaller, very slow moving class.

Focus Classes

As with the focus schools, access to one focus class per school was negotiated for further data collection. At Kikorangi, the head of mathematics suggested Ron as a willing teacher, and he agreed to participate. Whero invited me to continue researching at the school, leaving the decision of a focus class to be negotiated between the individual teachers and me. I informally observed all six mathematics classes and spoke to all of the teachers. After these discussions, three teachers volunteered: the teachers from the accelerated class, the lowest streamed class and a class in the middle. Even though I was tempted to study the two extreme classes, I had already negotiated access to Kikorangi, and needed a class that would facilitate comparison between the two schools. For this reason, I decided on Room 6, Charles Forrest's[14] "middle stream" Year 5/6 class. But by the time I returned to the school to collect the video and audio recordings, the nature of Room 6 had altered slightly: the school had decided to reconfigure the two middle classes, the ones with averagely achieving students, into separate Year 5 and Year 6 classes, with the Year 5s in Mr. Forrest's room. These two focuses mathematics classes could be considered atypical because both of their teachers were male. However, even though there are many more female than male teachers in New Zealand primary schools, male teachers are more prevalent in Years 5 through 8 than during the early years; at Whero, half the Year5/6 teachers were male.

14 Pseudonym

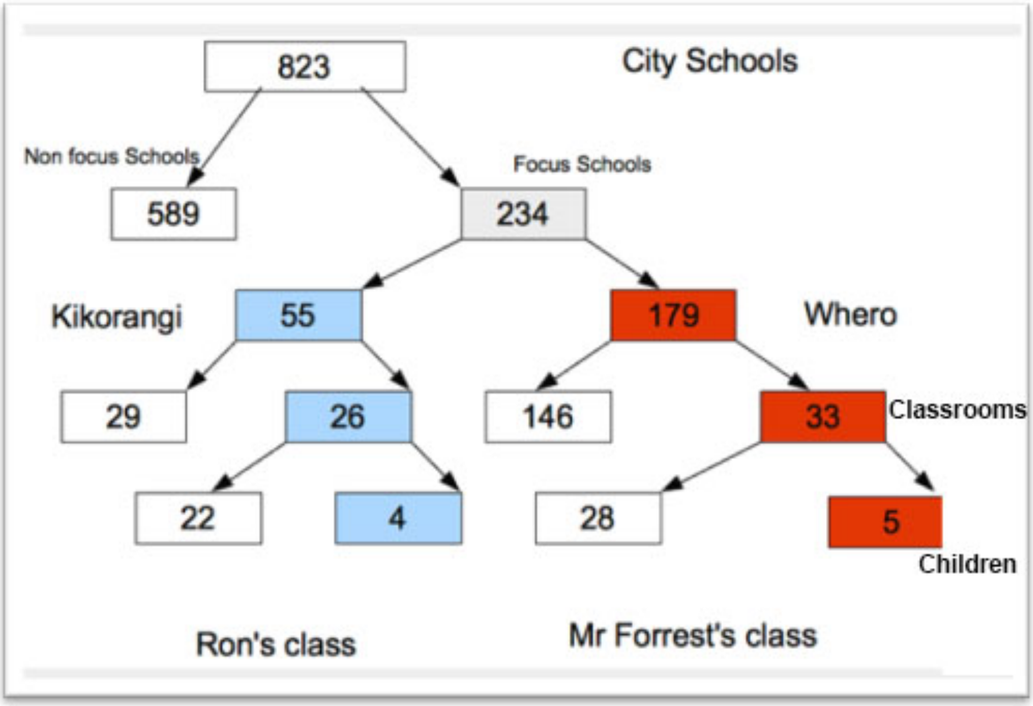


Figure 3.3: Student sample with nested sub-samples

Figure 3.3 summarises the student sample and illustrates how the sub-samples nest within the large sample (Miles & Huberman, 1994). The large sample of 823 children from the 17 City Schools incorporates the 234 at the two focus schools, 55 from Kikorangi and 179 from Whero. From Kikorangi, 26 children were in Ron's class, four of whom formed part of the focus student group. From Whero, 33 children were in Mr. Forrest's class, five of whom were part of the focus group.

Focus Students

I applied a purposive approach to choosing this embedded sample. The choice of focus students from each of the two focus schools was based on Nuthall's method of data collection and analysis (Alton-Lee & Nuthall, 1998; Nuthall, 2001, 2007). The criteria used when choosing the focus students were the following: parental and student consent; a gender mix; at least one student from each of the three mathematics groups (low, middle and high groups) in each class; and an ethnic mix of students in order to include as diverse a population as practicable.

Table 3.4: Focus student characteristics

School	Kikorangi	Whero
gender	2 female, 2 male	3 female, 2 male
group	2 high 1 middle 1 low	2 high 1 middle 2 low
ethnicity	1 Asian 1 Māori 2 Pākehā	1 Asian 4 Pākehā

All of the nine focus children chose their own pseudonyms for the study. The focus children at Kikorangi were Fred, George, Jasmine and Sammie. The focus children at Whero were Caroline, Chloë, Jack, Harry and Lilly.

Teacher Sample

The teacher sample was included to provide additional perspectives to this study of beliefs about the nature of mathematics and who is good at it, as well as beliefs about teaching and learning mathematics in the New Zealand school context. This information was collected to explore whether it was similar to that gathered from children at the two schools (Mertens, 2010; Yin, 2006). Furthermore, the data from this sample is worth considering in terms of what is known about the effects of teachers' beliefs on how they teach, present materials, choose tasks, interact with students, etc., which in turn influence the beliefs of the students in their classes (De Corte, Op 't Eynde, Depaepe, & Verschaffel, 2010). (For more detail on the teachers involved in this study see Appendix B.)

The teacher sample was made up of seven teachers from Kikorangi and 14 from Whero, who agreed to complete the teacher Maths Beliefs Questionnaire (now referred to as TMBQ) that was very similar to the one the children answered. The sample included the teachers of the two focus classrooms. In 2008, I returned to the two focus schools and observed the focus students in their new mathematics classrooms. The nine students were scattered among eight different teachers in 2008, three in Year 7/8 and five still in Year 5/6. These teachers completed the questionnaire before I entered their mathematics classes to observe; they also allowed me to interview them.

Consent Process

The consent process was based on the belief that ethical research presupposes that the participants voluntarily agree to take part based on clear and complete information of both the purpose of the research and what is required of them as participants (Christians, 2011). I discussed my approach to ethical research in Chapter 2, sub-heading Ethical Considerations. For this study, information sheets and consent forms were distributed to the participating schools. Permission was sought from Boards of Trustees, principals, teachers, children and their parents/caregivers. The children's information sheets and consent forms were read aloud and discussed with each group or class of students either by the classroom teachers or by me. The students were encouraged to ask questions about the research project and the process of data collection. Only students who had agreed to take part, and who had permission from parents/caregivers, were included in the study. All signatories knew that they could revoke their consent at any time during or after data collection (An example of a consent form is included in Appendix C).

The majority of the 17 schools involved in the first part of the data collection followed a consent process that entailed:

- Information sheets were distributed to schools, teachers and parents/caregivers
- Consents were collected from schools, teachers and parents/caregivers
- Information sheets and consent forms were distributed to children and discussed. Children signed the consent if they wished to take part.

Two other processes were followed at schools that had different consent protocols. These schools used the same procedure for distributing information to principals, teachers, parent/caregivers and children, as well as for collecting consent forms from principals, teachers and children. The differences lay in whether and how consent was received from adults responsible for the children.

A group of schools had the mathematics classroom teachers sign *in loco parentis* for their charges. One school used an "opt out" protocol where parents/caregivers were asked to sign forms only if they did not want their children to take part in the research.

The consent process used for data collection in the two focus classrooms was the same as that followed by the majority of the schools. An alternative class at each of the focus schools was organised for children who either chose not to take part or whose parents/caregivers refused consent to have them video-recorded. Because every member of the two focus classes had the requisite consents, the alternative arrangements were not needed.

However, some issues arose from the consent procedure despite the care taken to mitigate expected problems. There was no problem when the adults as parents or *in loco parentis* gave consent, but some students did not want to take part. I, therefore, followed the wishes of the students and respected their autonomy. However, when students chose to take part, but they did not have adult consent, either because the adult refused consent or had failed to return the forms, I was faced with a dilemma. In these cases, in terms of the University Human Ethics Committee approval, I was obliged to disregard the students' wishes; in effect, the children's voices and opinions were stilled. In these instances, I allowed the students to have their say on the MBQ, but the data were not included unless the required adult consent was received within the following month. In each of these cases, I sent a follow-up note home with the children. A second quandary surrounded the concepts of confidentiality and anonymity, and I began to question for whom were these principles ethical. The children in almost every classroom where we discussed the information sheets and the consent forms were extremely disappointed that they would not be identified by name and by school. They seemed to feel that by not being given their true names, their contributions were not really being acknowledged. They were giving me their time and were prepared to give carefully considered answers, yet I was not respecting them enough to recognise them as real individuals (Emond, 2005; Hill, 2005). This question about anonymity was the most common question, and I had to explain my interpretation of the reasons in almost every classroom I visited. This was less of a problem in the two focus classrooms because the students selected their own pseudonyms; thus they felt they retained some control and power over their contributions.

Data Collection

The methods utilised in this study were the following: questionnaires, observations which included both field-notes and video recordings of the two focus classrooms, collection of classroom work/tasks (artifacts) as well as interviews with the nine focus students and their teachers. However, once the first data collection phase was underway, two problems arose with how the students answered one particular question on the MBQ, the *Alien Task* (Q33) that led me to include the drawing task for the students at the two focus schools (See the section: Drawings, p. 81). My decision to include drawings as an important data source is described and discussed in Chapter 5. The following data map (Figure 3.4) summarises the data collection procedures, the samples involved and the times they took place.

In 2007, I collected mathematics beliefs questionnaire data from 17 schools and drawings from the children at the two focus schools. In addition, I observed mathematics lessons at the focus schools and recorded the teaching and doing of mathematics in the two focus classrooms. In 2008, I returned to Kikorangi and Whero to observe and interview the focus children. All of these children completed the questionnaire for a second time, and eight agreed to draw new maths pictures to see if there had been any major change in their beliefs over the year. At this time, I interviewed the focus children's current mathematics teachers as well as Ron and Mr. Forrest.

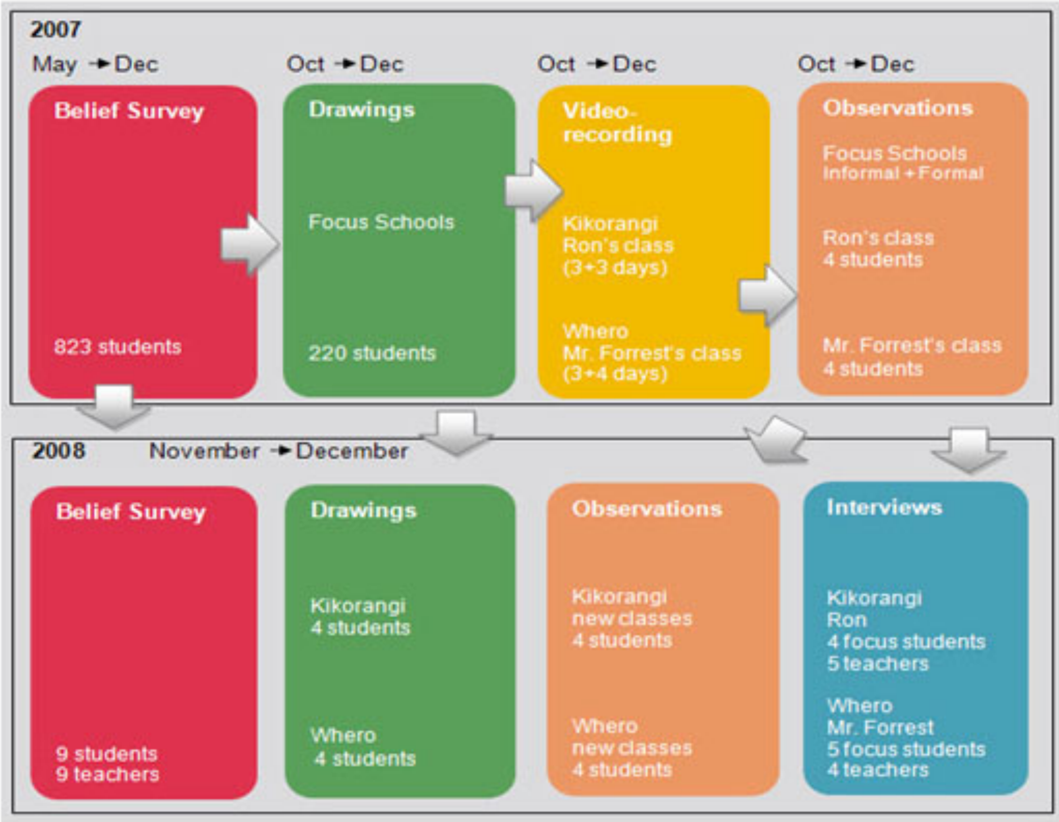


Figure 3.4: Data collection map

Questionnaires

Questionnaires were chosen as a data collection strategy for gathering mathematics beliefs information from a large sample of students and their teachers. It was an expedient strategy for a single researcher with limited time and money, and it had the potential for easy analysis (Johnson & Turner, 2003). If well designed, the questionnaire would be easy to administer and not stressful for most of the children to answer (Greene & Hill, 2005; Punch, 2002). However, much of the criticism leveled against the use of questionnaires focuses on the issues of self-report, problems associated with giving the researcher what the respondents perceive the researcher(s) want, low response rates (Cohen et al., 2007; Mertens, 2010), and problems with analysis of open questions (Johnson & Turner, 2003; Mertens, 2010). In addition a questionnaire is a one-off strategy limited by time and context; at another time, the same participants may answer the questions differently (Creswell, 2008; Davidson & Tolich, 2003b). There may also be issues with the statistical techniques applied Likert-scales as if they were interval rather than ordinal scales (de Winter & Dodou, 2010; Jamieson, 2004; Norman, 2010), which are discussed later in this chapter. The criticism of questionnaires—self-report, wanting to please the researcher, and the potential difficulty with analysis of open-questions—is no different from those against any other qualitative data collection strategy, such as interviews. Low response-rate was not an issue in this case because the researcher administered the student questionnaires at schools.

Initially, the hope was to find an instrument that had been trialled and would be applicable both in terms of the research questions and the New Zealand context. Ideally, this instrument would include a mixture of open and closed items to reflect the complementary methods approach employed by this study (Johnson & Turner, 2003). Another option was to find a questionnaire that could be adapted to fit the New Zealand context, which contained some questions related to this study. As a result, a variety of questionnaires, scales and questions were reviewed, some in mathematics and others related to more general beliefs about knowledge and knowing (Barlow & Reddish, 2006; Bong, 2004; Chalke, 2007; Fennema & Sherman, 1976; Franks, 1990; House, 2006; Jinks & Morgan, 1999; Schoenfeld, 1985; Schommer-Aikins, 2002; Schommer-Aikins et al., 2000; Schommer-Aikins et al., 2005; Schommer-Aikins & Easter, 2006; Seaman, Szydlik, Szydlik, & Beam, 2005; Skaalvik & Skaalvik, 2006; Tuan, Chin, & Shieh, 2005; Vanayan, White, Yuen, & Teper, 1997). These questionnaires or scales neither matched the range of beliefs that were of interest nor were they appropriate for the New Zealand classroom context: I found none of these surveys asked children questions about ethnicity and gender beliefs that were of interest based on the prevailing achievement discourses discussed in Chapter 1. Other surveys and scales included very narrow items such as beliefs about speed and success; and many of them were geared to the United States context.

There was some tension between wanting to access what children and teachers believe and how they might explain or construct their beliefs in their own words, and pre-determined statements chosen by adult academics and/or researchers as representative of these beliefs, a tension between using open and closed questions. Asking the children to agree/disagree with pre-constructed statements is not the same as empowering them to construct their own statements from their "own frame of reference" (Bogdan & Biklen, 2003, p. 3). The solution, in this case, was to try to balance the types of questions by including closed items, which are easier to analyse, with open ones that allow the individuals to use their own words and expressions in order to explain what they think. An additional concern based on my experience was that New Zealand primary children of this age are not as familiar with closed questions as some of their foreign counterparts. By age 9 or 10, most of them have not been exposed to Likert-scales and the only experience they have had with closed-questions is on a handful of assessment tasks such as the PAT (Reid, 1983) and e-asTTle (http://e-asttle.tki.org.nz/) tests.

Development and trials

I designed two versions of the Maths Belief Questionnaire (MBQ), one for children (MBQ) the other for teachers (TMBQ). The two versions of the questionnaire were designed to be very similar in order to compare the responses from these two groups. Copies of these questionnaires are included in Appendix D.1and D.2.

The goal of the questionnaires was to discover what these children and teachers believe about the world of mathematics and how the participants viewed themselves and others as either being part of, or excluded from this world. In the initial design phase, the survey items were divided into *epistemological beliefs*, both those related to personal epistemology and to the nature of mathematics, *self-beliefs* or how individuals judge themselves as being good/not good at mathematics, and *self-efficacy beliefs*, what sorts of mathematics tasks or types of mathematics they believe they are good at, (Bandura, 1997) with some items straddling more than one category. From the beginning of the design phase, it became obvious that certain questions overlapped belief categories. For instance, if Tui identified herself as female and Māori, then Student Questions (SQ) 6 (*Girls are good at maths.*) and 8 (*Māori students are good at maths.*) could be read either as *self-belief* or *epistemological belief* responses. As the project evolved, the focus shifted slightly to the following three categories for interpreting the responses: beliefs about the *nature of mathematics* (epistemological beliefs, e.g., SQ 33: *If you met an alien who had never done maths, how would you describe maths and what it is about?*), the *individual and mathematics* (me and the world of maths, e.g., SQ 4: *I can do maths.*), and others and mathematics (e.g., SQ 5: *Boys are good at maths.*). A further iteration of belief categories including beliefs about the *nature of mathematicss*, about the *self and mathematics*, about *mathematics ability*, and about the *learning environment* (LE, e.g., SQ 3: *Everyone can learn maths.*) was developed after conducting a factor analysis on the closed item (Chapter 4). (See Table 3.5).

Table 3.5: Three categorisations of mathematics beliefs by question

Student Questions	Teacher Questions	Question types	Original Beliefs	Revised Beliefs	Factor Beliefs
2	1	closed, Likert, 5-point	Personal Epistemological	Epistemological	nature
3	2	closed, Likert, 5-point	Personal Epistemological	Epistemological	L E[15]
4	3	closed, Likert, 5-point	Self-belief	Self-belief	self
5	4	closed, Likert, 5-point	Personal Epistemological / Self-belief	Myths and self-belief	ability
6	5	closed, Likert, 5-point	Personal epistemological / self-belief	Myths and self-belief	ability
7	6	closed, Likert, 6-point	Personal Epistemological / Self-belief	Myths and self-belief	ability
8	7	closed, Likert, 6-point	Personal Epistemological / Self-belief	Myths and self-belief	ability
9	8	closed, Likert, 6-point	Personal Epistemological / Self-belief	Myths and self-belief	ability
10	9	closed, Likert, 6-point	Personal Epistemological / Self-belief	Myths and self-belief	ability
11	10	closed, Likert, 5-point	Epistemological, Nature	Epistemological	nature
12	11	closed, Likert, 5-point	Epistemological, Nature	Epistemological	nature
13	12	closed,Likert, 5-point	Self-efficacy	Other	self
14	13	open	Self-efficacy	Self-belief	L E
15	14	open	Personal Epistemological	epistemological/ int?	L E / nature
16	15	open	Epistemological, Nature	Epistemological	nature
17	--	closed, Likert, 5-point	Self-belief	Interest	L E
18	16	closed, Likert, 5-point	Self-belief	Self-belief	self
19	18	closed, Likert, 6-point	Self-belief	Self-belief	self
20	17	closed, Likert, 6-point	Self-belief	Self-belief	self
21	--	closed, Likert, 5-point	Other	Other	L E
22	19	closed + open	Self-belief	Self-belief	self
23	20	open	Epistemological, nature of	Epistemological	nature
24	21	closed + open	Personal Epistemological	Self-belief	self / group
25	22	closed + open	Personal Epistemological	Self-belief	self / group
26	23	open	Epistemological, Nature	Epistemological	nature
--	24	open	Epistemological, Nature	Epistemological	nature
27	25	open	Self-belief, Epist	Self-belief	self
28	26	open	Self-efficacy	Other	self/ L E
29	27	open	Self-efficacy	Other	self / L E
30	28	open	Self-efficacy	Self-belief	self/ nature
31	29	open	Self-efficacy	Self-belief	self / nature
32	30	closed, 3-point	Self-efficacy/ self-belief	Self-belief	self
33	31	open	Epistemological, nature of	Epistemological	nature

15 Learning Environment

To find items that would answer the research questions, I adapted activities, ideas and questions from the literature (Barlow & Reddish, 2006; Flockton, Crooks, Smith, & Smith, 2006; House, 2006; Seaman et al., 2005; Stodolsky, Salk, & Glaessner, 1991; Vanayan et al., 1997; Young-Loveridge, Taylor, Sharma, & Hāwera, 2006) as well as my experience as a teacher. Questions (SQ) 4, 30 and 31 are similar to activities I used in my classrooms, while the questions about genetic predispositions towards mathematics (SQs 24 and 25) were suggested by conversations both with adult literacy students and pre-service teachers. SQ items 2 through 10 are an attempt at teasing out identity beliefs, beliefs about who naturally inhabits the world of mathematics. These items are also associated with popular discourses about the inclusionary/exclusionary nature of the domain, as well as discourses about certain groups, identified either by gender or ethnicity, that excel or fail in this field. Eight questions come directly or were adapted from the National Education Monitoring Project (NEMP) mathematics survey (Flockton et al., 2006). Thus, the items in the final version of the questionnaire came from an eclectic range of sources (See Appendix E).

The first trial of the student questionnaire was very informal. Six children, borrowed from neighbours and my supervisors, answered the first version. Four of the children were in the target age range of 9-10, one was a year older and another, a year younger in order to check the readability of the items. Four of the children gave me extensive feedback on what they thought worked and what did not. This initial version of the Student Belief Questionnaire comprised 30 items, three of which were closed questions (See Appendix F). The closed items included three categories for the closed responses (true, false, not sure, or yes, no, not sure). The second version, as a result of the trial and advice from my supervisors, included 18, 5-point Likert-scale items and four three-category ones.

The second version was trialled in two classrooms in a local primary school. Consent was signed by parents and children. After this trial, I discovered that the children did not understand the language commonly used on Likert-type questions; as a result, I had to use more child friendly words such as 'heaps' in the place of ‘a great deal’ and ‘suck’ instead of ‘awful’. In addition, a 'don't know' option was added to six questions. Question 2 was dropped as the children found it difficult to discriminate between questions 2, 3 and 4 (‘Everyone can do maths’, ‘Only some people can do maths’, and ‘Everyone can learn maths’). The final student version had 19 closed questions, which included 17 Likert-scale items – 11 with a 5-point scale and six with a 6-point (including a 'Don't know' category)– plus four 3-point items, and 14 open questions. See Appendix D for the wording of the questions. The reliability of the closed questions was checked and found to be acceptable with a Cronbach’s Alpha = .73 (Bryman & Cramer, 1997; Cohen et al., 2007; Field, 2006).

The Teacher Belief Questionnaire was edited to correspond to the student one where possible (See Table 3.5.). A version similar to MBQ version 2 was first trialled on colleagues and later at the same school as the student trial. The only other change resulted from some teachers’ reluctance to fill in their ages; as a result, the item was edited to include age brackets, which they could circle.

On the whole, the closed questions, which included the Likert items, were designed to ask the respondents to judge how well they rated themselves and others, in terms of 'being good at maths', being able to learn it, do it, how difficult they find it, as well as the questions that attempt to tease out the popular discourses/myths surrounding gender and ethnicity and mathematics. The open questions ask what, who and how questions. The SMBQ consisted of a combination of 21 closed questions, 14 open questions and general information ones, such as school, class, mathematics teacher, gender, ethnicity, age, school year (grade). The TMBQ version comprised 19 closed questions, 16 open questions and general information questions including gender, ethnicity, age, classes taught, years of experience and willingness to take part in the next stage of the research project. The teacher version was very similar although the language was slightly less colloquial: ‘suck’ was replaced by ‘awful’, and ‘heaps’ with ‘a great deal’. Other differences included changing the focus in order to be appropriate for the different audiences, e.g., SQ15: *What kinds of maths activities* *do you like doing best?* with TQ 13: *…. do you like teaching best?;* and SQ 33 and TQ 31 ask the respondents to describe mathematics, but the student version has the question imbedded in a narrative (*If you met an alien who had never done maths, how would you describe maths and what it is about?).*

Administration of the MBQ

The MBQ was administered at 17 schools. At 14 of these schools, I administered the questionnaires on my own, at two schools some of the mathematics teachers helped, and at one a colleague assisted. The research was explained to the children, and they had time to ask questions and to discuss concerns. A reader-writer was available for any child who had difficulty with literacy or fine motor skills. Children were encouraged to ask questions at any time during the activity, and were allowed to talk to friends. At the end of the task, the questionnaires were checked for inadvertent gaps and where possible the authors were asked for clarification if necessary.

The conditions for the administration differed from school to school. In the majority of schools, I visited individual classrooms where the children who wanted to take part and who had permission completed the questionnaire in the classrooms. At other schools, the children taking part in the research went to the staffroom, the library, a spare meeting-room or classroom, and completed the questionnaires away from those not taking part. A third method of collection involved three or four classes being sent to the school hall, or in one case to a single classroom.

Observations and Fieldnotes

At different points in this research journey, I employed different types of observations from informal general information collecting to more detailed recording of what specific teachers and students did or said at particular times. Most of these observations took place in 2007 during the administration of the MBQ, the classroom observations at the two focus schools, and during the recording of mathematics lessons in the two focus classrooms. In 2008, I returned to the two focus schools in order to observe the focus students in their new mathematics classes. This presented the opportunity to have a slightly longer focus in order to discover whether their beliefs and performances had changed over a year in another context, with different teachers, different experiences and perhaps different peers.

During Phase One of this study, when I began visiting schools, classrooms and staffrooms in order to explain the research project, discuss the consent process and field questions, I took notes about the schools, and the sorts of concerns that were mentioned by school administrators, the teachers and the children. While involved in administering the questionnaires, I observed how students responded at the different sites and in different contexts. In part these observations were influenced by the different experiences of the students: some were in their usual classrooms with a familiar class teacher in attendance, others were in small pull-out groups, or very large noisy ones in unfamiliar locations like a boardroom, staffroom, library, the hall, etc., with an unfamiliar adult whom they were expected to trust.

Once the Second Phase of the research began, I spent an extended amount of time at the two focus schools. In order to understand children's and their teachers' experiences of the mathematics classroom, I needed to familiarise myself with the norms and rituals of these classrooms (Cobb, 2007; Nuthall, 2005, 2007). In order to become familiar with the primary mathematics classroom, I observed its choreography, routines, tasks and roles. I needed to understand how the primary mathematics classes worked before I could begin to interpret the mathematics beliefs data I was collecting. During this process, I asked myself the following questions: What do teachers and children do in these classrooms? How do they behave? What is the curriculum? How does it work? To begin with, I sat in classes and just tried to follow the elements that made up the mathematics class, jotting down notes and sketching student and teacher movements around the classrooms. At Whero, these observations and fieldnotes assisted me in deciding which of three classrooms would become the classroom of interest; that is, the one that included children achieving at a similar level as those at Kikorangi focus class. By the time I began the more detailed study of the two focus classroom (Phase 3), I was familiar enough with the workings of Year 5 and 6 mathematics classrooms to become more strategic with both my observations and note taking. I wanted to understand the particular routines of the focus classes and how the ability/achievement groupings worked within these classrooms.

During these observations, I struck all of the challenges Creswell refers to:

> remembering to take fieldnotes, recording quotes accurately for inclusion in fieldnotes, determining the best timing for moving from a nonparticipant to a participant, and keeping from being overwhelmed at the site with information, and learning how to funnel the observations from the broad picture to a narrower one in time (2007, p. 139).

The role of the observer differs depending on the purpose and type of research being conducted: according to Johnson and Turner (2003), the roles range from "complete observer" through "observer-as-participant" and "participant-as-observer" to "complete participant" (p. 313), depending on the kind of observing and how naturalistic the setting. At the beginning, I was an outsider looking in and, at other times, much more of a participant observer (Creswell, 2007); however, when I attempted to return to acting as a detached recorder of behaviour to accompany the video-recording in the focus classrooms, I was less than successful because both the children and their teachers had accepted me as an almost-insider, someone who belonged, and they treated me as an extra helper, even teacher, in the classroom, someone with whom to chat, to share experiences, or from whom to get help.

Drawing task

As previously indicated, two issues arose from the MBQ that led to the addition of a drawing task as a data collection method. Many students, even those who had the literacy skills to complete the questionnaire, were disinclined to write much, especially for the *Alien Task* 'what is maths' question. The second problem was that their responses to the Alien Task presented a very limited picture of their mathematics epistemological beliefs, describing mathematics predominately in terms of number. My dilemma was whether they really believed that mathematics was limited to number, or whether they would describe the subject in broader and/or different terms given a different type of task. In order to explore the possibility of engendering different kinds of responses, I introduced a drawing task. This decision was influenced by one of the children asking if he could draw instead of write the answer to the original question, and by others who added drawings to illustrate their responses. Only the children at Kikorangi and Whero, the two focus schools, completed the drawing task. I present a detailed description and discussion of this drawing task as well as the justification for its inclusion in Chapter 5: *Reading the drawings*.

Recordings

As part of my study of how children's and teachers' beliefs about mathematics play out within the classroom context, I recorded a series of mathematics lessons in the two focus classrooms. I was particularly interested in the children's experiences of being mathematical, of doing and learning mathematics within ordinary primary school classrooms (Collins & O'Toole, 2006; Nuthall, 2007) For the recordings I followed the method developed by Graham Nuthall which utilised cameras, microphones and recording equipment developed by Roger Corbett at the University of Canterbury (Nuthall, 2007). Cameras were set up on the ceiling of the classrooms: one focused on the mat or main teaching area of the classroom; the others pointed at areas where maths groups usually worked. Each child and the teacher wore a personal microphone that they could turn on or off as they saw fit. In order to familiarise the children and teachers with the equipment, I set it up a week before recording began. The following week, I began recording lessons to habituate the children and teachers to how all the equipment and monitors worked. The recordings that were used for data collection took place in the final week of recording once the children were used to having strange equipment in their classrooms. Only the two teachers and the focus students had live microphones; however, neither the teacher nor any of the children knew which of the identical looking microphones were operating at the time of the recording.

Artifacts

In order to make sense of the classroom recordings, of what the children were saying and doing, and of my observations, I collected copies of the tasks the children were working on in their mathematics classes. A set of tasks was made for each of the 33 students in Mr. Forrest's mathematics class at Whero. In contrast, the members of Ron's class at Kikorangi were more casual about completing work in their exercise books: they used scraps of paper, the backs of other subject worksheets, or anything they could lay their hands on, which often did not find its way back to maths books or folders. Their teacher was more interested in student engagement in the classroom and activity than in neat exercises in books. Ron concentrated on wandering around the classroom and recording the skills his students were achieving competence in. As a result, my collection of the children's mathematics work was more complete for Mr. Forrest's class than for the Ron's.

Interviews

In 2008, I audiotaped interviews with the individual focus children and their teachers. The decision to use interviews was based on the belief that they would be helpful in clarifying the written responses, drawings and/or recorded data by confirming or contradicting, expanding or explaining these data (Cohen et al., 2007; Johnson & Turner, 2003). They would also help me to flesh out the impressions gained about students' and teachers' mathematics beliefs and classroom interactions. All of the focus children responded to questions about themselves and mathematics, related to both their 2007 and 2008 maths experiences. They also commented on their drawings and on video-cued clips (O'Toole, 2005) of themselves in their 2007 mathematics classrooms. In order to show the video clips to the individual children, I took a laptop loaded with the appropriate videos to the interviews. The open interview questions were designed so there were opportunities to ask for clarification and expansion. The interview procedure and a question guide for the focus students is included in Appendix H. The two focus teachers, Ron and Charles Forrest, were asked about their mathematics beliefs in general, their responses in the questionnaire and their experiences with teaching mathematics (Appendix H.1). They were shown video-clips of each of the focus students in their 2007 classrooms and were asked to respond to them Appendix H.2 (e.g., Do you remember George? What sort of student was he? Can you remember this activity?, etc.) and H.3. The focus student's 2008 mathematics teachers were interviewed, as well, but without the use of video-cued clips. (See Appendix I for the 2008 questions). All of the interviews were professionally transcribed, and I checked the transcripts by comparing them to the original soundfiles.

Summary of the dataset

In order to manage a very large data set, I excluded all student participants who were either younger than 8 or older than 11, as well as those who were neither in Year 5 nor Year 6. Because I decided to focus on the children's mathematics beliefs, I eliminated the teachers from the non-focus schools. Rather than include all of the drawings, I included only those from children who had also completed the MBQ so that I could compare the drawings with the *Alien Task* responses. Even though I recorded two weeks of mathematics lessons at each of the focus schools, the first week's recordings were used to familiarise the children and teachers with the recording equipment as well as the process of filming mathematics classes, and thus these recordings were excluded but were part of the background landscape of the study. These data were a way for me to become acquainted with the field and the context of mathematics classrooms. They remained in the background to be consulted if elements in the focus data were unclear. Table 3.6 summarises the reduced dataset that became the focus of this exploration of mathematics beliefs. The final dataset for analysis consisted of a combination of questionnaires, drawings, recordings (both audio and video), observations, interviews and artifacts (classroom maths work).

Table 3.6: Final dataset

method		date	participants	number
SMBQ		2007	17 schools	823
		2008	2 focus schools	9*
TMBQ		2007, 2008	2 focus schools	21
Drawings		2007	2 focus schools	180
		2008	2 focus schools	8*
Video- and audio recordings plus notes		2007	Kikorangi	3 lessons
			Whero	4 lessons
Observations, notes	Post-recording	2007, 2008	Kikorangi	5 lessons
			Whero	4 lessons
Interviews		2008	Focus students	9
			Focus teachers	2
			Teachers	9
Artifacts		2007	Focus classrooms	Kikorangi 4
				Whero 5

* These children completed the task both in 2007 and 2008.

Making sense of the Data

The process of systematically dealing with and making sense of the data was similar for all of the sources: the MBQ and TMBQ, drawings, recordings (video and audio), artifacts, observations and interviews. The data were collected, sorted, reduced, interpreted and written about (then re-sorted, re-reduced, re-interpreted, etc.)(Bogdan & Biklen, 2003; Creswell, 2007). This process, however, was in no way linear; instead, it was flexible, recursive and iterative (Braun & Clarke, 2006; Davidson & Tolich, 1999; Schoenfeld, 2007; Tashakkori & Teddlie, 2008). Elements of collection, analysis and writing ran concurrently at times and sequentially at others. Creswell uses the metaphor of a spiral to describe this system of data analysis (See Figure 3.5), which seems to reflect the processes I used in this study.

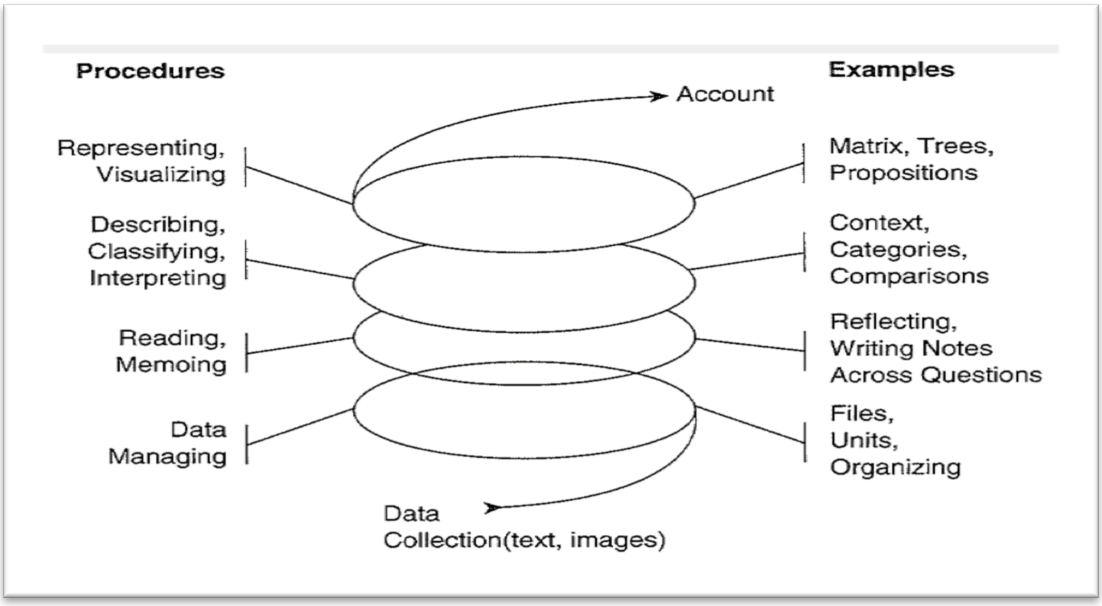


Figure 3.5: Data Analysis Spiral (Creswell, 2007, p. 150)

My spiral seemed much less controlled than Creswell's as it looped both up and down, sometimes missing a revolution or repeating, rather like a stuck record. Every time a new bit of data was collected, I explored it, tried to see patterns, coded it in a variety of ways, made notes, and tried to find different categories, metaphors and themes. Once patterns emerged, these informed the next episode of collection, reduction or categorisation; at other times, new insights forced me to return to earlier analyses and explore them again (Pawson, 2008; Tashakkori & Teddlie, 2008). An example of this resulted in the addition of the drawing task, which is discussed in Chapter 5. An additional illustration was the Maths Beliefs Framework (MBF) that emerged from the statistical analysis of the MBQs (Chapter 4) and became a tool for analysing data from some of the other sources, the drawings for instance (Chapter 5). Another way of looking at how one type of analysis or understanding of data informs or suggests another is in terms of intertextuality where the understanding of one text depends both on the text itself (interview, drawing, questionnaire response, etc.) as well as on other texts (for example, interview and video-recording of doing mathematics in the classroom) (Rose, 2007).

During the process of making sense of the data, I was cognisant of the problem of imposing meaning on the data rather than understanding what was 'there', that is, what the children, teachers and my observations were saying. To mitigate this problem, Mertens suggests that "the researcher begins with specific observations and allows categories of analysis to emerge from the data as the study progresses" (2010, p. 225). Rose takes a different approach towards this issue by suggesting strategies for interpretations:

> 1) looking at your sources with fresh eyes; 2) immersing yourself in your sources; 3) identifying key themes in your sources; 4) examining their effects of truth; 5) paying attention to their complexity and contradictions; 6) looking for the invisible as well as the visible; and 7) paying attention to details (2007, p. 158).

Initially, I tried to follow Merten's advice of going from the specific and allowing more general categories to unfold over time, which was a similar process to that suggested by Rose's strategies 2 and 3. When I later returned to re-analysing these data, I was able to look at the data afresh and incorporate the rest of Rose's strategies.

However organised the process of analysis may seem, with its reduction of data, initial coding, subsequent coding, and emerging patterns and themes, in reality it is much messier. Pawson sees the process of dealing with data, especially where a complementary methods approach is involved, as akin to witchcraft as it attempts to make order of "the weave and weft of the disorderly threads of social inquiry" (2008, p. 120). For me, through an iterative process of analysis, re-analysis, reading and interpreting, patterns, metaphors and themes slowly emerged as if by magic.

Making sense of the Questionnaires

The data from the questionnaires were entered into FileMaker Pro7, scrutinised for accuracy, and later copied into SPSS (Cohen, 1988). The analyses of the data were completed in SPSS19. Background information such as gender, ethnicity, age, school deciles, school class and achievement data were explored and compared to scores and means on the closed questions. A decision was made to treat the data as parametric despite the controversy surrounding the use of Likert-items in this way (Cohen et al., 2007; Jamieson, 2004; Mertens, 2010). This decision was based on the convention of using Likert-items as interval data (Dancey & Reidy, 2004; Field, 2000; Larson-Hall, 2010). I also considered the very convincing arguments on doing so as presented by Norman who compared results from the 'proper' Spearman's correlation (rho) suitable for non-parametric data with the Pearson's *r*, the 'wrong' technique, only to discover that the results were almost identical thus concluding that "parametric methods can be utilized without concerns for 'getting the wrong answer'" (Norman, 2010, p. 625). In addition, Norman pointed out that even though individual Likert items are ordinal, "sums across many items are interval" (p 629) implying that the argument against using parametric techniques with Likert scales lacks logic. De Winter and Dodou also compared results from the non-paremetric with parametric techniques on Likert items, in this case, Mann-Whitney-Wilcoxon with the parametric *t*-test; again the results were almost identical (2010). In addition, I explored the data by using histograms and box plots to check whether the data were normally distributed, another requirement according to statistics purists (Field, 2009; Norman, 2010).

Means were compared by using independent *t*-tests where two groups were involved. Effect sizes were also considered following the recommendation made by the American Psychological Association (American Psychological Association, 2010; Field, 2009) by calculating Cohens' *d* (Cohen, 1988). Initially, the effect sizes were calculated manually; finally, they were checked using an on-line effect size calculator (Becker, 2000). Cohen considered a d=0.2 as small, d=0.5 as moderate and d=0.8 as large. However, in this analysis a d=0.4 is considered an acceptable effect size (Hattie, 2009); even though, as Schagen and Hodgen (2009) point out, judgements about effect size should be contingent on the specific context. One-way analysis of variance, ANOVAs, were used to compare means where more than two groups were involved. Where significant differences between groups were indicated, the Scheffé post-hoc procedure was applied to identify precisely which pairs, if any, had significant differences of means. The Scheffé, the most conservative of the post-hoc tests (Stevens, 1999), was chosen because of size of the sample (N=823) to counteract the problem of a type I error. Effect sizes were calculated for each of these pairs.

In addition, a principal component analysis was conducted in order to identify mathematics belief factors that were compared within and across the databases (Field, 2009). The scores for the four mathematics belief components were summed, and the means on each of these belief factors or subscales were compared. A personal mini-factor was also identified and compared across groups and in terms of demographic information. In this case, gender and ethnicity means were compared. These factors are described in detail in Chapter 4.

Initially, the open-questions were explored by using an open coding strategy which later became more focused (Charmaz, 2006; Mertens, 2010). The 'what is maths' *Alien Task* was coded for all 823 participants while the other open-questions were coded only for the children at the two focus schools. An example of coding categories and how they changed is recorded in Table 3.7. I report and discuss the results from the statistical explorations and procedures in Chapter 4, *The Landscape of Beliefs*, and for some of the open questions in Chapters 5 and 6.

Table 3.7: Coding categories for SQ 33: Alien Task

Initial codes		**Final codes**
content	number	numbers symbols
	other strands	measurement geometry statistics problem solving thinking
affect	affect	positive negative
		attitude
other	difficulty	easy hard
		teaching/learning
		metaphor
	"dunno"	"dunno"

The initial coding categories were very broad and included *content* which was later expanded to include *number* and *other strands* (e.g., geometry, etc.), *affect*/feelings and *other* which included any reference to the difficulty/easiness of mathematics, as well as the "dunno" response for those that either had nothing to say on the item or who wrote they did not know what maths is or how to describe it. The expanded, final categories were much more specific and informative.

Making sense of the Drawings

The drawings were analysed and interpreted in a number of ways. In the initial analysis I compared the results from the *Alien Task* with the drawings. The *Alien Task* was coded first, then the drawings, and finally the categories were compared and amalgamated. An example of this process follows (Figure 3.6 and Table 3.8):

Student 11.1.24's[16] *Alien Task* response:

> *"boring, toeasy [too easy], stoped [stupid]"*

16 Student identity codes: the first number indicates the school, the second the classroom, and the third the student.

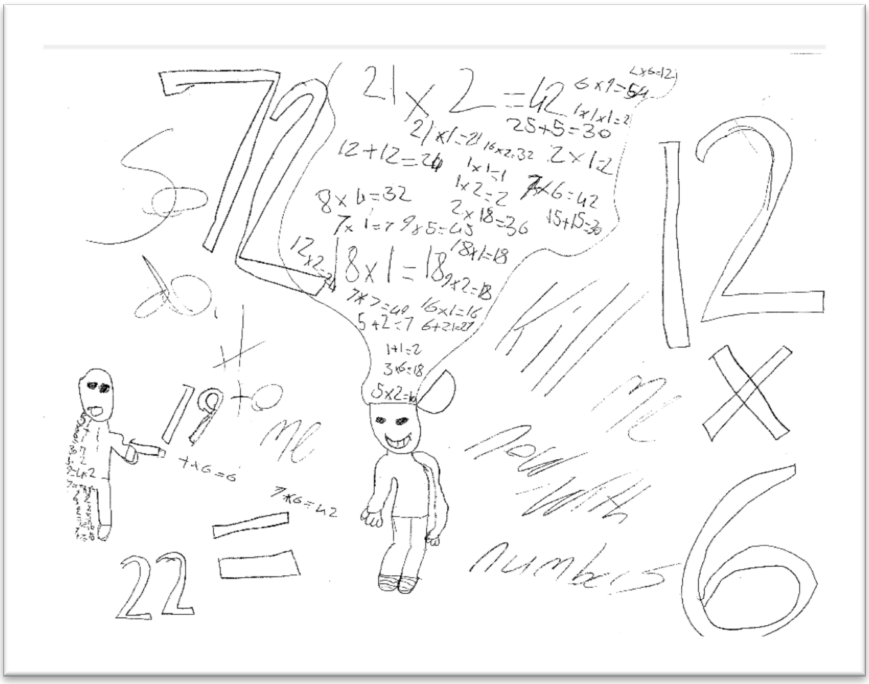


Figure 3.6: Student 11.1.24
[Not to be reproduced without permission.]

Table 3.8: Codes for Student 11.1.24 comparing Alien Task (AT) with drawing

Codes	AT		drawing		
content	x		✓	number✓ symbols✓ algorithms✓	
metaphor	✓	boring✓	✓	brain✓ kill me✓	processing✓ exploding✓
affect	✓	-ve✓		✓	+ve/-ve?
difficulty	✓	easy✓		?	

Key: ✓ = present, x = absent, ? = unknown
+ve = positive response
-ve = negative response

The coding on the *Alien Task* indicates no mention of mathematics *content* or strands as opposed to the drawing which includes numbers, symbols and algorithms. Both responses reference *metaphors*, “maths is boring” in the *Alien Task* and brains associated with processing algorithms as well as exploding in the drawing. For the *affect* category, the written response is negative while it is unclear in the drawing with the smiling boy. A similar lack of clarity is present for the *difficulty* category, the *Alien Task* indicates easiness while this is unclear in the drawing.

For the second iteration of the drawing analysis, I applied the Mathematics Beliefs Framework developed from the factor analysis and based on the closed-questions. The framework identified beliefs about 'self as doer of maths', beliefs about 'others as doers of maths' (achievement or ability beliefs), beliefs about the learning environment, and beliefs about the nature of mathematics. This framework is discussed in Chapter 4. Table 3.9 illustrates this framework as applied to Student 11.1.24's drawing. From this analysis, I could infer that the student's drawing clearly referenced three of the types of mathematics beliefs, and that the framework worked as a method of coding and categorising these beliefs. Conversely, it wasn't clear whether the 'learning environment' was referenced at all.

Table 3.9: Coding 11.1.24 drawing in terms of mathematics beliefs framework

self	✓
ability	✓
learning environment	?
nature	✓

Key: ✓ = present, ? = unclear

The third analysis and reading of the drawings took a more qualitative approach by exploring themes that emerged from multiple readings of the drawings. Themes that were relevant in terms of the student 11.1.24's drawing were "What is maths?", in particular, the subthemes of 'maths as number', 'maths as problem solving' and 'maths and brains', 'identity' and perhaps 'the maths classroom'.

Although I used three different types of analysis in order to make sense of the children's drawings, they spilt over into each other as themes appeared and disappeared across iterations. After completing these analyses, I returned to the drawings and looked at them through the eyes of Nuthall's three worlds of the classroom (2007) as well as through Freeman and Mathison's Framework for analysing visual data (2009). The analysis of the drawings is covered in detail in Chapter 5, Reading the drawings.

Making sense of the data collected from the Focus Schools

In the process of interpreting the focus children's and their teachers' mathematics beliefs within the contexts of their schools and classrooms, I examined the data set that comprised a combination of observations with their accompanying fieldnotes, interviews, video-recordings and artifacts. The MBQ and drawing responses were also used as a backup in order to confirm or contradict interpretations.

Episodes from the video footage were used during the interviews as a vehicle for discussion with both the children and the two focus class teachers. Like the drawing data, the interviews were analysed in various ways in order to make sense of beliefs and experiences. During the first iteration of interview analysis, I used an open coding system keeping the research questions in mind. The Mathematics Beliefs Framework (MBF) was used next, and finally the coding was checked against emergent themes from the other data sources. An interview extract illustrating this multiple coding system is included in Appendix J.

Conclusion

In this chapter, I have presented a detailed description of the research design by introducing and discussing the sampling procedure, as well as the data collection and data analyses processes. The embedded nature of the student sample was described as the focus of the study shifted from the large group of 823 children from 17 primary schools to nine children at two focus schools. The collection of data from the questionnaires, observations, recordings and interviews was explained as were the methods I used when analysing these data. These descriptions of the complementary data collection methods and analysis processes form part of the dependability audit trail (Chapter 2, Negotiating research quality) which, by including the specific elements of the decisions and processes, work towards demonstrating the dependability and credibility of this study (Creswell, 2007; Lincoln & Guba, 1985; Schoenfeld, 2007).

In the next three chapters, I present, analyse and discuss the final data set. In Chapter 4, *The landscape of beliefs*, the range of beliefs about mathematics is explored, based on responses to the closed questions on the MBQ. I look at definitions and dimensions associated with mathematics beliefs and try to analyse variables such as achievement levels, SES, gender and ethnicity that may affect these beliefs.

Chapter 4: The landscape of beliefs

The metaphor of the landscape suggests the relationship between elements, between the foreground and the background, between the multiplicity of beliefs that unfolds as one moves through the chapter. It is the background as well as the structures that are placed and movements that take place against it. Landscape can also be viewed in figurative terms to indicate a comprehensive mental view of a domain, in this case, beliefs about mathematics.

The intention of this chapter is to use statistics to explore and develop a baseline of primary mathematics beliefs in the form of a framework that could act as a filter for understanding and interpreting beliefs that are either espoused by the children or enacted in the focus schools' mathematics classrooms. In order to understand the variety of mathematics beliefs held by Year 5 and 6 children, I examined their responses on the Student Maths Beliefs Questionnaires (MBQ) in terms of the following questions:

> What do children believe about the nature of mathematics?
>
> What do children believe about themselves and others in relation to mathematics?

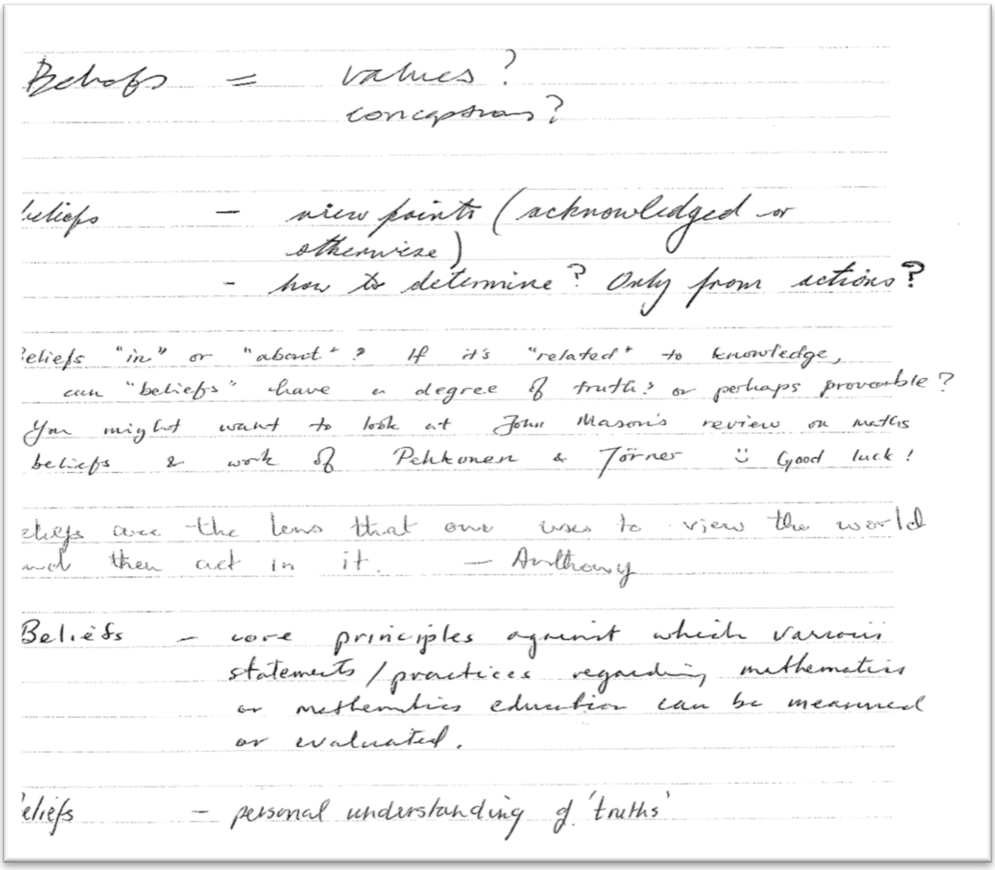
I use statistics techniques as a lens for exploratory data analysis, as a tool for exploring patterns of responding in order to further my understanding of the landscape of mathematics beliefs (Behrens & Yu, 2003; Dancey & Reidy, 2004). I acknowledge that I have not applied statistical procedures in the traditional manner as a means of hypothesis testing. During the process of developing a framework of mathematics beliefs, I conducted a factor analysis (a principal component analysis) and compared the extracted factors with aspects of beliefs covered in the research literature. I also explored the means calculated for each of these factors in terms of school and student characteristics. I investigated these data by using descriptive statistics, t-tests, analyses of variance, effect sizes and regression analyses. The Scheffé *post hoc* test was chosen because it can deal with unequal group size, and because it is conservative which is essential when taking into account the large number of participants (Pallant, 2007). In the cases where the assumption of homogeneity was violated, Brown-Forsythe *f*-ratios were reported and Dunnett T3 *post hoc* test was applied (Field, 2009).

In this chapter, I draw from the general and domain specific research literature in order to describe definitions of beliefs and the factors or dimensions associated with these beliefs definitions. I report on the factor analysis process and findings, then explore the student patterns of responding on each of these factors as well as on a mathematics personal minifactor (MPM). Finally, I discuss the Mathematics Beliefs Framework (MBF) that I developed as a lens for understanding this landscape of mathematics beliefs.

Belief definitions

Epistemological beliefs, also referred to as beliefs about knowledge and knowing, are recognised having an influence on learning and teaching (Bendixen & Feucht, 2010; Hofer, 2002, 2005, 2008; Muis, 2004). In addition, beliefs affect motivation, interest and engagement both in general and in particular subject areas (Kloosterman, 2002; Leder & Forgasz, 2002). In mathematics, in particular, these beliefs may either facilitate or impede individuals' capacities to link mathematical concepts from everyday experiences with those covered in the mathematics classroom (Presmeg, 2002). However, a precise definition or agreement about the nature of these beliefs is difficult to determine because of the range and variety of definitions found in the research literature (Cross, 2009; De Corte et al., 2002). This diversity of definitions and conceptions is illustrated in Figure 4.1, which includes examples articulated by the small group of academics wrestling with definitions. These definitions range from "the lens that one uses to view the world and then act in it" (Anthony Fernandes, 2011) which is very similar to Philipp's lens for "interpreting the world" (2007, p. 257), through viewpoints, values, conceptions, "core principles" to notions of truth and knowledge. These writers also raised the issues associated with how beliefs are "determined" as well as evaluating and measuring beliefs. McLeod and McLeod

(2002) suggest a reason for the range of different types of definitions and lack of agreement is that certain definitions may be appropriate for different audiences; for example, those for the general public as opposed to more sophisticated and extended definitions for experts.


Beliefs = values?
conceptions?

eliefs – view points (acknowledged or otherwise)
– how to determine? Only from actions?

eliefs "in" or "about"? If it's "related" to knowledge, can "beliefs" have a degree of truth? or perhaps provable?
You might want to look at John Mason's review on maths beliefs & work of Pehkonen & Törner ☺ Good luck!

eliefs are the lens that one uses to view the world nd then act in it. — Anthony

Beliefs – core principles against which various statements/practices regarding mathematics or mathematics education can be measured or evaluated.

eliefs – personal understanding of 'truths'

Figure 4.1: Definitions of beliefs supplied by academics attending Mathematics Education Research Group of Australasia Conference, 2011

In attempting to find a clear working definition of beliefs, in general, one needs to acknowledge the difference between belief and knowledge. Knowledge implies some sort of objective, publicly or socially accepted and constructed truth, depending on the theoretical perspective of the definer (De Corte et al., 2002; Goldin, 2002; Hart, 2002; Lincoln et al., 2011), while beliefs are internal, personal conclusions about the veracity of something based on the individual's experience. Cross adds the influence of social context in her definition of "beliefs as embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups" (2009, p. 329). Mason points out that beliefs may be the claims one makes when trying to justify rather than precursors of one's actions or positions (2004). Not only is there a problem with defining beliefs, but there is also a myriad of terms applied to beliefs, for instance personal epistemological beliefs (Hofer & Pintrich, 2002), individual constructs (De Corte et al., 2002), intuitive beliefs (Tsamir & Tirosh, 2002), subjective conceptions (Op 't Eynde, de Corte, & Verschaffel, 2002), subjective knowledge (Goldin, 2002; Hart, 2002) or "the personal assumptions from which individuals make decisions about the actions they will undertake" (Silva & Roddick, 2001, p. 101). Another problem associated with defining beliefs are the opposing traditions of viewing the construct as general versus domain-specific or both (Hofer, 2002, 2008; Leder et al., 2002a). An additional issue is whether beliefs are conceived of as developmental (W. Perry, 1970) as opposed to made up of dimensions or factors (Muis, Bendixen, & Haerle, 2006; Op 't Eynde et al., 2002; Op 't Eynde, de Corte, & Verschaffel, 2006; Schommer-Aikins, 2002; Schommer-Aikins et al., 2005).

In addition to the range of terms assigned to and definitions of beliefs, some researchers have acknowledged that beliefs may include more than one approach. Pintrich describes the various approaches to the study of beliefs as focusing on aspects of beliefs such as "the certainty of knowledge, the simplicity of knowledge, the source of knowledge and justifications for knowing" (2002, p. 390), especially for those who adopt a global perspective. He also acknowledges that other models may include an "individual's cognitions and beliefs about the nature of learning, intelligence, instruction, classrooms, domain-specific beliefs and disciplines, and beliefs about self" (p. 391). Hofer combines some of the general beliefs definitions with a developmental notion in her definition of personal epistemology as "... an identifiable set of dimensions of beliefs about knowledge and knowing, organized as theories, progressing in reasonably predictable directions, activated in context, operating both cognitively and metacognitively" (2005, p. 98). She includes notions of predictable development with aspects of beliefs, as well as the idea of beliefs as organised or grouped, contextual and related to thinking.

The theory of beliefs about knowledge and knowing as developmental began with Perry's study of Harvard undergraduates in the 1950s and 1960s with his discussion of beliefs moving from naïve to more sophisticated (Perry, 1970). In the 1990s, Schommer-Aikens, building on Perry's research, developed a different approach and hypothesised five independent types or dimensions of beliefs: "the stability of knowledge", "the structure of knowledge", "the source of knowledge", "the speed of knowledge acquisition", and "the control of knowledge acquisition" (2002, pp. 104-105). A factor analysis undertaken by Schommer-Aikens on her epistemological beliefs survey isolated four beliefs: "structure and stability of knowledge as well as speed and control of learning" (2002, p. 105). Other researchers who attempted to replicate a factor analysis on this instrument with different populations identified different factors. One example, a study of university students in an introductory psychology course was undertaken by Schraw, Bendixon and Dunkle who found "innate ability, certain knowledge 1, incremental learning, certain knowledge 2, and integrated thinking" (2002). Wood and Kardash, who also used a sample of university students, isolated a different set of factors, namely "speed of knowledge acquisition, structure of knowledge, knowledge construction and modification, characteristics of successful students, and attainability of objective truth" (2002, p. 246), thus expanding the list of factors and questioning their generalisability.

Both of these theories of beliefs traditions, the developmental (Hofer, 2002; King & Kitchener, 2002; Perry, 1970) and the dimensional (De Corte et al., 2010; De Corte et al., 2002; Schommer-Aikins, 2002; Wood & Kardash, 2002), seem to view beliefs in terms of binary opposites as naïve/sophisticated or appropriate/inappropriate. Muis criticises these value-laden terms preferring to use the more neutral availing/non-availing beliefs pairing which she defines as follows: "availing beliefs are associated with better learning outcomes, and non availing beliefs have no influence on learning outcomes or negatively influence learning outcomes" (2004, p. 323). Despite the differences in approaches and terms, certain sorts of beliefs are seen as more helpful than others, or as indicators of higher achievement for learners.

Beliefs about mathematics

Multiple approaches to defining and studying beliefs within the domain of mathematics add to the problem of finding a common set of understandings or definitions. Goldin (2002) identifies eleven different types of mathematical beliefs:

> 1). Beliefs about the physical world, and about the correspondence of mathematics to the physical world (e.g., number, measurement);
> 2). Specific beliefs, including misconceptions, about mathematical facts, rules, equations, theorems, etc (e.g., the law of exponents, the quadratic formula, the idea that 'multiplication always makes larger');
> 3). Beliefs about mathematical validity, or how mathematical truths are established;
> 4). Beliefs about effective mathematical reasoning methods and strategies or heuristics;
> 5). Beliefs about the nature of mathematics, including the foundations, metaphysics, or philosophy of mathematics;
> 6). Beliefs about mathematics as a social phenomenon;
> 7). Beliefs about aesthetics, beauty, meaningfulness, or power in mathematics;
> 8). Beliefs about individual people who do mathematics, or famous mathematicians, their traits and characteristics;
> 9). Beliefs about mathematical ability, how it manifests itself or can be assessed;
> 10). Beliefs about the learning of mathematics, the teaching of mathematics, and the psychology of doing mathematics;
> 11). Beliefs about oneself in relation to mathematics, including one's ability, emotions, history, integrity, motivations, self-concept, stature in the eyes of others, etc.
> (2002, pp. 67-68)

Goldin's list of beliefs ranges from the philosophical (beliefs 3, 5, 6 and 7), through the concerns of those professional inhabitants of the world of mathematics (beliefs 1, 3, 4, 8), through various notions of mathematics identity—self and others (beliefs 8 and 11)—to the sorts of beliefs that are prevalent in the school mathematics classroom (beliefs 2, 4, 9, and 10). However, much of the research in the domain of mathematics that focuses on belief factors, which are also referred to as aspects or dimensions, concentrates on a smaller, and sometimes more general, set of mathematics beliefs categories related to knowing about and learning mathematics, which also includes beliefs about the self and the context of the mathematics classroom. McLeod, who views beliefs along with attitudes and emotions as part of the affective domain, examines beliefs about mathematics, about self, about mathematics teaching and about social context (D. McLeod, 1992; Romberg, 1994). Kloosterman identifies beliefs about "a) mathematics as a discipline, b) the self as a mathematics learner, c) the role of the mathematics teacher, and d) other beliefs about mathematics learning" (2002, p. 247). Cobb, Yackel and Rasmussen explore beliefs as influenced by the social norms and practices of the mathematics classroom, such as the nature of school mathematics, the roles of learners, and teachers (Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). A focus on the class context is also taken up by Op 't Eynde, de Corte and Verschaffel who developed a three-factor model of students' beliefs about mathematics which include beliefs about mathematics education, about the self and about the social context of the class (De Corte et al., 2002; Op 't Eynde et al., 2002). This framework is covered in more detail in Figure 4.2.

1. Beliefs about mathematics education
a) beliefs about mathematics as a subject
b) beliefs about mathematical learning and problem solving
c) beliefs about mathematics teaching in general

2. Beliefs about the self
a) self-efficacy beliefs
b) control beliefs
c) task-value beliefs
d) goal-orientation beliefs

3. Beliefs about the social context
a) beliefs about social norms in their own class
   _ the role and the functioning of the teacher
   _ the role and the functioning of the students
b) beliefs about socio-mathematical norms in their own class

Figure 4.2 A framework of students' mathematics-related beliefs (Op 't Eynde et al., 2002, p. 28)

However, in a more recent study of 14 year olds' beliefs about mathematics, after conducting a principal component analysis, the same researchers identified a four-factor model: "Beliefs about the role and the functioning of their own teacher, beliefs about the significance of and their own competence in mathematics, mathematics as a social activity and mathematics as a domain of excellence" (2006, p. 65). Although these factors overlap with their original model, the differences indicate how difficult it is to develop a model that could be useful and relevant in all cases.

Factor Identification and Analysis

The following subsection describes the process of developing a framework of beliefs about mathematics based on the data from the MBQ. Before the factor analysis was executed, all items were aligned so that they were in the positive form with high and low scores indicating the same intent over all items. The Student analysis was based on an acceptable sample size of 823 (Field, 2009; Garson, 2009). The Kaiser-Meyer-Oklin and Bartlett's Test of Sphericity indicated sampling adequacy for the questionnaire: Student KMO value was .828 and Bartlett's p= .000 (Dancey & Reidy, 2004; Field, 2009; Pallant, 2007). In order to investigate whether certain items from the Maths Belief Questionnaire would cluster together in a way that would link to particular constructs or dimensions of beliefs, a principal component analysis (PCA) with Varimax Orthogonal rotation was performed using SPSS version 18.0 on all 19 closed-items. After the PCA was completed, factor scores were calculated for each participant.

Student Questionnaire (MBQ)

The analysis revealed four components with eigen values above 1 accounting for 22.6%, 13.4%, 7.9% and 6.1% of the variance, respectively (50% of the total variance). The four factors with the item loadings over .300 are summarised in Table 4.1.

Even though Q13 had a factor loading greater than .5 (Dancey & Reidy, 2004; Field, 2000, 2009) on both the Factor 1: *Self* (beliefs about own mathematics ability) and Factor 3: *Learning Environment*, it was included in *Self* because it seems to make more sense in terms of a real-world construct. An examination of the scree plot supported the decision to retain all four components (See Figure 4.3) because it indicates four components above eigen value 1 before the line flattens out. For the remainder of this chapter I refer to the components as factors.

Table 4.1: Rotated component matrix for the Student MBQ

	Self	Ability	Learning environment	Nature
Q18. How good do you think you are at maths?	**.824**			
Q22. I am good/ok/not good at maths	**.766**			
Q4. I can do maths.	**.711**			
Q19. How good does your teacher think you are at maths?	**.693**			
Q20. How good does your family think you are at maths?	**.666**			
Q32. Do you find maths hard, easy or in-between?	**.581**			
Q13. How much do you like doing maths at school?	**.559**		.504	
Q9. Pacific Island students are good at maths.		**.788**		
Q8. Māori students are good at maths.		**.745**		
Q10. Pākehā/European students are good at maths.		**.699**		
Q6. Girls are good at maths.		**.629**		
Q7. Asian students are good at maths.		**.628**		
Q5. Boys are good at maths.		**.621**		
Q21. How good do you think your teacher is at maths			**.628**	
Q17. How often do you do very interesting things in maths at school?			**.558**	
Q3. Everyone can learn maths			**.519**	
Q11.There are many ways to work out a maths problem.				**.775**
Q2. Everyone can do maths.				**.734**
Q12. There are many answers to a maths problem				**.398**

Note: major loadings for each item are in bold.

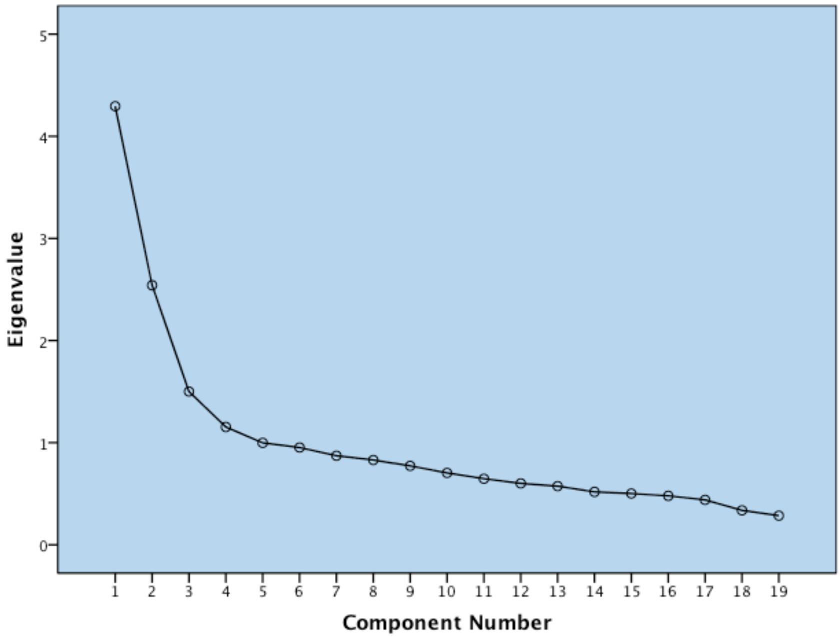


Figure 4.3: Scree plot for Student Belief Factors

A factor analysis is a fairly objective statistical procedure. One gives the statistics programme the necessary data, sets the perimeters and presses the "ok" command. Naming the factors is different; it is a subjective activity where the hand, and perhaps biases or at least the theoretical perspectives, of the researcher is evident. Another researcher may look at the same items grouped within a factor, and come up with a completely different descriptor.

Factor 1 includes seven items that relate to individuals' beliefs about themselves as doers and learners of mathematics, how they position themselves within the world or domain of mathematics. This factor covers items about achievement such as how good the students themselves and others view them at mathematics, how difficult they find the subject, and to what extent they enjoy it. I labeled this factor *Self*.

Factor 2 is related to beliefs associated with group identity and includes six items that consist of beliefs about ability/ lack of ability to achieve in mathematics, in terms of gender and ethnicity: in other words, their beliefs about certain groups of people being good or not at mathematics, being natural inhabitants of this world or not. I labeled this factor *Ability*

Factor 3 includes three items associated with the teaching and learning mathematics, such as beliefs about the teacher's ability, about learning and interest, suggesting a relationship between teaching and making mathematics interesting. I labeled this factor *Mathematics Learning Environment (LE).*

The final Factor 4 also has three items, two of which focus on how mathematics works and one which reflects beliefs about the inclusionary/exclusionary nature of mathematics. This factor was labeled *Nature of mathematics (Nature).*

Although these four factors are not identical to those considered in the initial design of the study, there is enough overlap to confirm the spirit of the original constructs. The items included in *Self* are identical to beliefs about the "individual and the world of mathematics", *Ability* matches "others and the world of mathematics" while *Nature* includes items from the epistemology category. It is the third factor, the *Learning Environment* that differs, yet it makes sense in terms of students' classroom experience—learning is affected by the teacher who also, in turn, affects how interesting mathematics classes are (Mason, 2004; Muis, 2004).

Discussion

The four factors extracted through the factor analysis align with belief factors discussed in the research literature. Factors 1 and 2 seem to reflect different aspects of identity beliefs, those associated with the *self* and those with *ability* of groups in relation to mathematics. The factor which encompasses *self* as a doer and learner of mathematics bears a similarity to "[b]eliefs about oneself in relation to mathematics" (Goldin, 2002, p. 68; Kloosterman, 2002; McLeod, 1992; Op 't Eynde et al., 2006; Yackel & Cobb, 1996). This *Self* factor also includes beliefs about enjoyment of the subject, which may relate to motivation and interest in mathematics (Kloosterman, 2002). The second factor is related to beliefs about innate mathematics ability associated with membership of groups based on gender and ethnicity. These beliefs are likely to be influenced by individuals' experiences of doing and learning school mathematics. This factor seems to overlap with two of Goldin's types of mathematics beliefs, "about individual people who do mathematics", and beliefs "about mathematical ability" (2002). Even though other researchers discuss ability beliefs as associated with being born with a "maths brain" and being fast at solving problems (Schommer-Aikins, 2002; Schommer-Aikins et al., 2005; Schraw et al., 2002; Wood & Kardash, 2002), none of them seem to associate ability with gender and /or ethnicity.

Factors 3 and 4, those associated with beliefs about the *learning environment* and about the *nature of mathematics*, also relate to factors or dimensions discussed in the research literature. Factor 3 is related to notions of beliefs about the social context of the classroom, and of students and teachers doing mathematics in the classroom (Goldin, 2002; King & Kitchener, 2002; Kloosterman, 2002; D. McLeod, 1992; Op 't Eynde et al., 2002; Op 't Eynde et al., 2006; Schommer-Aikins, 2002; Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). For example, Op 't Eynde et al's category of "mathematics as a social activity" (2006, p. 65) includes an item very similar to the MBQ item: "Everyone can learn maths". The *nature of mathematics* factor which deals with the nature of mathematics seems to have more in common with Kloosterman's "mathematics as a discipline" (2002, p. 249) than with Goldin's "[b]eliefs about the nature of mathematics" (2002) which is concerned with the philosophical underpinnings of the domain rather than with teachers' and students' understandings of mathematics. Factor 4 also seems akin to beliefs about mathematics education which includes an item on multiple ways of problem solving (Op 't Eynde et al., 2002). On the other hand, factor 4 appears to contrast with the absolutist, naïve, or non-availing beliefs about knowing and knowledge that are discussed by Perry (1970), Schommer-Aikens (2002; 2005) as well as by D. McLeod's beliefs about mathematics (1992) and Muis (2004). This final factor suggests that this group of participants espouse "availing beliefs", for example, the notion of multiple methods of solving problems as opposed to one, correct method.

The MBQ findings

Having extracted four factors to use as a means of exploring the children's responses on the MBQ, I returned to this dataset for the second round of analysis. The responses from the large sample of children were initially explored by looking at the background information of the participants in terms of school characteristics, student characteristics and data. The relationships between demographic characteristics, such as school SES, as well as student gender and ethnicity, and achievement data were also explored in order to understand the landscape and context that may influence the students' beliefs about mathematics (See Appendix K). The following subsection presents the results from comparing and analysing the means and standard deviations of the *Self, Ability, Learning Environment* and *Nature of Mathematics* belief factors in terms of schools, SES, gender, ethnicity, age, school year and achievement levels.

Results derived from the mathematics beliefs factor scores

Belief scores for *Self, Ability, Learning Environment (L.E.), Nature of mathematics* and Total beliefs were calculated for each student by adding the scores on the items within each factor. Means and standard deviations for each belief were computed for individual schools (Table 4.2), school decile (SES), as well as for individual characteristics, such as gender, age, ethnicity, school year, and achievement, and are reported in Table 4.4. *T*-tests, ANOVAs and effect size were used to determine which of these characteristics could account for significant differences in beliefs about mathematics. Multiple regression analyses were also explored to check whether any sets of characteristics could be used to predict mathematics beliefs scores. However, because the models derived from the multiple regression analyses accounted for such small percentages of the variance (between 1 and 8%, depending on the factor), they are not included in the following summary of results (See Appendix L).

As part of the process of examining the whole landscape of mathematics beliefs from a variety of perspectives, the means (and standard deviations) from all 17 schools were explored. In Table 4.2, the highest means scores for each factor appear in bold and the lowest in italics.

Table 4.2: Means and standard deviations on four belief factors by school

School	N	Self	Ability	L. E.	Nature	Total
1	24	21.50 (5.62)	*15.63 (7.44)*	12.25 (1.70)	9.96 (3.18)	*59.33 (9.73)*
2	86	23.07 (4.93)	21.16 (6.14)	12.66 (1.49)	9.66 (2.96)	66.56 (10.25)
3	34	22.62 (5.41)	20.09 (6.43)	12.15 (1.79)	10.62 (1.95)	65.56 (10.47)
4	14	22.29 (3.79)	20.57 (6.21)	12.86 (1.46)	10.29 (1.86)	66.00 (9.40)
Kikorangi	56	22.55 (4.92)	20.05 (6.11)	12.82 (1.31)	10.05 (2.81)	65.48 (10.22)
6	50	23.10 (4.41)	18.38 (7.46)	**12.98** (1.51)	10.64 (1.85)	65.10 (9.44)
7	28	23.32 (5.56)	18.14 (5.30)	12.36 (1.57)	9.93 (3.20)	63.75 (10.10)
8	10	**23.80** (5.41)	21.30 (3.09)	12.30 (1.16)	*8.20 (2.44)*	65.60 (7.37)
9	29	23.34 (5.66)	20.07 (5.70)	12.45 (1.79)	10.55 (2.68)	66.41 (9.95)
10	33	21.39 (5.04)	**22.24** (5.56)	12.91 (1.68)	10.79 (2.86)	67.33 (10.04)
Whero	179	21.47 (5.16)	18.13 (6.15)	11.96 (1.93)	10.60 (2.39)	62.16 (9.71)
12	32	22.50 (5.21)	20.91 (6.25)	12.72 (2.07)	**11.59** (3.00)	**67.72** (10.91)
13	74	22.78 (4.92)	18.49 (7.23)	12.61 (2.03)	9.66 (2.89)	63.54 (10.46)
14	44	21.30 (6.15)	17.95 (7.36)	*11.50 (2.53)*	11.14 (2.77)	61.48 (13.83)
15*	38	22.29 (5.78)	19.18 (5.39)	12.63 (2.17)	9.68 (2.82)	63.79 (8.63)
16	20	23.40 (5.13)	18.45 (6.22)	12.60 (1.23)	10.10 (2.77)	64.55 (10.56)
17*	72	*21.01 (6.06)*	20.03 (6.14)	12.43 (1.85)	8.65 (3.22)	62.13. (10.39)
Total	823	22.22 (5.28)	19.27 (6.44)	12.41 (1.85)	10.16 (2.79)	64.03 (10.35)

School 12 had the highest factor total (M = 67.72) and School 1 the lowest (M = 59.33). The two focus schools, Kikorangi and Whero are also highlighted in order to see where they fit within the range of mathematics beliefs. (A comparison of belief factors for these two schools in included in Appendix L.) Kikorangi is very close to the overall mean on Self, above the mean on *Ability*, just above the mean on *LE*, just below on *Nature* and above the total school mean. Whero, on the other hand, has means below the overall school means on all the factors except

Nature. Significant differences in mean scores of the 17 schools were found for *Ability, LE, Nature* and the Total belief scores. The results of the ANOVAs are summarized in Table 4.3.

Table 4.3: ANOVA of mathematics beliefs factors differentiated by school

	F	**df**	**p**
Self	1.21	16, 806	.250
Ability	2.65	16, 518.314	**.001***
L.E.	2.58	16, 528.661	**.001***
Nature	3.65	16, 485.633	**.0001***
Total	2.13	16, 806	**.006***

* Significance at p<.05

Because of the number of schools involved, it was not practical to run *post hoc* tests on the school means ANOVAs; instead, effects sizes were calculated between the highest and lowest means for each belief factor. Effect size (Cohen's *d*) for each of these differences is above .4: Self=.49, for Ability = 1.01, for L.E. = .71, for Nature = 1.24 and for the Total belief mean = .81 indicating substantive differences between these results (Hattie, 2009). Although interesting differences are observable between schools, it was less complicated to make sense of the differences in means by grouping schools in terms of characteristics, such as SES.

Table 4.4 records patterns that will be analysed in more detail later in the chapter headings *Results based on school characteristics* and *Results based on student characteristics*.. In terms of the Total belief score, boys had higher means than girls, 11-year olds had higher scores than other age groups, middle decile schools were higher than low or high decile, Māori students had higher means than other ethnic groups, and Year 5 had higher means than Year 6. Students who scored highly on the achievement measures, PAT and NumPA, had higher Total belief means than others.

Table 4.4: Means and standard deviations on four belief factors by student and school characteristics

		Self	Ability	L. E.	Nature	Total	n
School Decile	Low	21.66 (5.76)	19.37 (6.36)	12.51 (1.94)	9.64 (3.24)	63.18 (10.27)	166
	Middle	**22.78** (4.92)	**19.96** (6.61)	**12.73** (1.65)	10.07 (2.76)	**65.54** (10.13)	328
	High	21.95 (5.33)	18.53 (6.24)	12.04 (1.92)	10.50 (2.52)	62.96 (10.45)	329
Gender	F	21.73 (5.33)	18.93 (6.76)	12.48 (1.71)	10.25 (2.66)	63.34 (10.74)	412
	M	**22.77** (5.18)	19.63 (6.09)	12.35 (1.94)	10.06 (2.93)	**64.81** (9.88)	406
Ethnicity	Asian	23.76 (4.46)	18.47 (6.43)	12.31 (1.54)	10.22 (2.64)	64.76 (8.66)	51
	Māori	22.58 (5.40)	**20.67** (6.00)	12.56 (1.90)	9.68 (2.78)	**65.49** (9.41)	149
	Pākehā	21.95 (5.33)	18.98 (6.52)	12.35 (1.87)	10.33 (2.72)	63.59 (10.76)	584
	P I	22.72 (4.86)	19.33 (6.24)	12.83 (1.60)	9.53 (3.61)	64.42 (9.33)	36
School Year	**Year 5**	**22.50** (5.05)	19.19 (6.47)	12.50 (1.76)	9.96 (2.93)	**64.15** (10.33)	397
	Year 6	21.98 (5.47)	19.36 (6.40)	12.33 (1.93)	10.32 (2.63)	63.95 (10.36)	424
Age	8	22.86 (5.84)	20.52 (5.52)	12.19 (1.94)	7.57 (3.63)	63.14 (9.07)	21
	9	22.43 (5.11)	19.05 (6.59)	12.54 (1.73)	9.91 (2.82)	63.93 (10.56)	348
	10	22.03 (5.34)	19.39 (6.26)	12.34 (1.92)	10.50 (2.55)	64.19 (9.95)	425
	11	24.42 (4.55)	20.11 (7.47)	12.26 (1.59)	9.89 (3.93)	**66.68** (10.50)	19
PAT	Low	21.62 (5.19)	20.01 (6.89)	12.71 (1.52)	9.45 (3.33)	63.79 (10.70)	91
	Middle	21.79 (4.96)	18.91 (6.38)	12.21 (1.85)	10.27 (2.32)	63.19 (10.10)	222
	High	**24.44** (4.45)	18.09 (6.49)	12.40 (1.67)	**10.75** (2.29)	**65.68** (9.05)	88
NumPA	Low	21.24 (5.26)	20.73 (5.78)	12.62 (1.65)	8.88 (3.39)	63.47 (9.24)	66
	Middle	22.22 (5.32)	19.39 (6.36)	12.44 (1.84)	10.18 (2.76)	64.20 (10.53)	598
	High	**24.05** (4.61)	18.02 (6.32)	12.05 (1.93)	**11.14** (2.57)	**65.26** (8.98)	42

Results based on school demographic information

In this subsection, comparisons of means on the beliefs factors were examined in terms of school socio-economic status/decile (Table 4.4). Schools were divided into three socio economic status categories, low (deciles 1-3), medium (deciles 4-7) and high deciles 8-10). Means for each category on the four belief factors were compared. In order to explore the difference between these means, one-way ANOVAs were conducted (See Table 4.5). School SES affected the pattern of responding on all of these factors. Students attending schools in the middle band had higher means on the total beliefs score (M = 65.54, SD=10.13) than did those in either the lower (M = 63.18, SD=10.27) or higher (M = 62.96, SD= 10.45) decile groups (Table 4.4).

Table 4.5: ANOVAs of the mathematics belief factors differentiated by School SES

SES	***F***	**df**	**p**	***post hoc***	**Cohen's *d***
Self	3.21	2, 820	**.041***	ns	
Ability	4.09	2, 819	**.017***	m&h (.018) m&l (.56)	.22 .09
L.E.	12.17	2, 820	**.0001***	l&h (.027) m&h (.0001)	.24 .39
Nature	5.04	2, 524.936	**.007***	l&h (.01)	.30
Total	5.86	2, 820	**.003***	m&h (.006)	.25

* Significance at p<.05

The ANOVAs in Table 4.5 demonstrate significant differences on all of the beliefs factor means even though the Scheffé *post hoc* test indicated no significance difference on *Self*. Students at middle decile schools had significantly higher means scores than did the students from higher decile schools for *Ability, LE* and Total beliefs. Students at low decile schools had higher means scores than those from high ones on both Learning Environment and Nature of mathematics factors.

Results based on student characteristics

In order to explore patterns that might help me make sense of data in the MBQ, I looked at student characteristics such as gender, ethnicity, school year, age and/or achievement levels. I was interested in the extent to which using statistical tools would help me understand differences in the ways groups scored on the belief factors. I used a series of independent *t*-tests and one-way analyses of variance (ANOVA) to explore whether there were any significant differences between these groups on the mathematics belief factor means even though I acknowledge that lack of significance or no difference between groups is also important. These procedures were selected despite the possibility of Type 1 errors because this is an exploratory rather than hypothesis testing study. I also included effect size as a measure of comparison. The results for these five variables are reported in the following subsection.

In order to determine whether gender could account for difference in beliefs about mathematics, *t*-tests were performed on each of the belief factors. The means and standard deviations for girls and boys are included in Tables 4.4 (p. 109) and 4.6. *T*-test analyses of the factor scores for boys and girls indicated that the only significant difference was on the *Self* and the Total means. Boys had significantly higher means than girls in both these cases, but the effect sizes were relatively low (d = .20 and .14, respectively). These results indicate that boys were more confident than girls about their identities as doers of mathematics and as successful mathematics students; otherwise the means were more similar than different.

Table 4.6: Mean, standard deviation, t-value and significance for belief factors differentiated by gender

gender	Female (N=352) Mean(SD)	Male(N=350) Mean(SD)	*t*-value	p	Cohen's *d*
Self	21.73 (5.33)	22.77 (5.18)	-2.82	**.005***	.20
Ability	18.93 (6.76)	19.63 (6.09)	-1.56	.120	.11
L.E.	12.48 (1.71)	12.35 (1.94)	1.00	.316	.07
Nature	10.25 (2.66)	10.06 (2.93)	.95	.341	.07
Total	63.34 (10.74)	64.81 (9.88)	-2.04	**.042***	.14

* Significance at p<.05

Although the students completing the questionnaires identified a large range of ethnicities, these were grouped into Asian, Māori, Pākehā/others, and Pasifika. The means for each of the belief factors were reported in Table 4.4 and compared in Table 4.7

Table 4.7: ANOVA of the mathematics belief factors differentiated by ethnicity

Ethnicity	*F*	df	p	*post hoc*	Cohen's *d*
Self	2.30	3, 816	.076		
Ability	3.02	3, 815	**.029***	M & Pak (.043)	M&Pak = .27
L.E.	1.20	3, 816	.308		
Nature	2.43	3, 135.520	.068		
Total	1.79	3, 201.293	.150		

* Significance at p<.05

Māori students had the highest total beliefs mean of 65.49 (SD = 9.41), and Pākehā students had the lowest (M = 63.59, SD = 10.76), but the difference between them was not significant. However, Māori student means scores were significantly higher than Pākehā ones for the *Ability* belief scores but with a moderately low effect size. A high score on *Ability* indicates beliefs that all of these groups of students, boys and girls as well as all ethnic groups, have the capacity to be good at mathematics. This score suggests that Māori students are more positive than their Pākehā counterparts when it comes to judging how capable different groups are at mathematics, based on gender and ethnicity.

There were no significant differences in means between Year 5 and Year 6 students on the mathematics beliefs factors.

Because there is a clear relationship between age and school year, the results were expected to be similar. No significant differences were found when comparing responses by age for the factors *Self, Ability* and *Learning Environment* means scores. There was, however, one unusual difference: significant variance was found for the factor *Nature* with a low mean of 7.57 (SD = 3.63) for 8 year olds and a high for 10 year olds (M = 10.50, SD= 2.55) (Tables 4.4 and 4.8).

Table 4.8: ANOVAs mathematics beliefs factors differentiated by age

Age	***F***	**df**	**p**	***post hoc***	**Cohen's *d***
Self	1.57	3, 809	.196		
Ability	.55	3, 808	.648		
L.E.	.98	3, 809	.404		
Nature	5.96	3, 60.376	**.0001***	8&9 (.05)* 8&10 (.009)* 9&10 (.017)*	8&9=.72 8&10=.93 9&10=.22
Total	.51	3, 809	.68		

* Significance at p<.05

The Dunnett T3 *post hoc* test indicated that significant means variance lay between ages 8 and 9, 8 and 10, as well as between 9 and 10. Strong effect sizes of .72 between ages 8 and 9 and of .93 between 8 and 10 were found (Table 4.8). An exploration of the items included in the Nature factor suggests that children with higher scores on Nature have more sophisticated, "availing", or rather less naïve, beliefs about mathematics (Baxter Magolda, 2002; Hofer, 2002; Muis, 2004; Perry, 1970; Schommer-Aikins, 2002). This suggest that they may have developed these more availing beliefs with age and perhaps with more exposure to school mathematics.

Most schools provided information on which stages of the Numeracy Project their students were working within; some schools also had PAT (Mathematics) scores available. The beliefs means in terms of both of these measures are reported in Table 4.4. Students in the highest bands on both of these measures had higher total means scores than the other two bands.

Table 4.9: ANOVAs of mathematics belief scores differentiated by PAT results

PAT	***F***	**df**	**p**	***post hoc***	**Cohen's *d***
Self	10.46	2, 398	**.0001***	l&h (.001) m&h(.0001)	.58 .56
Ability	1.97	2, 398	.141		
L.E.	2.71	2, 398	.068		
Nature	5.24	2, 219.341	**.006***	l&h(.008) l&m (ns)	.46 .29
Total	1.95	2, 398	.143		

*Significance at p<.05

High achieving students on the PAT (mathematics) had a significantly higher mean on the *Self* factor (M =24.44, SD = 4.45) than did either the middle group (M = 21.79, SD = 4.96) or the low (M = 21.62, SD = 5.19). The effect size was above .50 in both cases (See Tables 4.4 and 4.9). Higher achieving students also had a higher mean than the other two groups on the *Nature of mathematics* factor (M = 10.75 (2.29), 9.45 (3.33) and 10.27 (2.32); however, Dunnett's T3 *post hoc* test indicated this difference was only significant between the high and low achieving groups. Similar results were found in Table 4.10 which is based on the NumPA achievement levels with significant differences on the same two belief factors.

Table 4.10: ANOVAs on mathematics belief factors differentiated by NumPA

NumPA	*F*	df	p	*post hoc*	Cohen's *d*
Self	3.65	2, 703	**.027***	l&h (.027)	.57
Ability	2.45	2, 703	.087		
L.E.	1.30	2, 703	.274		
Nature	8.30	2, 121.186	**.0001***	l&m (.011) l&h (.0001)	.42 .75
Total	.39	2, 703	.679		

* Significance at p<.05

High achieving students on the NumPA have significantly higher means than the low group for the *Self* factor (M = 24.05 and 21.24, SD = 4.61 and 5.26, respectively), with a moderate effect size of d= .57 (Tables 4.4 and 4.10). High achieving students also had significantly higher means than either the middle of low achievers on the *Nature of mathematics* factor with a high effect size of .75 between the low and high means and a moderate one of .42 between the high and middle ones (Hattie, 2009). One explanation of the *Nature* results is connected to why older children may have higher factor means; children who are older, in general, may be studying mathematics at a higher level and thus have more availing beliefs (Krista Muis, 2004). Another non-age related explanation could be that children who do better at mathematics may have more availing beliefs based on their experiences of doing mathematics at a higher level than that of some of their peers. The relationship between achievement and beliefs about self as a doer of mathematics could be associated with the children's realisations that they have been identified as competent at school mathematics, and thus also identify themselves in the same way.

By changing the focus slightly and concentrating on the belief factors rather than the individual and school characteristics, another perspective emerged. For the *Self* factor, gender, school decile, as well as the two achievement measures (PAT: Mathematics and NumPA) accounted for significant differences of means while ethnicity, age and school year did not. For the *Ability* factor, ethnicity and school decile could account for significant differences of means, but none of the other characteristics explained different patterns of responding. For the *Learning Environment* factor, school decile was the only characteristic that accounted for significant differences of means. For the *Nature of mathematics* factor, age, both achievement measures, and school decile accounted for significant differences of means, but there were no significant differences associated with gender, nor with ethnicity. This belief factor perspective will be discussed in the final section of this chapter.

Mathematics Personal Mini factor

The preceding sections of this chapter have dealt with Maths Beliefs Questionnaire (MBQ) responses in terms of school and student characteristics that may or may not have affected or explained the different beliefs about mathematics. In these subsections I reported on and compared means for the four extracted mathematics beliefs factors. However, the way these factors are configured separates individuals' beliefs about themselves as doers of mathematics from their beliefs about how good they feel their own gender and ethnicity is at mathematics. Even though the *Ability* factor combines all the questions about gender and ethnicity, links between self-belief and gender/ethnicity beliefs about membership or identity within the world of mathematics needed to be explored in greater detail. For instance, the *Ability* factor cannot be used to help understand positions such as what a female Asian or male Pasifika child believes about her or his membership in this world of mathematics. In order to answer these questions, I calculated a Mathematics Personal Beliefs Mini factor (MPM) that included three questions. A different combination of questions was used for each category of student (See Table 4.11). Q4 :*I can do maths* was included for all the children. Q5: *Boys are good at maths* was included for all boys while Q6: *Girls are good at maths* for all girls. The appropriate ethnic statement from Qs 7, 8, 9, and 10 were given to the corresponding ethnic groups. An example is the Māori female group from the Student Database that includes Questions 4 (*I can do maths*), 6 (*Girls are good at maths*) and 8 (*Māori students are good at maths*).

Table 4.11: Student Questionnaire items included for the MPM belief factor

	Student items
Asian females	4+6+7
Asian males	4+5+7
Maori females	4+6+8
Maori males	4+5+8
Pakeha females	4+6+10
Pakeha males	4+5+10
Pasifika females	4+6+9
Pasifika males	4+5+9

Means were calculated for all students who identified both their gender and ethnicity. I reported and compared the student means for the three combined MPM items in terms of ethnicity, gender, school year, age, school decile and achievement data (See Appendix N, Table N.1).

Student achievement had no effect on the MPM as reflected in the means for both the NumPA and PAT (Mathematics). Age and school year also seemed to have no influence on the means; however, when these means were explored in terms of ethnicity, more differences became apparent (See Tables N.2 and N.3).

In general, boys had significantly higher means than girls on the MPM (M (SD) = 12.36 (2.09) & 11.98 (2.02), respectively, but the effect size was low. When comparing boys to girls within each ethnic group, the significant difference was only between Pākehā boys and girls with a moderately low effect size. On the other hand, even though Pasifika girls had higher means than boys and an effect size of .60, this difference was not statistically significant because of the small size of the Pasifika sample (Table N.2). As mentioned previously, overall, school year did not affect means of the MPM; however, Māori Year 5s had higher means on this score than Year 6s (M (SD) = 12.69 (1.74) & 11.97 (2.04)). Once again, Pasifika students, in this case in Year 5, had higher means than Year 6 students with an effect size above .40, but the difference in means was not significant.

On the whole, age did not account for different means on the MPM except for Māori students, but the effect sizes were below .40 (Table N.3). Means were also compared between ethnicities within gender with, for instance, ANOVAs indicating no significant differences in means for either girls or boys by ethnicity (Table 4.14). On the other hand, effect sizes above .40 indicate that the differences between the high means of the female Pasifika students and the other female ethnic groups is worth considering.

The minifactor acted as an additional way of exploring and understanding student identity beliefs about mathematics. To sum up:

- males had significantly higher mean scores than did females
- age and school year influenced Māori students' means but no other groups, while
- school decile and achievement levels seemed to have no significant effect on student means.

Making sense of the landscape

Through an examination of a landscape of the children's beliefs about mathematics by using a statistical lens, it was possible to begin focusing on elements that may have affected, or at least accounted for, the range of these espoused beliefs. The framework comprising the four-factor MBF, *Self*, *Ability*, *Learning Environment* and *Nature of mathematics*, derived from the factor analysis works as a way of making sense of the students' beliefs about mathematics. The framework differs slightly from the original conception of a three-factor model that was suggested by the initial research questions and the design of the questionnaires; however, in terms of the range of complexities within the school setting, including a fourth factor, the *Learning Environment*, makes sense (De Corte et al., 2010; Op 't Eynde et al., 2002; Yackel & Rasmussen, 2002).

Nevertheless, limitations of this method of model-making need to be acknowledged. A factor analysis can only be applied to closed questions; as a result, the very rich data provided by the open questions, by necessity, were excluded which may or may not have affected the make-up and intelligibility of the factors (Muis et al., 2006). Even more fundamentally, because the questionnaire items were designed to elicit responses to very specific research questions about mathematics beliefs, the factors were obviously influenced by the questions; in other words, these factors, although useful as a lens through which to analyse students' mathematics beliefs, are neither innate nor value-free. Even if these factors were extracted through a seemingly unbiased method of pushing metaphorical buttons in a statistics program, the factors are not neutral and objective because they are derived from a potentially biased and limited set of questions (Mason, 2004). In addition, I acknowledge that I was responsible for assigning the labels to these factors. Despite these shortcomings, the four-factor framework proved useful for grouping beliefs and exploring them in order to focus on relationships between mathematics beliefs and school as well as on individual characteristics. This framework also functioned in terms of relating the findings from this study to other New Zealand research.

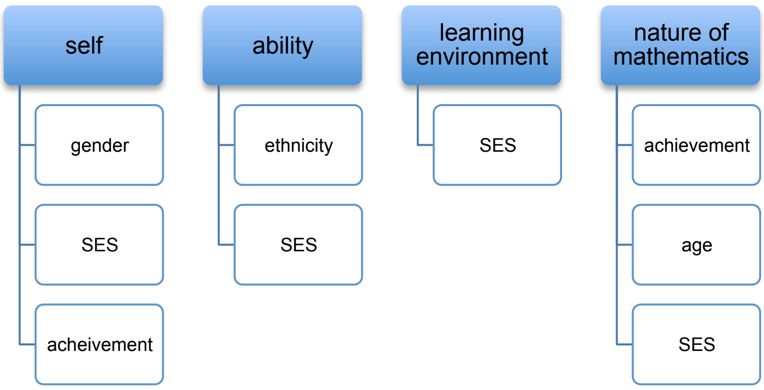


Figure 4.4: The four-factor framework of student mathematics beliefs (SMB)

Although no causal inferences could be made, school SES, gender, ethnicity and achievement levels seemed to account for some of the differences between means on the MBF (See Figure 4.4, p. 109). The relationship between gender and self-beliefs about mathematics proved to be particularly interesting and was similar to both the NEMP Mathematics (Crooks et al., 2010; Flockton et al., 2006) and TIMSS results which found “boys were more confident in their mathematics ability than girls” (Caygill & Kirkham, 2008, p. 43). Other studies report similar results even when boys’ achievement levels were equal to or lower than those of girls (Carr, Steiner, Kyser, & Biddlecomb, 2008; Vanayan et al., 1997). The New Zealand TIMSS results found that Asian students were more confident about their mathematics abilities than Pākehā, Pasifika and Māori children (Caygill & Kirkham, 2008). However, the results from NEMP (Crooks et al., 2010; Flockton et al., 2006) as well as this study indicate a different finding that Māori students were more positive than Pākehā. In terms of the relationship between the *Self* factor and school SES, NEMP Mathematics (2010, 2006) report that students from low decile schools were more positive than other groups while this study found that the students who attended mid-decile schools had the highest scores. TIMSS (Caygill & Kirkham, 2008) studies appear to ignore the relationship between SES and mathematics confidence.

On the other hand, TIMSS (Caygill & Kirkham, 2008) results are similar to this study in that they establish a link between achievement and confidence. Students who achieve within a high band have the highest self-beliefs while those in the middle are in turn higher than the low band. In addition, TIMSS reports that SES and mathematics achievement was closely related – the higher the decile the higher the achievement results – which suggests a relationship between SES, achievement levels and Self-belief about mathematics (See the link between SES and achievement in Appendix J and Table J.4).

The literature that explores beliefs, even those studies that include comparable groups, components or dimensions of beliefs about mathematics do not analyse the difference between individuals' beliefs in terms of group, school or individual characteristics (Kloosterman, Raymond, & Emenaker, 1996; McLeod, 1992; Op 't Eynde et al., 2002; Op 't Eynde et al., 2006). Instead, the focus is on understanding the *differences in beliefs* as a way of explaining *differences in achievement and engagement* for students (Greer, Verschaffel, & de Corte, 2002; Kloosterman, 2002; Leder et al., 2002b).

Although only the children's beliefs are discussed in this chapter, it was important to consider how this framework might apply to the teachers involved in this study. The teacher sample was introduced in Chapter 3 (p. 68), and the teacher version of the MBQ (the TMBQ) on pages 74-78. Some of the results from the TMBQ are reported in Chapter 6. A reason for expanding the model in this way was the recognition of the importance of the reflexive relationships between child, adult and context (Prout, 2002). De Corte et al (2010) write of the relationship between the culture and norms of the mathematics classroom. The notion of teachers as pivotal, but not all-powerful, needs to be considered because teachers establish the tasks, the ways of doing mathematics, and of being mathematical, in individual classrooms, and thus both influence and are influenced by the mathematics-related beliefs of their students (Li, 1999).The MBF, with slight adjustments, also proved useful as a framework for looking at teacher mathematics beliefs especially when analysing their interview responses as well as the video footage of the mathematics classrooms reported on and examined in Chapter 6: *Narrowing the Focus*.

Conclusion

From this exploration of the landscape of the children's beliefs about mathematics, a four-factor model emerged that was not only useful as an analytical framework for understanding the range of the responses on the Maths Belief Questionnaires, but suggested a framework for approaching and making sense of the qualitative data derived from the open-questions, interviews, observations and drawings. Rather than remain static, the four mathematics belief factors evolved, through an iterative process of analysing these data, into overlapping themes that helped me decipher the complex world of mathematics beliefs at Kikorangi and Whero Schools: beliefs about the self as doer of mathematics, as successful member of the world of mathematics, about the groups of people who belong to this world, about the mathematics classroom environment including teaching, learning and doing maths within social contexts, as well as about the nature of maths. I report on and discuss the complex worlds of mathematics beliefs at the two focus schools in Chapter 5: *Reading the pictures* and Chapter 6: *Narrowing the focus*.

Chapter 5: Reading the Pictures[17]

> But the most exciting, startling and perceptive critics of visual images don't in the end depend entirely on a sound methodology, I think. They also depend on the pleasure, thrills, fascination, wonder, fear or revulsion of the person looking at the images and then writing about them. Successful interpretation depends on a passionate engagement with what you see (Rose, 2007, p. 4).

In this chapter, I concentrate on an area of the landscape of beliefs inhabited by the children of Kikorange and Whero, the two focus schools. I begin this chapter with a discussion of the dilemma that led to the inclusion of a visual task, and I explore the literature associated with visual methodologies as well as with reading and interpreting art. In the second half of the chapter, I describe the data, methods of analysis and discussion of the children's maths beliefs drawings.

The dilemma

Q33: *If you met an alien who had never done maths, how would you describe maths and what it is about?*

Student: Can I draw it?

(School 2, boy 21.11, Year 5, age 9)

A dilemma inherent in accessing beliefs is that an individual's beliefs are neither easily observed nor analysed. Traditionally, beliefs have been accessed either by asking individuals what they believe through questionnaires and/or interviews, or by observing their behaviour (Hofer, 2002; Lester, 2002). Both of these methods have their limitations. Inferring (distilling the essence of from the information at hand) and interpreting (explaining) beliefs, which are private, internal thoughts, from classroom observations alone seems problematic if not impossible (Lester, 2002). Self-report measures such as questionnaires and

17 Some of the ideas, analyses and discussion incorporated in this chapter have been partially reported in other publications (Solomon, 2012a; Solomon & Grimley, 2011).

interviews also have limitations in that respondents may answer in ways they think the researcher expects (Creswell, 2003), which could be exacerbated in research with children where there is a power differential (Balen et al., 2000/2001; Greene & Hill, 2005; O'Kane, 2008; Punch, 2002). Another problem with questionnaires as Wetton and McWhirter point out is that

> [t]hey fail to tap into the children's own images and the language the children themselves use. Rather, they provide answers to questions which adults have posed in adult language with predetermined answers in language which adults have chosen (1998, p. 265).

An additional dilemma when working with young participants in the area of beliefs about mathematics is that some of them may not be able to articulate exactly what they believe while others may not know what they believe at all (Young-Loveridge et al., 2006). Consequently, I chose to combine the following methods of accessing beliefs (Lester, 2002; McDonough, 2004) by using self-report measure, as well as observations at different times. I also attempted to make sure that the methods were of interest to the participants and allowed them control over how much they shared with the researcher (Christensen & James, 2008a).

Because I was interested in what the children had to say about their beliefs in their own language and images, I had included open questions in the MBQ. During Phase 1 of my data collection, where 823 Year 5 and 6 students completed the MBQ reported in the previous chapter, I discovered two limitations that required me to reconsider the methods I was using for collecting this information. Although a reader-writer was available for anyone who requested help, many of the children were reluctant to write much on the open questions, or genuinely struggled to do so, which concerned me because the method I was relying on assumed adequate literacy skills. After I had entered all of answers to the *Alien Task* into a database, a second limitation became apparent. The majority of these children portrayed mathematics in terms of number: 688 (84%) described mathematics as number, and only 35 mentioned other areas of the curriculum while 32 chose a "dunno" response or left the question blank. The issue became whether this was their genuine belief about the nature of mathematics as narrow and restricted to one area of the New Zealand Mathematics Curriculum (Ministry of Education, 1992) which also included the strands of measurement, geometry, algebra and statistics. This narrow construction of the nature of mathematics may have been the result of their own prior beliefs, their teachers' beliefs or the emphasis placed on numeracy because the majority of the schools (16 of the 17) were involved in the implementing the Numeracy Project (Ministry of Education, 2006c, 2006d).

While grappling with these competing explanations, I wondered if the question were asked differently, would different sorts of answers eventuate or would the children still view mathematics as number? The solution to these problems came in part from the children, from student 2.21.11's suggestion ("Can I draw it?"), and the19 other children who chose to illustrate the *Alien Task*. Two other students suggested in words that a drawing would be a good way to answer the question as in student 17.5.5's "Diew picis to kqslant" ("Draw pictures to explain" what maths is.). In order to solve this dilemma, I introduced a drawing task at the two focus schools: eight classes of children were asked to draw "What maths or doing maths means to you".

Visual images

In this section, I report on some of the literature that deals with visual images. The section follows my peregrinations through what was for me completely novel, unchartered territory; in particular, it covers the advantages of using a drawing task especially with children, the ways of looking at the visual, the fields that have a tradition of using visual methodologies, as well as the challenges associated with using, analysing and interpreting visual data.

Advantages of using a visual task

Drawing is essentially an image-making activity that produces a visual image. An image-making task was selected because children often choose to communicate their experiences, feelings and beliefs by drawing (Christensen & James, 2008a; Veale, 2005). When children choose to draw as a social activity, they often display distinctive, idiosyncratic images representing their unique experiences (Greene & Hogan, 2005). Children may choose to draw in order to "reconstruct and assimilate the experiences they have had" (Barnes, 1987, p. 1); in other words, to make meaning of the world. As Freeman and Mathison (2009) explain, "Images are a rich source of understanding the social world and for representing our knowledge of the social world" (pp. 109-110). Consequently, drawings provide researchers with another window for looking at or accessing children's lived experiences (Anning & Ring, 2004; Barnes, 1987; Christensen & James, 2008a; Einarsdottir, Dockett, & Perry, 2009; Golomb, 1992; Hanes & Weisman, 2008; Horstman, Aldiss, Richardson, & Gibson, 2008; Hubbard, 1989; Kendrick & McKay, 2004; Kostenius & Öhrling, 2008; Veale, 2005). Drawings may also be a vehicle for envisioning other "things that are not in themselves visual" (Cox, 2005; Mirzoeff, 1999, p. 5), in the case of this study, beliefs. Furthermore, by considering different sign systems, for example both images and words (Vygotsky & Cole, 1978), researchers are able to view the world in different ways with multiple interpretations of both belief and experience (Short, Kauffman, & Kahn, 2000).

Another advantage of using an image-making task is that "[s]tudents' artistic expressions can provide teachers with additional ways of determining what they understand about facts and concepts as well as how they understand them" (Sidelnick & Svoboda, 2000, p. 176). Children may also find drawing useful for explaining concepts and experiences that they have difficulty putting into words (Golomb, 1992; Horstman et al., 2008). Classroom teachers and researchers frequently have difficulty trying to discover what individual children know, understand or believe about a topic or task, particularly in cases where their language or verbal communication skills are inadequate; in this case, the drawing task was used to try to mitigate this problem.

Using a drawing task as a method of collecting data has the advantage of being easy to administer and fitting comfortably into the normal daily activities and routines of the primary classroom as part of the classroom culture, something done both for fun and as part of the curriculum. Because many students see drawing as fun, image-making is often viewed as both an in- and out-of-school activity, outside the formal context of the classroom (Christensen & James, 2008a; Veale, 2005). To avoid student anxiety associated with being evaluated, this task was presented as a low-stakes, low-stress activity where peer conversations were acceptable, rather than as an "art" task. The children had the freedom to draw using diagrams, words, colours, pens or pencil and to choose not to complete the task but to work on another activity instead.

Looking at art

A challenge associated with visual-images, in this case the drawings of beliefs, is that of looking at and "reading" the images. Depending on one's "angle of repose" (Richardson & St. Pierre, 2005, p. 963)(See Chapter 2, Negotiating research quality), one views and understands visual images differently. If one looks at a piece of art, one brings one's history and experience of looking at art, one's cultural biases, one's sense of aesthetics, one's values and personal beliefs about what art means and how to read or interpret it (Bal, 2006a; Banks, 2001). One looks at the piece as a whole, at its cultural and historical context and value. One tries to understand what it means, what the artist's intentions are, or perhaps what one thinks the creator was trying to communicate to her or his audience (Banks, 2001). Art can illustrate, illuminate, enliven stories, represent belief systems; it can threaten, shock, gladden, lift the spirit or plunge the beholder into despair. It may appear on the street, in galleries, churches and museums, on paper as in this study, in books, in gardens or on people.

If one approaches art and visual images as visual texts, then reading these texts incorporates elements of decoding and of sense-making/comprehension/ interpretation. Reading art is a learnt skill with its own sets of language and conventions that are influenced by the reader/viewer's experience (Arnheim, 1970; Banks, 2001); one may read a piece through the lens of the art historian, the art critic or the humble amateur (Bal, 2006b). One hones one's skills as for reading and analysing literature; in the same way, one learns to recognise the language of reading art, the symbols and the metaphors. One does not necessarily expect every piece to be completely representational, for every element to correspond directly to part of the physical world. While some observers may prefer representational art, others seeing themselves as more sophisticated may denigrate certain examples as "chocolate box" art. What may count as aesthetically pleasing and valuable is influenced by the viewer's culture; what I find moving or pleasing may be an anathema to one from another cultural tradition. I have learnt that I view and speak about the primitivism of bushman rock art differently from the naïve style of Rousseau, that Picasso's blue period was not the result of his limited colour sense, that I do not have to identify actual things or concepts in abstract paintings, and that I can accept Banksy as a wonderful artist while still disliking most other graffiti. These are a few of my biases and learnt responses to viewing art.

On the other hand, when looking at a piece of art created by a child, adults generally apply a different set of criteria in approaching the piece. Historically, children's drawings have been used in intelligence tests, in developmental psychology (where they have been analysed in terms of a cognitive or developmental deficits), or as a reflection of an emotional, usually troubled, state (Einarsdottir et al., 2009; Golomb, 1992; Hubbard, 1989; Rose, 2007). In the 1930s, Florence Goodenough developed the *Draw-a-man* test, which was revised in the 1960s by Dale Harris, and was used to measure children's intelligence in terms of how realistically they drew, and the number of details or elements they included in their drawings (Anning & Ring, 2004; Freeman & Mathison, 2009; Golomb, 1992; Malchiodi, 1998). The test lacked validity in that it was normed on a very narrow range of ages and showed cultural bias, but this did not halt its use (Golomb, 1992). Since the publication of Howard Gardner's book on multiple intelligences (Gardner, 1983), notions of what constitutes intelligence or intelligences have changed from narrow notions of fixed cognitive ability based on logical reasoning (often tested in terms of verbal, numerical and spatial reasoning) to a more multi-faceted flexible approach (Deary, 2013), reflected in tests such as Urban and Jellen's *Test For Creative Thinking-Drawing Production* that is used for assessing creativity (Urban, 2005).

The child’s work is often viewed as a scribble that lacks intention and thus is of insignificant value, or it is seen as something incomplete because it is created by a developing, and as yet, not quite competent individual (Cox, 2005). Those who follow a Piagetian developmental approach see children’s drawings as deficient in terms of adult skills, conceptions of perspective, composition and colour (Golomb, 1992; Hubbard, 1989). Art educators such as Lowenfeld (1939; Lowenfeld & Brittain, 1987) and Kellogg (1969) are examples of those who follow a stage theory of children’s art competence. They also judge children in terms of how realistic their work is, how closely elements mirror what they look like in real life, which seems to contradict the way one is taught to read adult art (Anning & Ring, 2004; Freeman & Mathison, 2009; Golomb, 1992; Malchiodi, 1998).

Drawings have been examined and interpreted in terms of understanding children’s personality traits, diagnosing mental illness, trauma, illness and behaviour. From my perspective, some of the interpretations made about individual children’s mental states based on their drawing seem dangerous, such as Machover’s body parts and their accompanying, often sexual or erotic explanations, without looking at possible alternative explanations of context, development, or the artist’s explanation (Golomb, 1992; Hubbard, 1989; Malchiodi, 1998; Pridmore & Bendelow, 1995; Wakefield & Underwager, 1998). Another popular diagnostic tool is the Burns and Kaufman’s (1972) *Kinetic Family Drawing* (KFD) which Golomb (1982) criticises both because it was not adequately normed, and because of the way the symbols included in the drawings are interpreted. She advises “…against using drawings as if they were an x-ray of the child’s heart and mind” (p.306). Hubbard also criticises Burns’ method of analysing KFD responses commenting, “In interpreting the behaviors and motives of children, adults are liable to approach the task from their own world views and conceptions; they are often *adultcentric*” (1989, p. 11) and use adult assumptions of the child’s meaning or intention.

Researchers of children’s art, such as Hubbard and Golomb, have a completely different perspective than those who impose their own interpretations about children's creations in terms of children's personalities and/or mental states. From Hubbard's experiences of working with children's drawings and consulting the creators' about their meanings, she comes to very different conclusions (1989). Golomb, who views "children's drawings as a creative search for meaning" (1992, p.11), investigated a large number of children's drawings and found commonalities or similar development trajectories for all children—the normally developing, the extraordinarily gifted child artists, those disadvantaged, children cognitively or intellectually impaired, those with mental problems, children from different cultures, as well as children with traumatic experiences, for instance concentration camp inmates.

Children's image making is frequently viewed as a valuable means of developing communication skills before they learn to read and/or write, or during the process of becoming a skilled reader and writer. In most primary schools, drawing and picture making is acceptable as a means of communication in the first few years of school. Children are encouraged to draw what they did, their reactions to a story, their observations, their experiences and as they develop writing skills to add words to the work. Thereafter, the written word gains importance, and image making is often relegated to time filling (Anning & Ring, 2004; Wright, 2010), a treat, decoration or the 'art class'. In other words, it is not seen as a valuable means of expression or communication in its own right, contrary to the manner in which adult art is viewed, where using multiple symbol systems to communicate is accepted. In contrast, certain art theorists consider the visual and verbal of equal importance. For Arnheim, art making is a cognitive activity which he calls visual thinking. He also sees art as capable of making "things visible that are invisible or inaccessible or born of fantasy" (1970, p. 254). He treats children's image making with as much respect as adults', as vehicles for the expression of visual thinking, reasoning and sense making. Similarly, Dewey, though not writing about children's art, explains his understanding of the differences between thinking and communicating through art and words: "Thinking directly in terms of colors, tones, images, is a different operation technically from thinking in words. … If all meanings could be adequately expressed by words, the arts of painting and music would not exist"

(Dewey, 1934, pp. 73-74). Ruth Hubbard's perspective is slightly different as she sees thinking as something active that needs articulating through many media and that "ideas may take form in images, movement or inner speech" (1989, p. 3). In addition she argues, "Drawing is not just for children who can't yet write fluently, and creating pictures is not just part of rehearsal for real writing. Images *at any age* are part of the serious business of making meaning—partners with words for communicating our inner designs" (Hubbard, 1989, p. 157) (author's italics). The equal importance of words and images for accessing what children know, understand and how they make meaning of their experiences is also seen as important by Wetton and McWhirter (1998).

For others, children's image making is important enough on its own without verbal accompaniment or comparison. "Art provides us with a prime means of learning, understanding and communicating" (Manners, 1987, p. i), or as a way to "reconstruct and assimilate experiences they have had" (Barnes, 1987, p. 1). Golomb frames children's art as a language "that children master in their quest to understand their world and to express their feelings" (1992, p. 305). Reading the language of children's drawings, what they are trying to express, can be a way of understanding the artists' worlds, their experiences, ideas, thinking and emotions (Anning & Ring, 2004; Malchiodi, 1998). The reader, however, needs to be aware that drawings are created within specific social and cultural contexts which influence how and what children draw (Malchiodi, 1998).

Visual methodologies

Visual images as data and the use of visual methodologies are acceptable and much more commonplace in socio-cultural, historical, sociological, and anthropological research than in education. This is the case particularly within an ethnographic research tradition (Banks, 2001; Pink, Kürti, & Afonso, 2004; Stanczak, 2007) as well as in media studies (Gauntlett, 2008) and more recently in health studies research (Wetton & McWhirter, 1998). The data used in these studies range from photographs, to film/video, maps, tables, graphs, figures, diagrams, tattoos, and street art to drawings and more traditional art forms (Prosser, 1998a; Rose, 2007).

Visual data may be read as mirrors of reality, documents from which information may be extracted, or as examples of how their creators constructed reality depending on the researcher's theoretical and/or methodological biases (Leeuwen & Jewitt, 2001). Visual images have creators and readers/ interpreters/consumers; they have narratives, tell stories which may or may not be the same as their creators' intentions (Bal, 2006b; Banks, 2001). Visual data may be collected, made or created by the researcher or by the research participants, which has implications for how the data are viewed and analysed. For some researchers, the images on their own are sufficient while others believe that the context of the image is also important if not essential (Bal, 2006a; Banks, 2001; Leeuwen & Jewitt, 2001). In either case, however, the experience and socio-cultural background of the reader affects the reading of the image.

Many researchers study and analyse visual data from a viewpoint of semiotics that is the study of signs (Banks, 2007; Harper, 1998; Kress & van Leeuwen, 2006; Leeuwen, 2001; Lister & Wells, 2001). Images and the making of images can be viewed

> as a process in which the makers of signs, whether child or adult, seek to make a representation of some object or entity, whether physical or semiotic, and in which their interest in the object, at the point of making the representation, is a complex one, arising out of the cultural, social and psychological history of the sign-maker, and focused by the specific context in which the signmaker produces the sign (Kress & van Leeuwen, 2006, p. 7).

When using a semiotics lens, the social and cultural context of both the signmaker and the sign are important during the process of making sense of visual images. Bal, in her discussion of reading images, takes a similar approach; however, she focuses on reading signs, on the reader who needs to decode and interpret in order to understand what is being read/viewed (2006b, p. 290). On the other hand, Mitchell's language is slightly different; he writes about iconography and decoding the language or grammar of images (1980) although Kress and van Leeuwen (2006) also refer to the term *grammar* or the rules and conventions associated with visual communication.

A completely different approach to visual methodology and the reading of images is used in health research with children where the *draw-and-write* procedure is commonplace (Backett-Milburn & McKie, 1999; Bak & Piko, 2007; Hantler, 1994; Horstman et al., 2008; McWhirter & Weston, 1994; Pluhar, Piko, Kovacs , & Uzzoli, 2009; Pridmore & Bendelow, 1995; Wetton & McWhirter, 1998). This research technique, which Doreen Wetton pioneered in the 1970s, had been used with over 22,000 children including a large number of under 5 year olds in the United Kingdom alone before the year 2000. Initially, only the written text the children included was analysed while the drawings were used to support the researchers' understandings of the language (Wetton & McWhirter, 1998); more recently drawings have been given greater if not equal consideration (Bak & Piko, 2007; Horstman et al., 2008; Pluhar et al., 2009; Pridmore & Bendelow, 1995). Some of the strengths of this procedure are that it works well with young children, and it has been validated as both a qualitative and a quantitative research tool (Wetton & McWhirter, 1998).

This *draw-and-write* protocol has been implemented and adapted to other areas of educational research such as mathematics (Kilpatrick, 2004; Kilpatrick, Carpenter, & Lomas, 2006). However, the approach has also been criticised, or, at least, researchers have been cautioned to be aware of the pitfalls of over-interpreting drawings, of ethical concerns surrounding coercion of young participants as is the case with other forms of research with young participants (See Chapter 2, Considerations with working with children) (Backett-Milburn & McKie, 1999; Bak & Piko, 2007; Horstman et al., 2008; Pridmore & Bendelow, 1995). In addition, Backett-Milburn and McKie (1999) alert researchers to issues of using what is essentially a qualitative methodology in a quantitative way. They are critical of researchers who use this protocol merely to quantify "pictorial content" (p. 392) rather than investigate drawings as influenced by the particular task and context which can be used as a way of exploring the "child's inner world or experiences" (p. 394).

Visual methodologies have also been used in the field of education within a wide range of theoretical traditions and methodologies. Dyson's research into school literacy is an ethnographic study (2003) as is Carruthers and Worthington's study of mathematical mark-making (2007). In contrast, Kostenius and Öhrling (2008) as well as Yuen (2004) approach their data from a phenomenological perspective looking at children's lived experience (Van Manen, 1990). Wright (2007a, 2007b, 2010) applies a *drawing-and telling* methodology which focuses on young children's narratives that is similar to Einarsdottir et al.'s (2009) focus on meaning making by exploring children's narrative during the drawing process. Much visual research in education is interested in the construction of meaning by applying a social constructivist lens (Kendrick & McKay, 2002, 2004; Sidelnick & Svoboda, 2000) and a focus on multiple sign systems (Kendrick & McKay, 2002, 2004; Short et al., 2000; Worthington, 2009; Wright, 2007a, 2007b). Walls uses children's drawings as one of the data sources in her study of children's sociomathematical lives by applying a symbolic interactionist framework (Walls, 2003, 2009). In some cases, visual data are collected and analysed without any clear articulation of a theoretical perspective although the "Draw a scientist/mathematician" research (Huber & Burton, 1995; Picker & Berry, 2000), I would claim could be read as phenomenological studies.

Whatever the researchers' theoretical approaches may be, Gillian Rose encourages them to develop a "critical visual methodology" that "takes images seriously", "thinks about the social conditions and effects of visual objects", and examines their individual "ways of looking at images" (2007, pp. 15-16). Banks suggests the visual researcher asks questions about the content, the creator and the context of the image. He also suggests that the owner and contextually accepted readings of the image need to be considered (2001). For Banks, the content of the image is its "internal narrative—the story, if you will, that the image communicates. This is not necessarily the same as the narrative the image-maker wished to communicate" (2001, p. 11). The visual researcher needs to adopt a systematic, theoretically-based method of reading the story and thus analysing visual data, a method that incorporates an awareness of the contexts of the images as well as of the researcher's own social and cultural position.

Challenges with using visual data

One of the difficulties with using visual data is that the mainstream, academic community continues to view the verbal as superior in both validity and veracity to the non-verbal, despite a long tradition of using visual methodologies and data in fields such as sociology, visual studies and health, to name a few (M. Banks, 1998). Some who fail to accept the visual believe that "… art, drawing and images of most kinds are thought of as being about expression, rather than about information or communication" (Wright, 2010, pp. 7-8) thus giving images less serious consideration than words. Prosser takes a different position towards visual data that are traditionally viewed as "acceptable only as means to record data or as illustration and subservient to that of the central narrative" (1998b, p. 99). The traditional view is that visual images are acceptable in the subsidiary role of supporting the verbal rather than important and to be trusted in their own right, and able to contribute to understanding on their own. These data cannot be trusted because they are created in a social context, and are contaminated by this process; the process is very difficult to separate from the maker and image itself, thus adding to the complexity inherent in analysing visual data (Prosser, 1998b). A third argument against image-based data addresses the dichotomy between the image as "record" as opposed to "construct" (Leeuwen, 2001, p. 6), the difference between image created by the researcher as opposed to that created by the participant or the researched. The criticism is leveled at researcher-recorded data in particular because of researcher bias in selection of data and composition of images (Banks, 1998; Leeuwen, 2001; Pink et al., 2004; Prosser, 1998a, 1998b). Other objections to the use of visual data are associated with the analysis of images, especially of complex, abstract drawings (Pridmore & Bendelow, 1995) and with data being over-interpreted or misinterpreted in the case of children's images (Punch, 2002).

Yet within verbal-based research, words are used in the collection and the recording of data produced by both the participants and the researcher within specific social and cultural contexts, in the same manner as images. Researchers who adopt visual methodologies view verbal and non-verbal data as equally valid (Kress & van Leeuwen, 2006; Mathison, 2007; Rose, 2007). "Images like any data, can be used to lie, question, imagine, critique, theorize, mislead, flatter, hurt, unite, narrate, explain, teach and represent" (Freeman & Mathison, 2009, p. 110). A data source is not completely disregarded because it may have been abused or may have the potential to be misused. Much of the criticism of image-based data seems to apply equally to a variety of other data sources, including interviews, written responses and observations. Like any other research method, if due diligence is taken with the research questions, methodology, ethical considerations, data collection and analysis, the use of image-based data should be no less legitimate, reliable or valid than any other.

In visual research that uses children's drawings, the researcher ought to be cognisant that a drawing activity is a one-off task influenced by the context—the time, the location, interactions with the people and the history—of that moment. What the children believed about mathematics when they were asked to draw was potentially affected by their peers, their teacher or how they were feeling, and what happened in class and in the playground that day, or perhaps even the day before and/or at home: the drawings were a response to the task but coloured by the creators' lived experiences. A drawing in response to the same task at another time or place may be very different.

Analysis of visual data

During the process of exploring the terrain associated with visual methodologies, I had difficulty in finding clear and coherent information from the literature about the practical aspects of analysing visual data. This lack of information about how to analyse drawings in a systematic, explicit and informative manner is noted by others writing about visual images (Backett-Milburn & McKie, 1999; Horstman et al., 2008). Even though many have studied images and other visual data, "there are remarkably few guides to possible methods of interpretation and even fewer explanations of how to do these methods" (Rose, 2007, p. 2). In this section, I present some of the methods employed by researchers in this field: the first group of methods deals with quantitative, qualitative and mixed methods approaches, the second gives examples from health studies, and the third from education.

In response to the problem associated with the lack of clear guidelines, Rose critically examines a range of methods for analysing visual images, such as compositional interpretation, content analysis, semiology (the study of signs), psychoanalysis and discourse analysis (Rose, 2007). Different disciplines and theoretical perspectives affect the approach adopted towards analysing visual data. Some researchers adopt quantitative procedures such as content analysis and semiotics (the study of signs and symbols); others within the traditions of phenomenology and ethnography use more qualitative ones; while a third group uses a mixed methods approach. Both Bal (2006b) and van Leeuwen and Jewitt (2001), semiologists, analyse visual images by looking at the signs and syntax of the images; however, Bal stresses that the reader's "own frame of reference" (p 298) affects the reading while van Leeuwen and Jewitt discuss semiology in terms of denotative and connotative interpretations as well as the social relationships between the elements of the image. Collier, writing about visual anthropology, argues for a "combination of artistic and scientific processes", the qualitative with the quantitative (2001, p. 58). For him, analysis is a four-stage process that begins and ends with open questions: in the first stage, one looks at the data all together and notes one's personal responses returning to the data at the end (stage 4) and viewing it in context in order to tease out the significance of the analysis. The two middle stages involve a quantitative approach by using an inventory of the image in order to conduct a systematic analysis using statistical procedures. For Collier, "patterns alone do not produce meaning and our search for it is complicated by our dual role as investigators and cultural beings. The cultural lenses through which we operate inevitably shape our analysis" (2001, p. 58). All these approaches to visual analysis explore images by using quantitative or a mix of methods while retaining a socio-cultural lens and an awareness of the influences of the reader's/viewer's own cultural background or stance.

Researchers who adopt a qualitative approach also use a sociocultural lens through which to read their visual data. One group of researchers describes their approach to classifying, analysing and interpreting as reflexive (Gold, 2007; Goldstein, 2007; Kostenius & Öhrling, 2008; Pink, 2007), a term that seems to have been colonised by visual sociologists and anthropologists (Stanczak, 2007), that often include iterative methods of analysis. For Goldstein this process includes analysis of "content, perception of content and context " (2007, p. 79) while Gold describes a cycle where data are analysed in terms of sociological questions which in turn engender additional questions (2007). Cox, in writing about children's drawings, also uses an iterative process of analysis while, or until, a theory develops (2005). Kostenius and Öhrling use a three-step approach to examining children's lived experiences through their drawings by "seeking meaning, theme analysis and interpretation with reflection" (2008, p. 26) p 26. Anning and Ring (2004) write of using content and style, similarities and differences in contexts and accompanying information from both young participants and the adults in their lives in order to develop categories from children's drawings. Analysis of visual data is clearly not a straightforward process, and all of these qualitative researchers are grappling with ways of developing or extracting themes from their visual data.

Although the *draw-and-write* protocol is widely used, especially in health studies (Backett-Milburn & McKie, 1999; Horstman et al., 2008; Veale, 2005), very few authors include detailed descriptions of how their data are analysed (Backett-Milburn & McKie, 1999; Pridmore & Bendelow, 1995). An example of this kind of omission is Punch's statement that "care had to be taken not to misinterpret the children's drawings and impose adult interpretation and analysis" (2002, p. 332) rather than explaining how she codes and analyses drawings. Initially, most *draw-and-write* analyses applied a quantitative content analysis process; however, Backett-Milburn and McKie criticise those who apply only a quantitative approach by arguing for qualitative interpretive methods (1999). More recently, mixed methods processes of coding and identifying themes based on content categories have become more accepted (McWhirter & Weston, 1994; Pluhar et al., 2009). Some researchers such as Hortsman et al. are more explicit about how they analysed drawings and their accompanying words by looking "both for what the child was trying to say and for the content" (2008, p. 1005) which was later classified into themes by the researchers.

Wright, an early childhood researcher who explores young children's creativity, is also explicit about her process of analysis (Wright, 2007a, 2007b, 2010). She uses a *drawing-telling* method rather than a *draw-and-write* protocol in order to understand children's meanings by looking at the "content" and "form" of the work. Her notion of form includes symbols, the story of the piece and the making of the piece, as well as how the creator uses her/his body in talking about, acting about and creating the drawing (2010, p. 23).

Visual methodologies and data have also been used in other areas of education, many of which seem to use an iterative, interpretive process. Like health studies, very few clear, detailed explanations of how these data are analysed are available (Kilpatrick et al., 2006; 2002; McDonough, 2004; Sidelnick & Svoboda, 2000). Some researchers analyse their data in terms of predetermined categories; for instance, Huber and Burton in their research on what children think scientists look like(1995). In mathematics, researchers have analysed visual data in order to devise their categories (Carruthers & Worthington, 2007; McDonough, 2002; Picker & Berry, 2000; Walls, 2003) through various interpretive approaches; for example, phenomenology (McDonough, 2002) and grounded theory (Carruthers & Worthington, 2007). It is also worth acknowledging Sam and Ernest's study on the "Public images of mathematics" (1999) which adopts an interpretative approach towards analysing images and developing themes even though these images are verbal rather than visual images.

The previous paragraphs have discussed analysis methods of visual data; however, it is difficult to untangle analysis, the breaking into bits, from interpretation, explaining the meaning, in many of the studies. One of the exceptions is Freeman and Mathison's framework for interpreting visual data that uses a series of questions to focus on the subject matter, the creation of the image, and the audience (Freeman & Mathison, 2009; Mathison, 2007). I used this framework as a starting point for my analysis of the children's maths drawings, and as a map for me to follow.

	Readings of Images		Questions to establish credibility of images
Focus on subject matter	Literal reading	Low Inference	What are the physical features of the image? Who or what is portrayed? What is the setting?
	Biographical reading		What is the relationship of the image to current practices? To identities? How is the image socially situated?
	Empathetic reading		What common experiences are invoked?
	Iconic reading		How does the image relate to bigger ideas, values, events, cultural constructions?
	Psychological reading	High	What are the intended states of mind and being?

Figure 5.1: Freeman and Mathison's framework for interpreting visual data (Mathison, 2007) (Table 1)

In Figure 5.1, I have included only the subject matter section of the framework because this was the focus of my analysis. This map suggested a way to begin with a literal, low inference reading of the drawings and work in series of iterations of readings that required higher levels of inference and interpretation.

Multiple readings of the data

The ways of reading, which includes both looking at and making sense of visual data are as varied as the research questions and the researchers' theoretical perspectives which can act as filters for analysis and interpretation. Whichever method or methodology is chosen, the researcher remains responsible for selecting images, devising categories for coding or making sense of them, and finally for distilling themes. From a complementary methods approach (Green et al., 2006a) and based on my research questions, I had the benefit of reading the drawings in multiple ways in an attempt to discover how the content the children included in their drawings might reflect their beliefs about what constitutes mathematics, how they view themselves as part of the world of mathematics, their unique representations of their individual experiences of the classroom, and their position in the classroom.

I explored and analysed the visual data multiple times over three years using a combination of quantitative and qualitative readings. The first reading was an attempt to solve the dilemma presented at the beginning of this chapter. To do this I analysed the content of the drawings and compared the elements to those included in the responses on the *Alien Task* (MBQ). The second reading included coding the drawings in terms of the factors extracted from the principal component analysis derived from the children's responses on the MBQ (Chapter 4). These factors were *Sel*f, *Ability*, *Learning Environment* and *Nature of mathematics*; however, in this chapter I refer to *others* as a shortened form for *ability of others*. The third reading included a thematic analysis. The readings complemented each other and presented me with numerous perspectives in my analysis of the complex content of the image-based data.

All of these readings of the drawings were influenced by two frameworks, Freeman and Mathison's framework for analysing visual data, especially the subject matter section (Freeman & Mathison, 2009; Mathison, 2007)(See Figure 5.1) as well as by Nuthall's three worlds of the classroom (Chapter 2). In terms of Freeman and Mathison's framework, I initially concentrated on a literal reading because it required the lowest level of reader interpretation, and the results confirmed my concern that the view of mathematics was largely limited to number. Once I began looking at how the students positioned themselves in the world of mathematics, I used the "biographical", "empathetic" and "iconic" levels by combining what was being presented on the page with other sources of data about the students and classrooms. The drawings that included affective elements or metaphors, which indicated an individual's state of mind, lent themselves to "psychological" readings (Freeman & Mathison, 2009; Mathison, 2007). Another influence on the readings, Nuthall's three worlds of the classroom, offered an alternative method of reading the same images by looking at which of these worlds the students chose to include in their drawings: the public world, the semi-private world of peer interactions and / or the children's private internal worlds of individual children (2007, p. 84).

I chose to exclude developmental, psychological trauma and deficit interpretations from the analyses of the children's drawings. These discourses are problematic when applied to a classroom task where the intent is to communicate beliefs about a topic or domain. Taken out of context, many of the drawings seem to indicate developmental delays or traumatic experiences; for example, some include burning brains, children vomiting and crying, and many use stick-like, cartoonish characters common in the drawings of much younger children. However, a conflicting and more appropriate interpretation is that this style of drawing is a culturally mediated, "… symbolic system learned in later childhood as a kind of shorthand and which is used when shapes or recognizable features are unimportant" (Johnson, 1993, pp. 154-155). A system of using cultural images, often derived from television, books, comics and computer games, is used to convey the essence of the picture quickly. Children often incorporate cartoon-like images, stick figures and images from popular culture when they draw, especially in informal situations when communicating with each other (Picker & Berry, 2000; Wetton & McWhirter, 1998; Wright, 2010).

Throughout the iterative process of examining the drawings, I needed to be aware of the challenges associated with multiple readings such as the interpretation of conflicting meanings; for example, in drawings where place (classroom or home), person (parent or teacher, self or other child) or emotions (positive or negative) were unclear, and of the trustworthiness of my analysis (Was I seeing what I thought I was seeing? Were my interpretations valid? Etc.). In order to mitigate these risks, I considered other evidence; for example, what the children wrote on their drawings and MBQ, what they said to me and others, their classwork, as well as my classroom observations and field notes.

The next three sections report the data analyses of the children's mathematics beliefs drawings. The first reading comprises a comparison of the *Alien Task* (p. 122) and the drawings, followed by the second, an analysis in terms of the four-factor mathematics beliefs framework (SMB) (Chapter 4, Figure 4.4), and the third with an analysis of themes.

The first reading: comparing the *Alien Task* with the drawings

Before implementing a drawing task as an additional or alternative way of accessing children's beliefs about mathematics, I undertook an analysis of the responses to the *Alien Task* (Q33 MBQ): *If you met an alien who had never done maths, how would you describe maths and what it is about?* The responses were coded into initial themes or elements; however, I was aware that this type of analysis, like any other, was coloured by my experiences, research questions and interests. Content analysis is sometimes touted as a value-free quantitative method, but Rose refers to it as "counting what you (think you) see" (2007, p. 54). To mitigate potential bias, I coded and re-coded data over time as well as checked coding categories with colleagues. The results are summarised in the following table:

Table 5.1: Summary of responses for the Alien Task

students N = 823	Number	Other strands	Teaching/ learning	Utility	"dunno"	drawing
number	688	35	172	81	32	11
%	83.6	3.9	20.9	9.8	4.3	1.3

Almost 84% of these students wrote about mathematics in terms of number while only 4% included other strands such as geometry, statistics or algebra. A sizeable group of these students wrote about mathematics in terms of teaching and/or learning (21%) while only 10% included notions of it as a useful thing.

In order to compare the written task *Alien Task* ("how would you describe maths and what it is about") with the drawing task ("what maths or doing maths means to you"), both tasks were coded in terms of what the children at the two focus schools wrote and drew in answer to what mathematics is, and what they believed belongs in the domain of mathematics. Many of the children wrote at least as much on their drawings as they did on their written responses: between 0 and 50 words (M = 13.71, SD = 9.80) for the written *Alien Task*, and between 0 and 86 words on the drawing task (M = 12.74 words, SD = 16.64). Twenty-eight children chose either to skip the written question or to write, “I wouldn’t” whereas everyone chose to complete the drawing task.

Categories emerged through multiple readings and coding, first of the writing and then of the drawing tasks, by attending to both common patterns and idiosyncratic responses. I tried, where possible, to compare my emerging categories with those developed by Young-Loveridge et al. (2006), Grootenboer (2003) and Sam and Ernest (1999). Young-Loveridge et al, whose research explored New Zealand primary school children’s perspectives on mathematics, included “mathematical content”, “processing”, “learning”, “problem solving”, “utility” and “enjoyment” (pp. 585-588). Grootenboer, in his phenomenological study of New Zealand children’s affective views, which included beliefs, attitudes and feelings, isolated three themes: two were associated with the “nature of mathematics”—“mathematics is about numbers and arithmetic” (p.3), which included the doing mathematics operations as well as “using your brain” (p. 4), and the “prominence of times-tables” (p.4)—and the third was associated with a range of feeling from positive to negative including boredom. Although Sam and Ernest’s study only included adult participants (over 17), they suggest that “most people’s image of mathematics is derived from their experience of learning mathematics in school” (p. 44) thus their coding categories were helpful: “absolutist or dualistic view” (p. 44), “utilitarian view” (p. 45), “symbolic view” (p. 45), “problem solving view” (p. 46), “enigmatic view” (p. 47), and metaphors which included “mathematics as a journey” (p. 47), “mathematics as a skill” (p. 49), “mathematics as a daily life experience” (p. 50) and “mathematics as a game or puzzle”(p. 51) (1999).

The children at Kikorangi and Whero, the focus schools, drew pictures to illustrate their beliefs about mathematics. In total, 182 children engaged with both tasks, 39 from Kikorangi and 143 from Whero. The responses to both tasks were coded and compared in Table 5.2.

Table 5.2: Comparison of analysis categories on the writing and drawing tasks

Alien Task	Number	%	Drawing	Number	%
Content	128	70	Content	163	90
Teaching/learning	48	26	Classroom context	136	75
Thinking	2	0.1	Thinking/learning	44	24
Problem solving	40	22	Problem solving	83	46
Utility	19	10	Utility	19	10
Feelings/affect	29	16	Feelings/affect	136	75
Attitude	11	6			
Difficulty (+/-)	48	25	Difficulty	48	25
Metaphors	29	16	Metaphors	86	47
Other	18	10	Games	17	9
Wouldn't	28	15	Books	41	23
			Computers	10	5
Total	**389**		**Total**	**775**	

The drawings included many more coded elements (775) than the *Alien Task* responses (389). Interesting differences emerged between the two tasks in the content area, the classroom context, where 75% of students included illustrations of teaching and learning on their drawings as opposed to 25% on the *Alien Task*, thinking (24% as opposed to 0.1%), feelings (75% compared to 16%), and the extensive use of metaphor in the drawing. In both tasks, the number of children conveying utility and how easy or difficult they find the subject was almost identical. This sort of aggregation of data can present a rather generalised picture of how students responded to these questions. By examining the patterns of how different groups of children responded to these tasks, I attempted to understand more of the detail within responses.

The children at Kikorangi wrote substantially more on both the tasks, the drawings and *Alien Task*, than did those from Whero (*M*= 19.49 and 16.28 as opposed to 12.13 and 11.78). Girls wrote more than boys on the *Alien Task* but not significantly more on the drawing task. Māori children wrote more on both tasks than did any other ethnic group.

The themes of affective elements, notions of mathematics as easy/difficult, metaphors, and, in particular, the metaphor of utility are explored in more depth in Table 5.3. In this table the children's responses on both tasks are examined in terms of school, gender, ethnicity, Year and achievement.

Table 5.3: Percentages of groups that included the following elements

		N	+ affect %		- affect %		Easy %		Difficult %		Metaphor %		Utility %	
			AT	D	AT	D	AT	D	AT	D	AT	D	AT	D
School	Kikorangi	39	**15**	**15**	**0**	**10**	13	5	10	8	**28**	**64**	**26**	**36**
	Whero	143	**12**	**50**	**4**	**27**	11	12	17	18	**13**	**43**	**6**	**4**
Gender	Female	85	**11**	**46**	1	**19**	11	8	17	14	12	39	8	11
	Male	96	**14**	**39**	5	**28**	12	13	15	18	20	55	12	10
Year	5	100	16	53	5	26	15	11	21	15	14	46	8	4
	6	82	9	23	1	21	6	10	7	17	18	49	13	18
Ethnicity	Asian	19	21	**47**	0	16	11	11	11	21	**32**	**53**	**32**	5
	Māori	22	5	32	0	9	0	5	0	9	9	46	0	23
	Pākehā	139	13	42	4	**27**	13	12	18	17	15	47	9	9
	Pasifica#	2	0	50	0	0	0	0	50	0	0	50	0	0
Achievement	unknown	8	25	50	25	25	25	0	13	0	0	38	0	13
NumPA	mid	154	11	41	3	23	12	11	17	16	16	47	10	11
stages	high	20	20	45	0	25	0	10	5	20	20	50	15	5
Overall		182	**13**	**42**	**3**	**24**	11	10	15	16	**16**	**46**	10	10

Key: numbers in **bold** are included in the paragraph below

#: Although the results from this group of students have been included, there are too few of them to draw statistical conclusions.

On the whole, a greater percentage of students included positive (42% vs. 13%) and negative feelings (24% vs. 3%) and metaphors (46 % vs. 16%) on their drawings than on the written *Alien Task* (AT). The areas that remained very similar were how easy/difficult they view the subject (easy at 10/11%, difficulty at 16/15%), and metaphors of utility (10%). Kikorangi and Whero have very different patterns of response for these items. The Kikorangi children included 15% of positive feelings on both their written and drawing tasks while the Whero children had 12 % on the *Alien Task* and 50% on their drawings. Both groups included very few negative feelings on their *Alien Task* (0% & 4%) and more on their drawings (10% & 27%). The differences between the two schools is quite marked with their use of metaphor; in both cases, the children included more metaphors on their drawings than the *Alien Task*, but Kikorangi included higher percentages than Whero (28 & 64% vs. 13 & 43%). Part of this difference can be seen in the use of metaphors of utility, which are included by a much greater percentage of students at Kikorangi.

I also examined the affective elements, notions of mathematics as easy/difficult, metaphors in terms of gender and ethnicity (Table 5.3). A higher percentage of boys included both positive and negative feelings about mathematics on the *Alien Task* and negative ones on their drawings than did girls while the girls incorporated more positive feeling on their drawings (46 % girls vs. 39% boys).

Another gender difference concerned the inclusion of metaphors where the boys included more than the girls on both tasks (boys = 20 & 55%, girls = 12 & 39%). Year 5 students included more positive feelings than did Year 6 ones. A higher percentage of Asian children included metaphors (32% on Q33 & 53% on the drawing) and the highest percentage of utility metaphors on the *Alien Task* than any other group. Asian children were close to Pākehā children when including positive feelings on their drawings. Finally, Pākehā had the greatest percentage of negative feelings towards mathematics on their drawings than any other ethnic group.

In order to respond to my concern whether the drawing task reflected a broader vision of mathematics than the written *Alien Task*, the "content" category was broken down into different strands or subsections (see Table 5.4).

Table 5.4: Content breakdown for the two tasks

Alien Task	**N = 128**	**%**	**Drawing**	**N = 163**	**%**
Number	111	**87**	Number	150	**92**
Symbols	37	**28**	Symbols	105	**64**
Measurement	5	4	Measurement	35	**22**
Geometry	5	**4**	Geometry	21	**25**
Statistics	1	1	Statistics	2	1
Other	7	5	Money	21	13
			Graphs	13	8
			Algebra	18	**11**

Key: numbers in **bold** are included in the paragraph below

In both tasks, children overwhelmingly identified number as the dominant strand of mathematics (87% and 92%), which reflects the current New Zealand emphasis on numeracy. However, the drawings present a more varied and extensive picture of the domain than the *Alien Task* data do. On both tasks, students included symbols associated with mathematics, 64% on the drawing and 37% in the *Alien Task*. A quarter of the drawings include geometric concepts as opposed to 4% of the written responses; a significant proportion also indicated measurement (22%) and algebra (11%). Statistics was the only strand equally excluded with only 1% of students making any reference to this area. Therefore, the analysis of both data sources revealed that the drawings portrayed a more comprehensive picture of what students believe the domain of mathematics encompasses.

A second (type of) reading: drawings and factors

This reading was an attempt to explore whether the MBF developed from the analysis of the responses the children gave on the MBQ (see Chapter 4 and this chapter p. 140) would work as a prism for analysing and making sense of the children's drawings (Richardson & St. Pierre, 2005). This particular framework comprises four mathematics beliefs factors:

Self, which includes how individuals view themselves as doers and learners of mathematics, including feelings about the domain, and how they position themselves within this world;

Ability, which covers the sorts of people, others, who do or do not inhabit this world, including notions of ability, and the relationships between ability and gender, and ability and ethnicity within this domain; however, in the chapter I've used the term *others* in order to emphasize the concept of *ability of other/s* rather than *ability* in general;

Learning Environment (LE) which includes beliefs about teaching and learning, what goes on in the mathematics classroom; and

Nature of mathematics, what is this subject, domain, world, and how does it work.

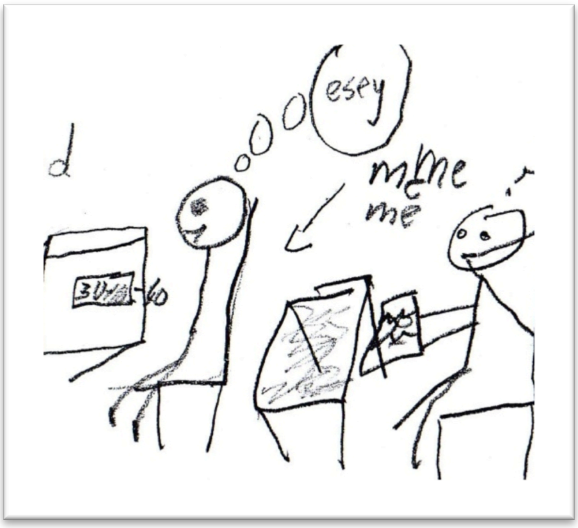
A concern with the application of a framework as a lens through which to read was that an interpretation was being imposed on the drawings rather than allowing what the artists had to say be heard unfettered. Another issue with using these factors as a mechanism for coding the drawings is that this framework, which was based on responses to a series of closed questions, was applied to an open task; as a result, it needed to be interpreted more flexibly.

The following details from seven different drawings illustrate how I coded drawings in terms of the four factors. (All the names of the children are pseudonyms.) By detail, I mean that I have only included part of the drawing. These examples also demonstrate how multiple factors are referenced within single images. Six of these drawings are included in full later in this chapter.

self and nature

Figure 5.2: Tom includes himself taming a new strategy, detail
[Not to be reproduced without permission.]

This figure shows Tom positioning himself as successful within the world of mathematics which includes strategies. He shows himself as capable of taming the new strategy. (Codes: *self* and *nature*)



self, others, LE and nature

Figure 5.3: Fred in the classroom with a friend, detail
[Not to be reproduced without permission.]

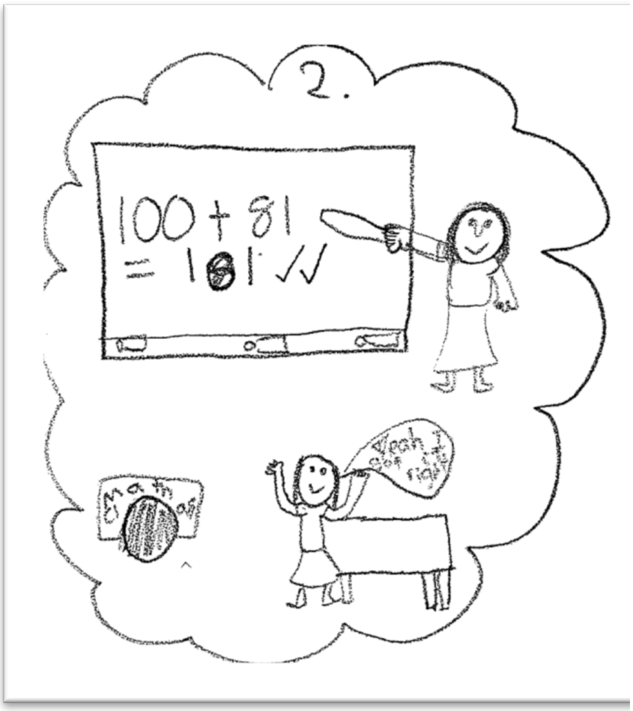
Fred finds mathematics easy while a friend struggles, an example of *self* and *others* (in this case, how others in the class cope) within the learning environment. This illustration also gives one a glimpse of the learning environment and the sort of problems (*nature of mathematics*) the class is working on. (Codes: *self*, *others*, *LE* and *nature)*.

self, LE and nature

Figure 5.4: Harry suffers from brainburn, detail
[Not to be reproduced without permission.]

Harry presents himself as struggling with mathematics in a stressful mathematics class.
(Codes: *self*, *LE* and *nature*)

In Figure 5.5, Ella depicts a teacher teaching in the classroom. She shows herself as a successful student. Another child is working away at a desk. Her belief about the *nature of mathematics* is that it is useful. (Codes: *self*, *LE* and *nature)*



self, LE, nature

Figure 5.5: Ella's illustration of the learning environment with explanation, detail

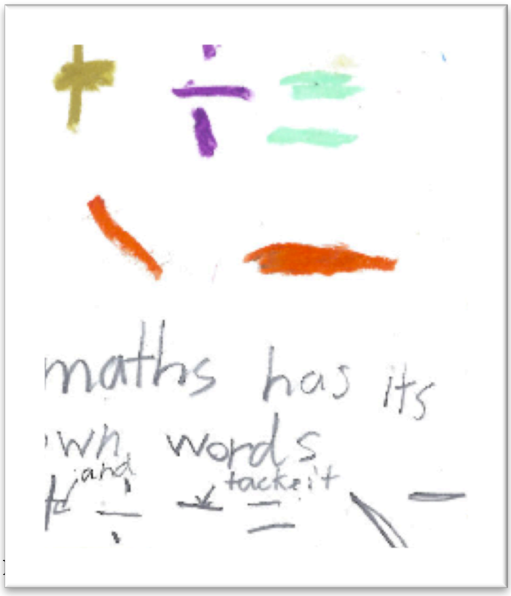
[*2. Number 2 is that a teacher is teaching the child so she can get a job involving numbers*]

[Not to be reproduced without permission.]

self, others, LE and nature

Figure 5.6: Govi's experience of his mathematics class with other(s), detail
[Not to be reproduced without permission.]

Govi positions himself in the mathematics classroom. He and a friend are successfully working out a maths problem together. In this case "working with friends" illustrates his experience with doing school mathematics. (Codes: *self*, *others*, *LE* and *Nature*)



nature

Table 5.7: Katia's maths with its own language, detail
[Not to be reproduced without permission.]

Katia explains her belief about the world of mathematics with its own words and symbols. (Code: *nature of mathematics*)

Almost all of the children (99%), as illustrated in Table 5.5, included some aspect of the *nature of mathematics* in their drawings, 50% included themselves (*self*), 20% other people (children, teachers, a parent, and unidentifiable, at least to this researcher, persons) (beliefs about *others*), and 37% included some elements of the learning environment (classrooms, teachers, working with other children, testing, playing maths games, etc.)(*Learning environment*).

Table 5.5: Percentage of students including four factors in their drawings

%		N	*Self*		*Others*		*LE*		*nature*	
			Yes	?	Yes	?	Yes	?	Yes	?
School	Kikorangi	39	18	10	26	8	21	8	100	0
	Whero	143	**59**	9	19	6	**41**	8	99	1
Gender	Female	85	51	4	15	4	31	9	99	1
	Male	96	50	14	**26**	9	**42**	7	99	0
Year	5	100	54	8	21	3	38	5	98	1
	6	82	45	11	20	11	35	12	100	0
Ethnicity	Asian	19	53	5	**37**	11	**47**	11	95	6
	Māori	22	41	9	18	0	18	9	100	0
	Pākehā	139	51	10	19	7	39	8	99	0
	Pasifika	2	50	0	0	0	0	0	100	0
Achievement	unknown	8	50	0	13	0	25	13	88	13
NumPA stages	mid	154	49	9	19	6	33	9	99	0
	high	20	**60**	15	**35**	15	**70**	0	100	0
Total	182	50	9	20	7	37	15	99	.5	Total

Key: numbers in **bold** are included in the paragraph below
? : in same drawings these elements could be read as present or not; it was not clear

The children at the Whero School had more examples of themselves (59%), and the learning environment (41%) in their drawings while those from Kikorangi had a higher percentage of others (26%). Boys included more illustrations of others (26%) and the classroom environment (42%) than did girls (15% and 31%). Asian students also included more examples of others (37%) and the learning environment (47%) than did any other ethnic group. The higher achieving group of students included more examples for each of these factors than did any other achievement group.

The second reading of the drawings was useful in so far as it aligned this analysis of the drawings with the factors, three of the original factors and one adapted (*other* in the place of *ability*), used in making sense of the children's questionnaire responses. On the other hand, the approach was limited because not all drawing elements were easy to interpret through this lens. In some cases, a human figure could be read as either the *self*/ the artist or *other*/a person, in general, involved in mathematics. Another limitation, because the images are often complex, is the difficulty in discovering and untangling where one factor starts and stops, or overlaps with another. In these cases, I coded the drawings in multiple categories and used a "?" when elements were unclear. Even though the drawing task itself was more closely aligned to the *nature of mathematics* than any of the other factors, the *self*, *others*, and *learning environment* factors were all illustrated in a variety of ways.

Although these two quantitative approaches to reading the drawings answered my concern arising from the children's written responses on the *Alien Task*, they did not reflect the richness, coherence and integrity of the drawings (Rose, 2007). They acted as a broad brush-stroke, thus oversimplifying the content and ideas contained within the individual drawings. In the third reading of the drawings, I addressed this shortcoming by undertaking a qualitative analysis of the visual data.

The third reading—through a qualitative lens

The drawings presented as examples in this next section were all analysed in terms of their mathematical content, yet each tells a complicated story about individual beliefs, feelings and lived experiences of the mathematics classroom. Because of the myriad of ideas and elements that percolated from multiple readings, I found it very difficult to group the elements in comprehensive, mutually exclusive themes. No sooner had I identified a single, clearly identifiable theme such as 'nature of mathematics', then I realised that identity, affect and lived experience of the mathematics classroom could also be viewed as sub-themes of nature. In returning to the original research questions of this study, I decided to explore whether they would work as a way of guiding the analysis and discussion of the themes found in the children's drawings. Unfortunately, the questions needed to be slightly recast in order to accomplish this role. The "what is mathematics?" question matched the 'nature of mathematics' themes; however, the questions associated with *self* and *others* within the world of mathematics both needed to be included within the theme 'identity within the world of mathematics'. The final theme, 'the experience of the mathematics classroom', included elements of all the research questions:

> What is mathematics as experienced in the classroom?
>
> What is the individual's experience of the classroom and of others within the mathematics classroom?

One of my solutions to the difficulty of analysing and categorising mathematics beliefs was to explore the intriguing metaphorical elements in the drawings. If metaphor is considered fundamental to human understanding, as essential for abstract thought, and to describing and making sense of experience by comparing one thing to something else (Chapman, 2002; Gauntlett, 2007; Lakoff & Johnson, 2003), then investigating these metaphors became a way of accessing what these students believe and understand. An exploration of the metaphors about mathematics could have been included as a separate theme in its own right or as a sub-theme under each of the three main themes; however, because of their pervasiveness, I chose to refer to metaphors throughout this subsection. In fact, many of the themes and sub-themes incorporate the notion of metaphor in the way they are worded as in the 'nature of mathematics', ' maths as useful', 'holders of knowledge', etc. The researchers I discussed earlier in this chapter (p. 143) also identified themes in terms of metaphor. Sam and Ernest (1999) describe images of mathematics that encompass both the cognitive and the affective by including "all visual or metaphorical images and associations, beliefs, attitudes, and feelings related to mathematics and mathematics learning experiences" (p. 43), some of which they classify as myths such as "mathematics is just computation" (p. 43) and others as metaphorical images of a journey, a skill, "daily life experience" (p. 50) or a game (p. 51). Picker and Berry (2000) also use the term "image" when looking at drawings that included metaphors associated with mathematics and mathematicians such as "maths as coercion" (p.

75), “the foolish mathematician” (p. 79), and “mathematicians with special powers” (p. 84). Young-Loveridge et al. include students’ beliefs about the nature of mathematics through metaphors of utility and problem solving (2006).

Not all of the drawings could be analysed in detail because of the number of drawings collected. The drawings from the two focus classrooms, Ron’s class at Kikorangi and Mr. Forrest’s (Room 6) at Whero, have been examined closely; some are reported here and others in Chapter 6. In places, I also refer to drawings that I was unable to include because their quality, clarity or colour. The drawings that have been included in this subsection are examples of a range of responses that are interesting, unusual approaches or particularly clear examples of common themes.

The nature of mathematics: What is mathematics?

Almost all of the drawings (99%) included images associated with beliefs about the nature of mathematics, what it is, what the world of mathematics includes, how mathematics works as well has how mathematics makes the creators feel. All three of Nuthall’s world of the classroom (Nuthall, 2007) are reflected in these drawings that include the nature of mathematics; the public world influenced by both teacher and curriculum, shared beliefs among friends or within groups and individual, idiosyncratic beliefs about what mathematics is and how it works. Some of the drawings included very simple depictions (e.g., a series of numbers 1 – 10) while others grappled with complicated concepts, ideas and metaphors. The range of belief extended from very narrow to universal views; even though these images have been grouped and labelled as maths as ‘number’, ‘number plus’, ‘useful’, ‘everywhere’, ‘problem solving’, and ‘feelings’, these categories overlap, and many drawings contain more than one of these views.

Maths as number

The majority of the drawings included some notion of number, usually in the form of cardinal numbers on their own in the simplest depiction to the addition of symbols usually +, -, x and ÷. An example is Jane's drawing (Figure 5.8), although some of the drawings also include =, %, fractions and decimal points.

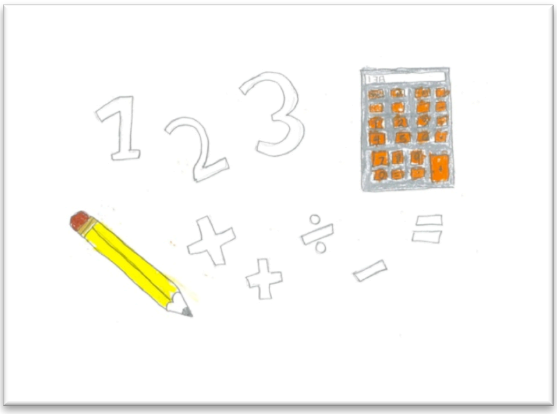


Figure 5.8: Jane
[Not to be reproduced without permission.]

Jane's drawing includes a combination of ordinal numbers and operations symbols as well as two mathematics tools: the calculator and the pencil.

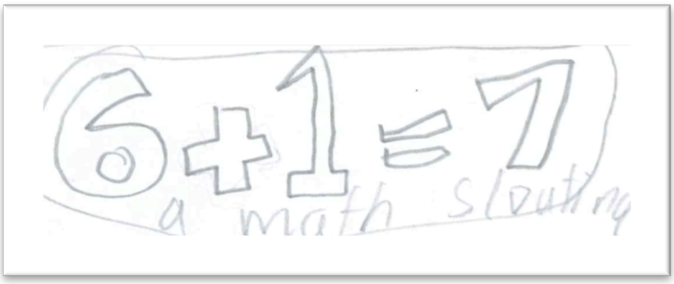


Figure 5.9: Jasmine's equation, detail
[*a math slouting (solution)]*
[Not to be reproduced without permission.]

In more complex drawing of maths as number, the children have included problems, which they label with terms like algorithms, or "slouting" (solution) as in Jasmine's Figure 5.9.

Only two children reference "story problems": "Sarah took 16 eggs then put back two, whats the anser?" and "Jenny has 2 apples, Fred has three how many have they got together" (examples from two different children's drawings).

Often a simple problem like “1+1” is used as a shorthand, almost like a comic book discourse that is understood by all children to depict or denote mathematics or the mathematics classroom. It acts as a metaphor for mathematics at a very basic level even though these Year 5 and 6 students are involved with much more advanced levels that include percents, decimals and fractions. An example of this is Mr. Forrest asking a question in Orange’s drawing (Figure 5.10).

Figure 5.10: Orange, detail
[Not to be reproduced without permission.]

Quite a few drawings include a number of problems that illustrate the students’ experiences of school mathematics in New Zealand where different problems are set for different groups of children. Most commonly the class is divided into three groups, thus three different levels of problems (1+1 champions, Ten champs and Mathematicians are the names of the three groups in Rosie’s drawing of school mathematics, Figure 5.11). The Mathematicians are working with decimals while the1+1 champions have the easiest problems.

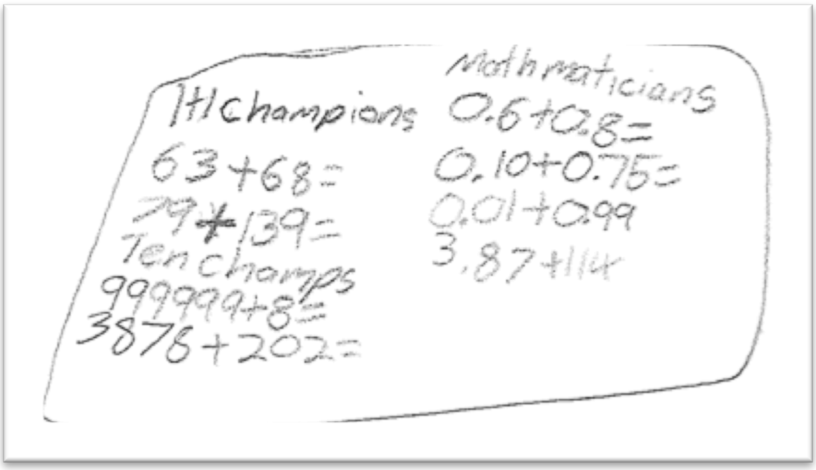


Figure 5.11: Rosie, detail
[Not to be reproduced without permission.]

Rosie's detail illustrates the different tasks assigned by the teacher to the three ability-sorted groups.

Maths as number plus other strands

After number, geometric images were the most common, many of them two-dimensional while others represent three-dimensional objects.

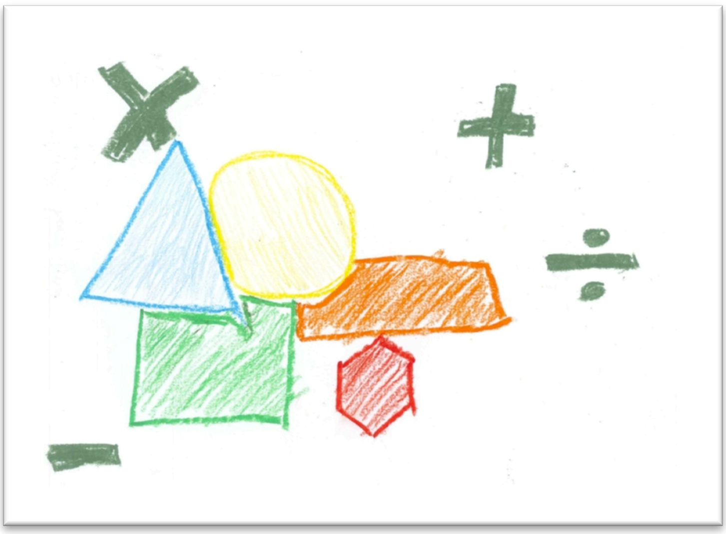


Figure 5.12: Destiny
[Not to be reproduced without permission.]

Destiny includes colourful 2-D geometric shapes with her operations symbols. On the back of the drawing, rather than write about her drawing, she explains: "I think maths is sometimes hard and sometimes easy. Also sometimes I can't be bothered doing maths that I do a really bad job of it". This description is an attempt to explain her beliefs about school mathematics and the consequences of her not being "bothered", or trying hard.

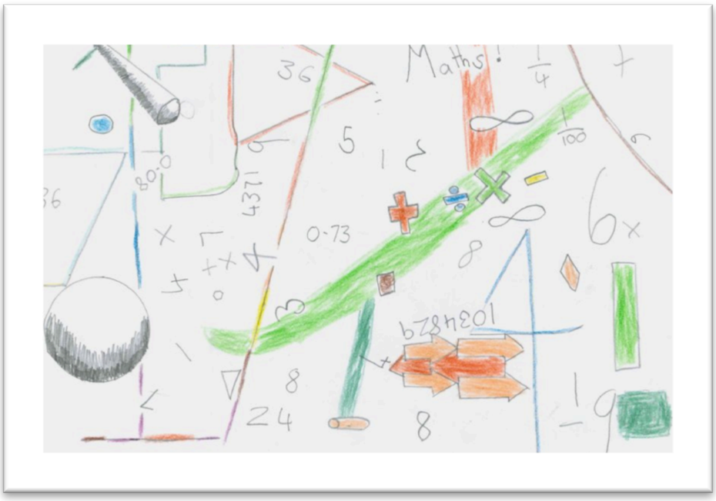


Figure 5.13: Michael
[Not to be reproduced without permission.]

Michael incorporates 3-D shapes, symbols, patterns such as the arrows, and the symbol for infinity, a sophisticated concept for children of this age, with his number strand. The references to algebra in some of the drawings are extremely interesting. Once again the students use a shorthand code to depict this strand of mathematics by including stray letters (6Z +3T as used by Steven or Graeme's F-E = O^2) or using "E=mc^2" as a common signifier for algebra.

Maths as useful

Metaphors or depiction of mathematics as useful or practical were almost as common as geometry in the drawings. Measurement, money and time have been included under utility even though they form part of the Measurement Strand (Ministry of Education, 1992) while the other strands are included in the previous paragraph. The following examples illustrate some of the range of examples within this category.

Ron's class at Kikorangi included images and explanations of utility much more than any other group of students. Most of these drawings included money (notes, coins and in one a cheque) and measurement with rulers, tape measures, centimetres, etc. One student explains his filing cabinet and map with "I think maths is for filing and maping out ". Another describes his drawing in terms of mathematics that helps him in life and in the mathematics classroom: "I put the math signs there because I use them in every day life. I put the paper money and coins there because maths helps me to add money. Maths equation board was there for helping me." Spud Murphy's beliefs about the nature of mathematics are only in terms "out of school" practicality. He wrote the following on the back of his drawing (Figure 5.14): "To use maths in the real world and to messure [measure] things to make them fit in ereas [areas] only just bigenough to fit."

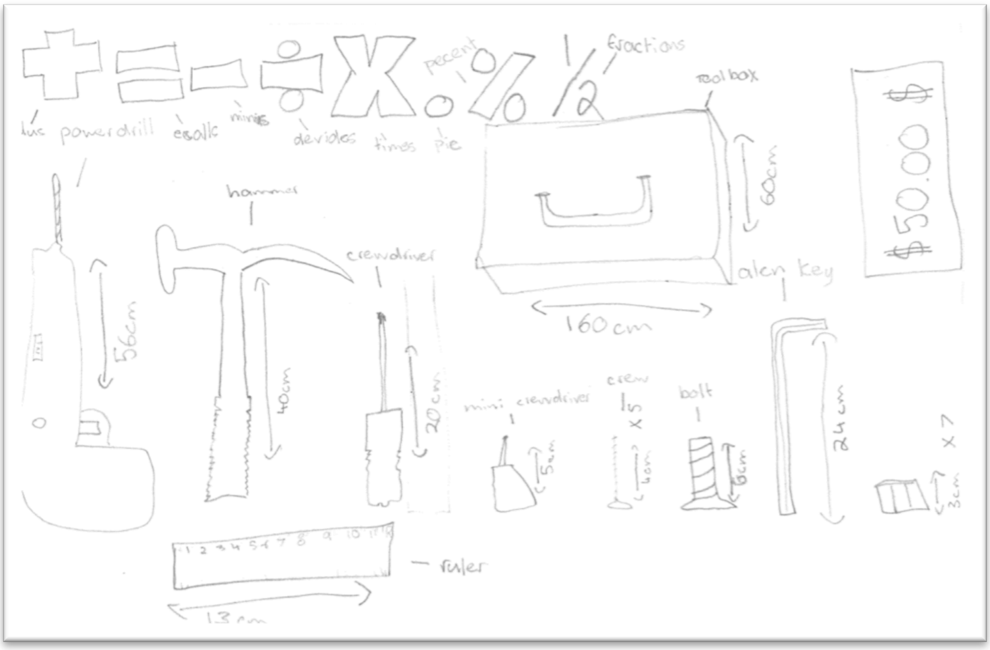


Figure 5.14: Spud Murphy
[Not to be reproduced without permission.]

Transcript of Figure 5.1
Line 1: plus, powerdrill ↓, ecalls(=), minis, devides, times, percents, fractions, tool box ↓,
Line 2: hammer, [s]crewdriver, minicrewdriver, crew, bolt, alen key
Line 3: ruler
Multiple measurements in cms

Margary Luna divides her useful elements in the following way: “Adults need maths for: Jobs, Money, Date, counting, Subtracting, measuring. Children need maths for: Money, Date, Measuring”. Ella (introduced in Figure 5.5) includes a long explanation on the back of her drawing about the usefulness of each of her images of mathematics in her numbered clouds (Figures 5.15 and 5.16).

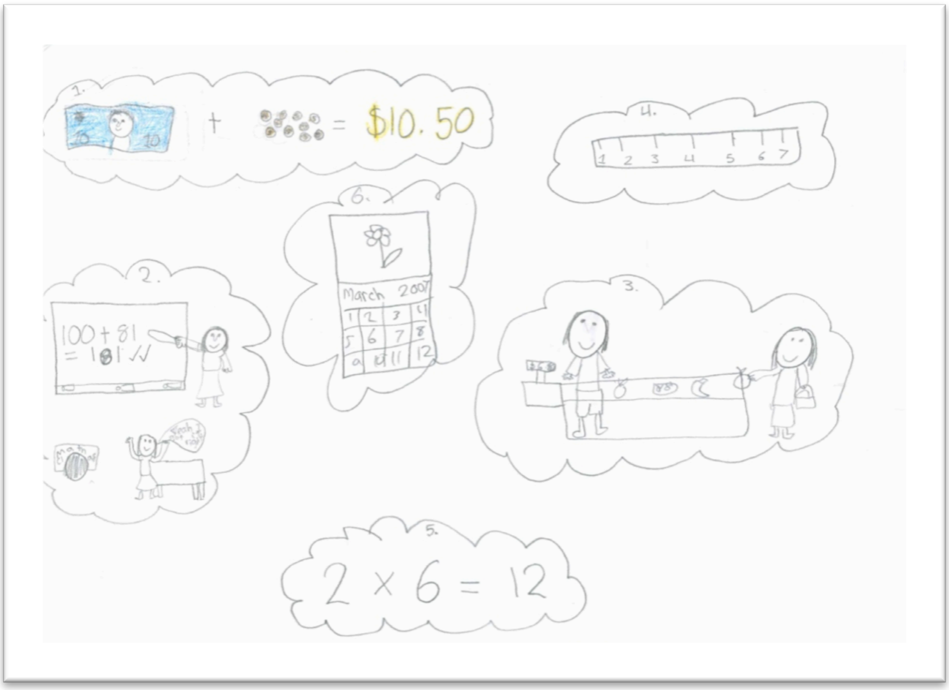


Figure 5.15: Ella, side a)

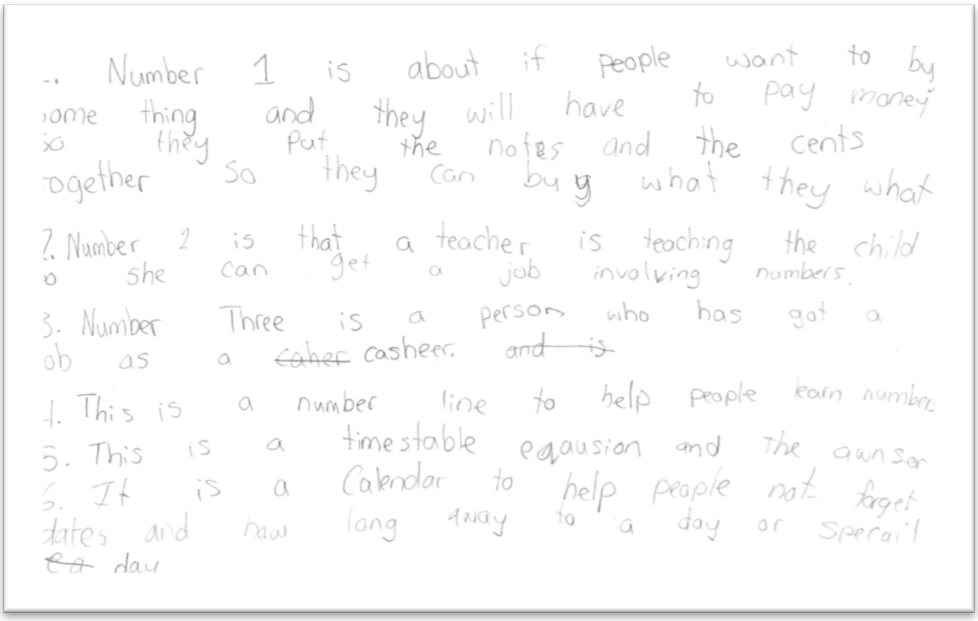


Figure 5.16: Ella, side b)
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Transcript for 5.16, side b:
1. Number 1 is about if people want to by some thing and they will have to pay money so they put the notes and the cents together so they can buy what they what
2. Number 2 is that a teacher is teaching the child so she can get a job involving numbers
3. Number Three is a person who has got a job as a casheer
4. This is a number line to help people learn numbers
5. This is a timestable eqausion and the awnser
6. It is a Calendar to help people not forget dates and how long away to a day or special day

Even her notion of school mathematics is in terms of being taught mathematics in order to find a job or to be able to use mathematics that might be necessary for a job.

In trying to explore why the Kikorangi children incorporated more images and words about the utility of mathematics than the Whero children did, I examined the teachers' questionnaire responses and found that four of the seven teachers at Kikorangi, one of whom was Ron, described mathematics in terms of utility. Conversely, eleven of the fourteen Whero teachers did not include any reference to utility of mathematics.

Maths as everywhere

Some children depicted mathematics in universal terms – as life, as something that underpins all of existence. For instance, Zach's picture includes a volcano, the sky, sun, and fishes in the sea with a sprinkling of numbers, algorithms and symbols (Figure 5.17). He explained his view to me, "Well, you know maths is everywhere. It's in the sky, in the volcano, and under the sea."

Figure 5.17: Zach’s “maths is everywhere”
[Not to be reproduced without permission.]

Katia (introduced in Figure 5.7) uses the sea as a metaphor to reflect her understanding of the never-ending universality of mathematics (Figure 5.18). She also views mathematics as a separate culture or world with its own language and symbols.

Figure 5.18: Katia’s “mix them all together that is maths”
[Not to be reproduced without permission.]

Lucy links her notion of mathematics to music, which is continuous but never boring in Figure 5.19.

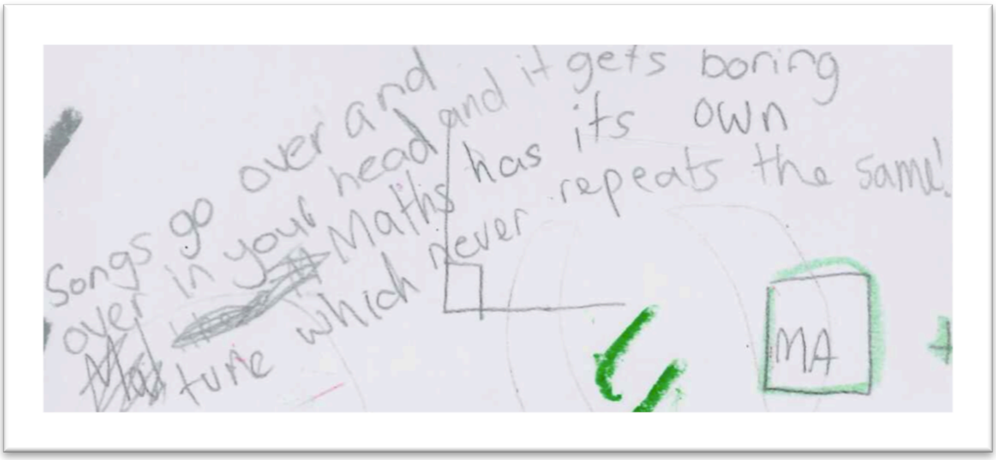

Figure 5.19: Lucy, detail
[Not to be reproduced without permission.]

Other students used geographic metaphors, such as "Numberland" or "Mathsland", usually as an interesting place to visit; however, at another extreme Luke draws it as a prison[18].

Maths as problem solving

For many of these children, mathematics is described in terms of solving problems. They often portrayed 'problem solving' with algorithms, equations or with the methods they have chosen in order to solve the problem, for instance Jack (Figure 5.20).

18 Luke's picture is not included because it was too difficult to replicate.

Figure 5.20: Jack, detail
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Or Hamish's more complex examples which he annotated with "when I think of hard maths I think of stuff like this even though that stuff I wrote isn't correct that's what I think of when I think of hard maths":

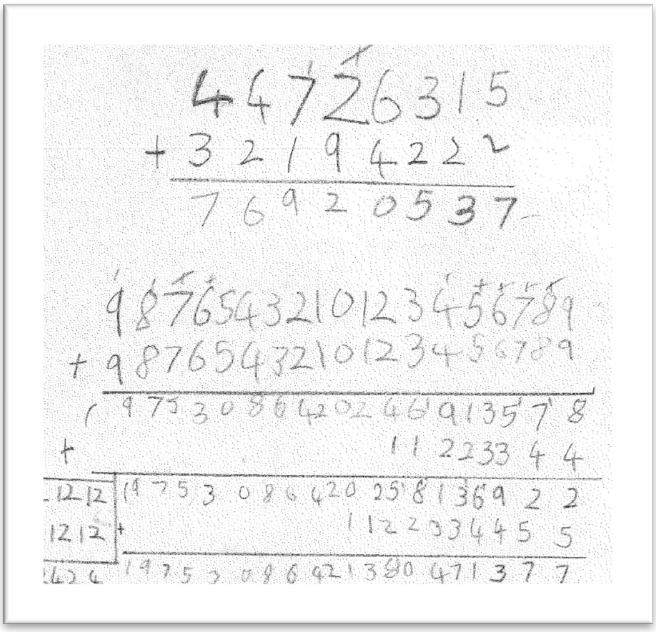

Figure 5.21: Hamish, detail
[Not to be reproduced without permission.]

Hamish has arrows running from his comment to Figure 5.21(not included in this detail) as well as to $E^2 = 4961$ and $49^{67} + 24^7 = 1976$ as additional illustrations of 'problem solving'. (For more detail see Figure 5.30.)

Ronan (Figure 5.22) tells a story of himself sitting at his desk, perhaps in class, going through the process of solving a difficult mathematics problem. This story includes each step of Ronan's thinking during the process of solving the problem. In frame 1) he is puzzled, 2) anxious with a nervous tap of his foot, 3) gets the idea, and 4), the only coloured frame, he has got it. By frame 3, he gets going on the solution, and in frame 4 he has solved it. This is his illustration of his "ahaa" moment.

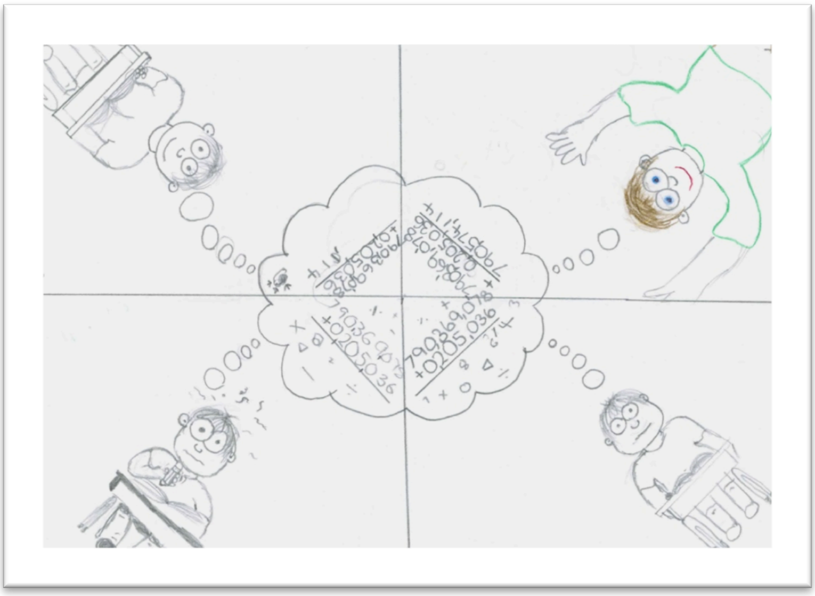

Figure 5.22: Ronan solving a difficult problem
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The drawings also included references to games and puzzles, many of which are mathematics classroom games like Greedy Pig or maths games day. Other drawings included references to role playing and/or computer games. Tom has adventures in "Mathsland" (Figure 5.23). He avoids falling into a new equation, dodges raining numbers and tumbling shapes, leaps over a hurdle, meets and tames a new strategy, all the time avoiding the evil textbook.

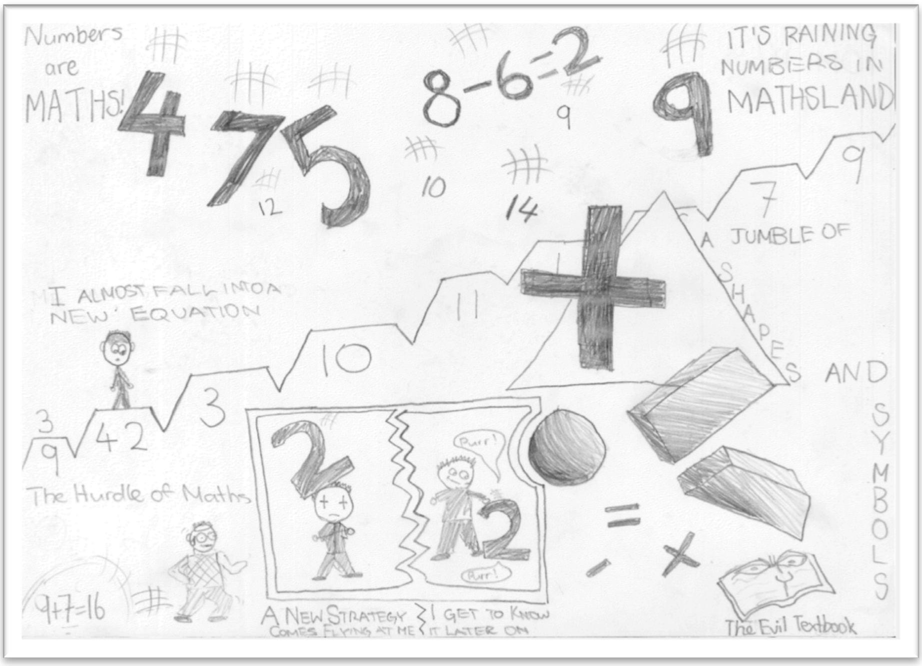

Figure 5.23: Tom's adventure
[Not to be reproduced without permission.]

A group of boys also include gaming references like becoming the "Plus King", Lyle's killing with numbers, and Damien's maths murders.

Related to notions of mathematics as problem solving were the drawings that feature maths as knowledge, as learning, and as a way to become cleverer. Many of these drawings included images of the brain as a signifier of working with numbers or increasing one's intellectual capacity through mathematics. Ellie incorporates a brain radiating knowledge (drawing not included) and Sammie links hers to thinking (Figure 5.24).

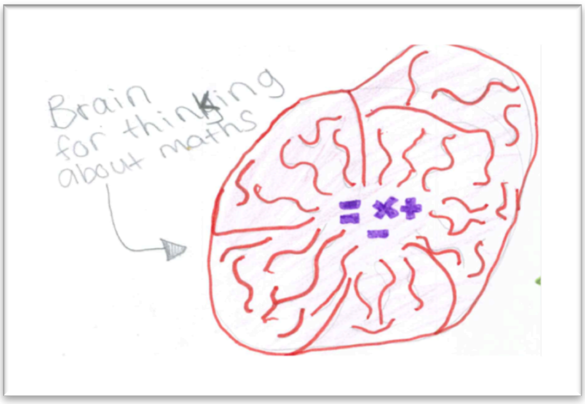

Figure 5.24: Sammie, detail
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A more detailed example of this is Miriama's (age 9, Year 5) drawing (Figure 5.25):

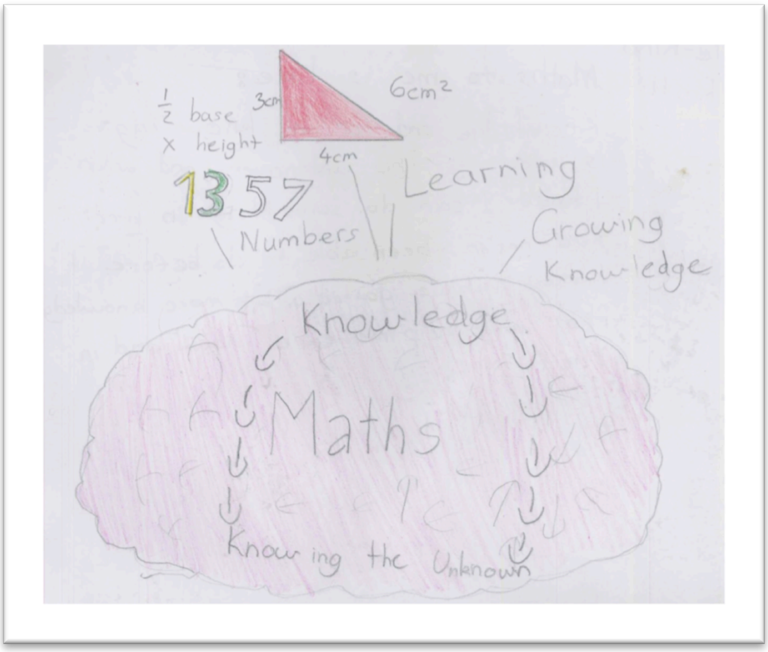

Figure 5.25: Miriama's brain
[Not to be reproduced without permission.]

This drawing not only includes number, geometry and the formula for calculating the area of a triangle but ideas about learning and "growing knowledge" which she articulates on the back with, "Maths to me is like: Knowledge and for me knowledge is knowing the unknown and when I know I can do something in maths I've never been able to do before I know I have gained a bit more knowledge that will help me excel now and in the future." Other students included brains filled with or exploding with numbers, symbols or mathematical concepts as in the following examples: Lucy's (Figure 5.26) and Lyle's (Figure 5.27) drawings.

Lucy includes a smiling girl whose brain is full of interesting mathematics concepts.

Figure 5.26: Lucy, detail
[Not to be reproduced without permission.]

Lyle's rather sinister drawing includes the text, "So do it to me Kill me now with numbers." His brain is spewing out multiple mathematics problems.

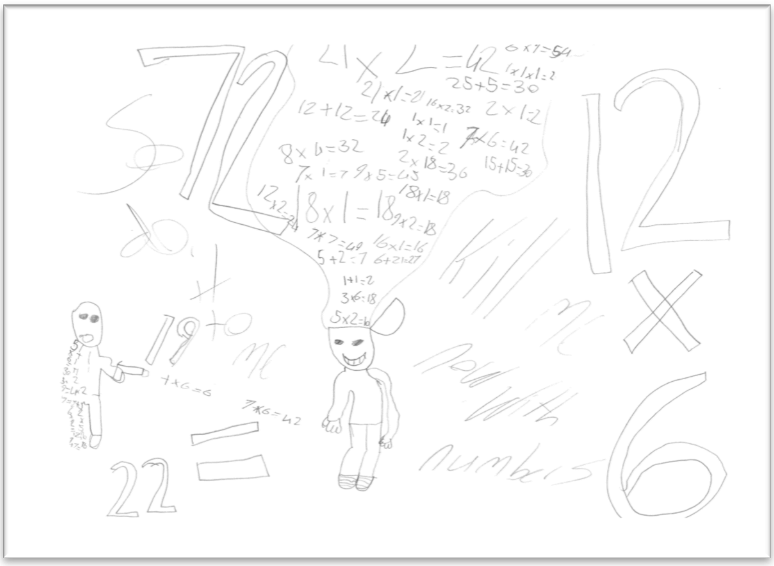

Figure 5.27: Lyle's "Kill me now with numbers"
[Not to be reproduced without permission.]

Most of the drawings that include the brain and/or head as an image of thinking and knowing are positive; however, a few others like Lyle's above depict mathematics more negatively as a source of "brainburn" and perhaps extreme stress.

Maths as something easy/hard, maths as fun/agony

The drawings not only contain the content or strands of mathematics (number, geometry, symbols, etc.) but also notions of mathematics being inherently easy, difficult, fun or agony. Although easy/hard and fun/agony seem to be presented as binary opposites, many of the students include both within their drawing as well as feelings of ambivalence towards mathematics. It is almost impossible to separate the ideas about difficulty from those of affect because for many of these children what they see as easy or challenging in an exciting way is fun; conversely, what they see as difficult or impossible may induce tears or brainburn. Interestingly, 70% of the drawings that included images or words associated with feeling were positive, while only 40% were negative and 42% ambivalent.[19] For Sophie (Figure 5.28), there are the bits of mathematics she

19 Some of the drawings included both positive and negative images and words.

likes and probably finds easy and those she does not.

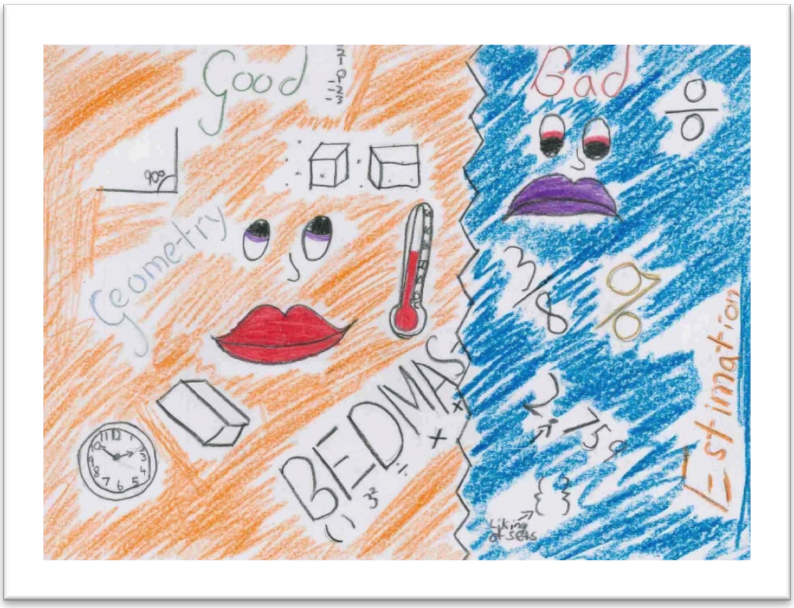

Figure 5.28: Sophie's good and bad
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Similarly, Brit is grappling with what she likes, can do and what she finds difficult. She refers to going to extra lessons, having to write things down and that "NO one can help me with my maths". For her, mathematicss is something you have to do on your own.

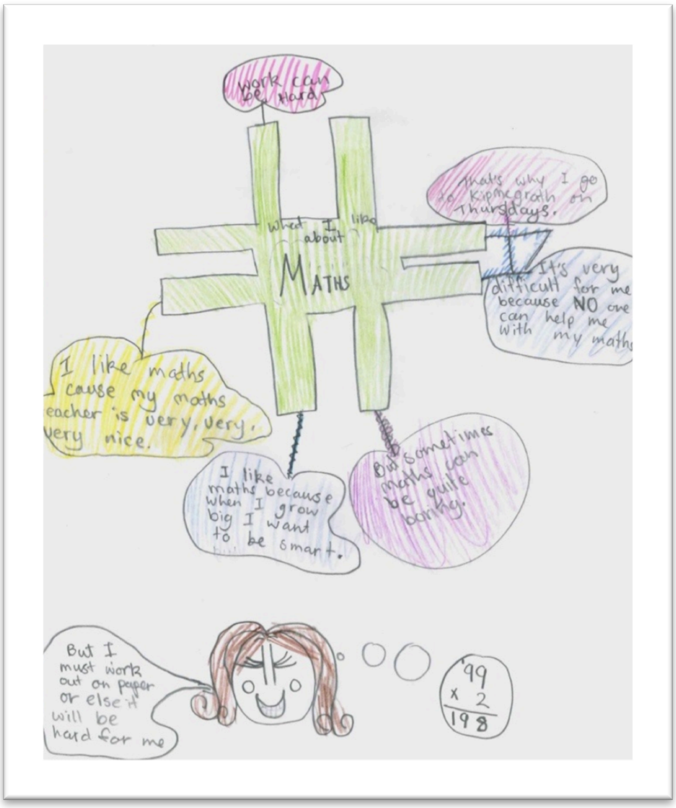
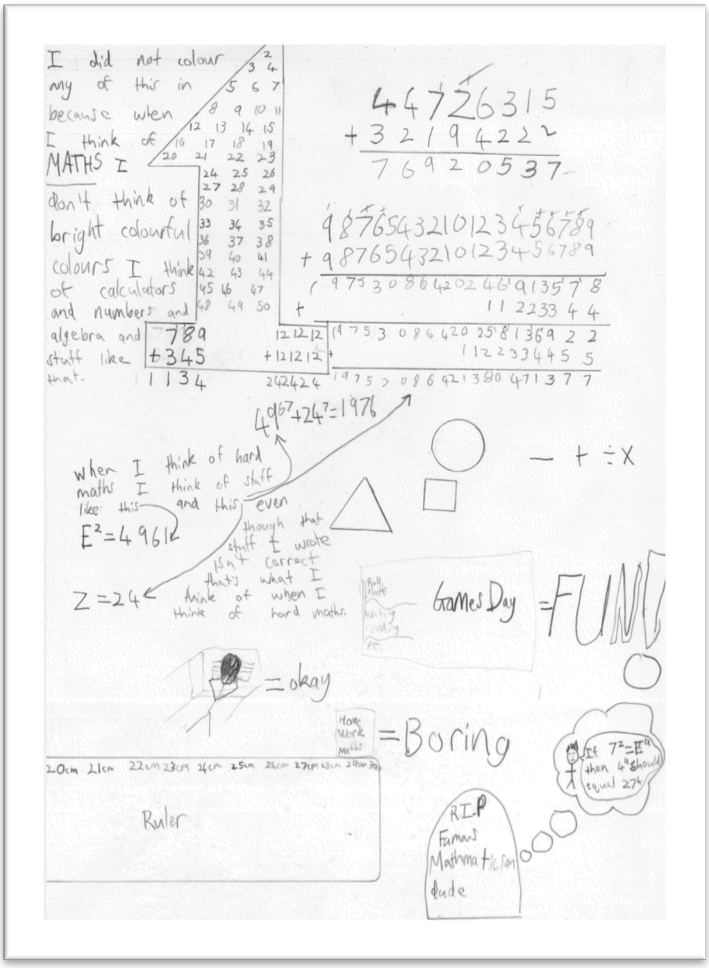


Figure 5.29: Brit's "work can be hard"
[Not to be reproduced without permission.]

Positive feelings are depicted through smiling faces, hearts, flowers, words like fun, awesome, enjoyable, exciting, mindblowing, fascinating, "maths is cool" and other cheerful images. For Chloë, it is a blend of bombs, fireworks and "Maths makes me feel as good as an icecream tastes" (Chapter 6). Lucy has wide-awake brains, smiling girls, tunes and maths pills: "Dosage. Take a lesson a day to get you going happily."

Hamish includes “fun with games”, but an explanation about “boring as in a subject without colour”, and a tombstone inscribed with “R.I.P Famous Mathmatician dude”(Figure 5.30). Other ambivalent mathematics doers use words like “ok”, “fun sometimes”, “easy to hard” and boring. Quite a few drawings include images of thumbs up and down to indicate their feeling about the subject.



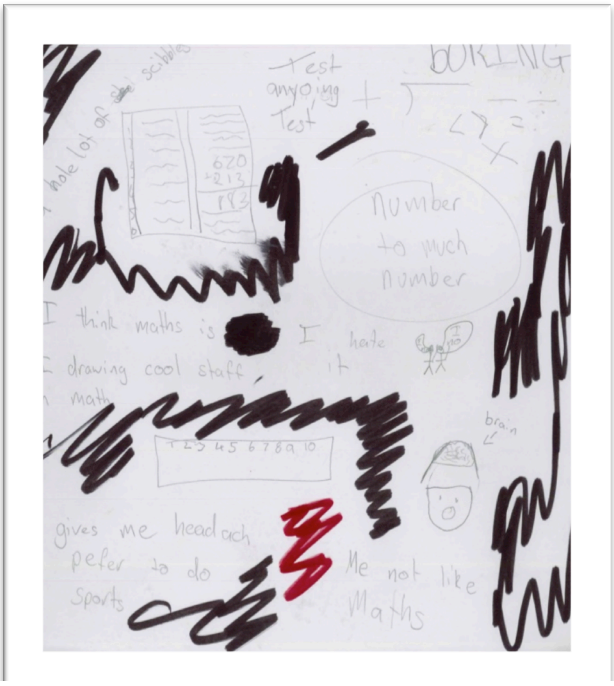
[Text:
I did not colour my of this because when I think of MATHS I don’t think of bright colourful colours. I think of calculators and numbers and algebra and stuff like that

When I think of hard maths I think of stuff like this ↓ [E^2 = 4961] and this↗ [49^{67} + 24^7 =1976] ↗[bottom line5.21] ↘ [Z = 24] even though that stuff I wrote isn’t correct that’s what I think of when I think of hard maths]

Figure 5.30: Hamish, fun and boring
[Not to be reproduced without permission.]

A few students like Sarah, Tane and Lizzy write on their drawings “not my favourite subject….I would rather be doing…”. Others hate particular activities like tests, homework and the textbook.

Hazel uses black and red to illustrate how much she hates mathematics, finds it both boring and difficult or headache inducing (Figure 5.31).



[“a hole lot of scribbles”, “boring”, “Test anyoing Test”, “number to much number”, “I think maths is ● I hate it” “I drawing cool stuff in math”, “gives me headache prefer to do sports”, “Me not like Maths”]

Figure 5.31: Hazel’s red and black drawing
[Not to be reproduced without permission.]

Joshua draws his reaction to the difficult mathematics in his classroom (Figure 5.32).

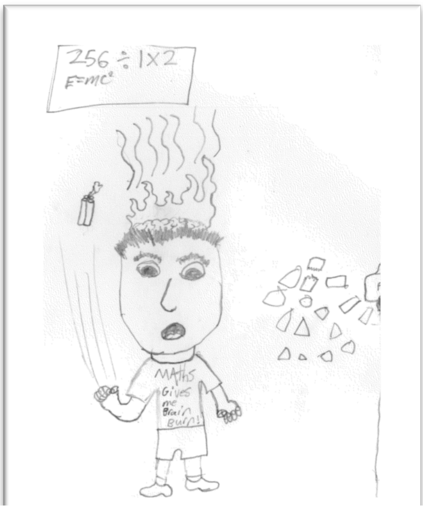


Figure 5.32 Joshua's brainburn
[Not to be reproduced without permission.]

He was the only child in his particular mathematics class to use the metaphor of brainburn to illustrate his response to the subject although children in other classrooms use this particular metaphor. In Mr. Forrest's class, Richie, Harry and Pink drew brains on fire. Overall, the range and complexity of the children's depictions of the nature of mathematics was extraordinary, covering very narrow to very broad ideas. It covered number, geometry, measurement, algebra, symbols, interesting imagery and use of colour to explain notions of mathematical thinking, difficulty and affect inherent within the nature of the subject or domain.

Identity within the world of mathematics

Of interest when considering student identity is how the children viewed themselves as part of the mathematical world and how they positioned themselves within the mathematics classroom. Some children identified themselves as good at mathematics and definitely members of this world. Others saw themselves as reluctant inhabitants or not natural members of this world. Identities are not formed in isolation; instead they result from a combination of discourses and contextualised experiences (De Corte et al., 2010) that are "produced through practices, relationships and interactions within specific sites and spaces" (Archer et al., 2010, p. 619). Mathematics identities are constructed within the public spaces of the mathematics classroom, in Nuthall's public world where all can see which group the children have been assigned to; however, the semi-private world of peer interactions and positionings as well as the private world of the individual child's internal conversations have as profound an influence on identity creation and maintenance as does the public world of the teacher (Nuthall, 2007).

From some of the previous examples, three children identified themselves as competent inhabitants of this world. Ella (Figure 5.15a) jumps up during the lesson saying "Yeah I got it right" in the classroom where the "teacher is teaching the child [so] she can get a job involving Number." She places herself as someone who can do mathematics in a utilitarian classroom. Ronan (Figure 5.22) positions himself as someone who can solve difficult problems. Tom (Figure 5.23) views himself as a hero on a quest in “Mathsland” where he is capable of leaping hurdles and taming new strategies.

Although Bob includes number in his drawings, the effect is different from many of the other number drawings because he places a boy with whirring eyes in the centre, explaining without any words how mathematics affects him: this is an expressive depiction of his lived experience of mathematics (Figure 5.33). The image of whirring eyes suggests confusion, and, as a metaphor not uncommon in cartoons and comic books, is easily understood by the children.

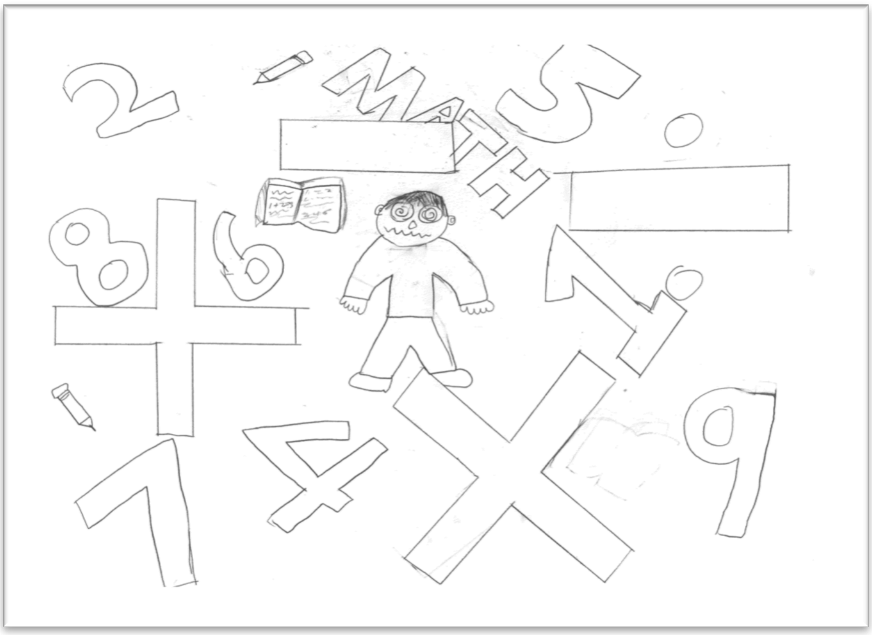


Figure 5.33: Bob’s whirring eyes
[Not to be reproduced without permission.]

The following examples all come from members of Mr. Forrest’s mathematics class. Neo identifies himself as good at mathematics by placing himself together with four other students who all like maths. The teacher says “Good” to Neo who responds with “AWSOME!!!”. Angus positions “me” in contrast to the “others” who are bored, daydreaming or clapping at his performance for the smiling teacher. Ziro views maths as exclusionary, some people belong like “me” and are welcomed into the “math shack” while “dum bum” is excluded.

Steven takes a similar position as a good-at-maths student with the teacher responding to his work (Figure 5.34). For Steven, “Maths is super cool” while the others find it boring or sleep-inducing.

Figure 5.34: Steven’s “me” versus “others”
[Not to be reproduced without permission.]

A very different identity is presented in Orange’s drawing where he positions himself, Harry and Pink as the “dunnos”[those who don’t know the answer] while “Smarty pants” the knower can answer the “1+1” problem. He and his friends identify themselves as the naughty Year 5 boys, or those who can’t do. Harry (introduced in Figure 5.4), one of this group, places himself and his friends as extremely stressed, suffering from brainburn because mathematics is too hard.

In a later drawing, Harry (now in Year 6, Figure 5.35) positions himself three different ways in his new mathematics classroom:

Figure 5.35: Harry, Year 6
[Not to be reproduced without permission.]

Harry explained this drawing to me: “All of these are me, sometimes confused (left figure), sometimes play fighting (positioning himself as the naughty boy), sometimes getting on with it”. In the right-hand depiction he is unhappily working away under the nose of the teacher. Here he has identified himself as the well-behaved boy doing what he is supposed to be doing.

Chloë (Figure 5.36) stands in a world where ‘getting it right’ is important and she is screaming with frustration:

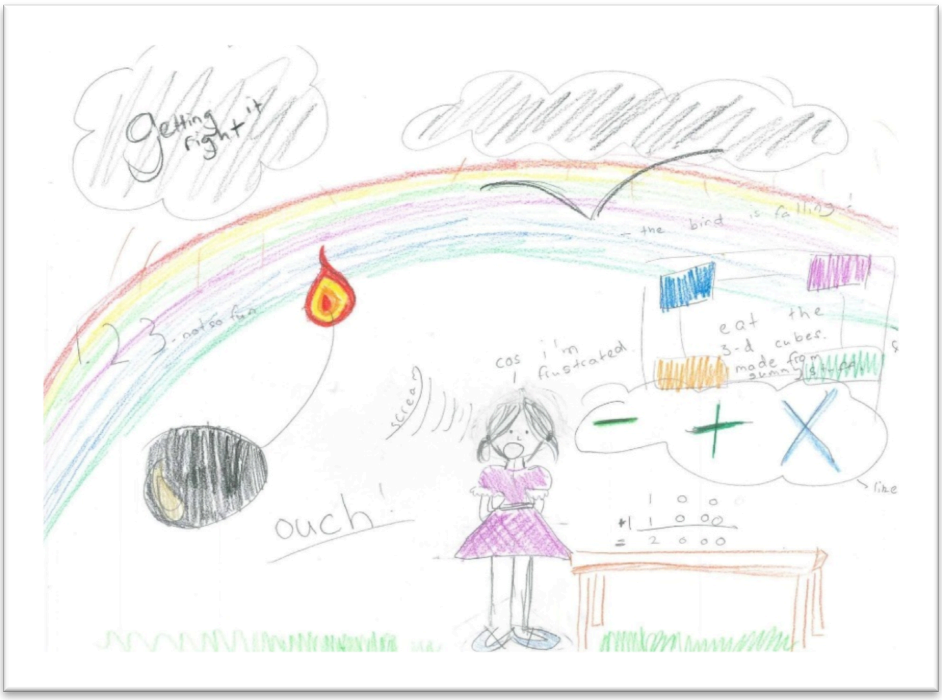


Figure 5.36: Chloë’s scream
[Not to be reproduced without permission.]

From Ron's class at Kikorangi, there is Sarah who explains in speech bubbles that her drawing shows "I am trying to do pluses with dise", and "I am trying to Do time Tables!" George, a very competitive student, positions himself as the plus and minus king on a pedestal above the rest. Phisic, his main rival, draws himself as a hero called "plus man" and as the evil "minus man". Fred, another classmate, identifies himself as the holder of knowledge within this class with, "Only I know the anser becose im relly good at It".

The children depicted a range of mathematics identities: those who definitely belong to this world of mathematics, those who sometimes belong, and those who feel alienated from this world.

The mathematics classroom

A common theme that many of the drawings included is that of the mathematics classroom or the mathematics learning environment. Of these, 37% made direct references to doing and learning school mathematics within a school environment. An additional group of 15% included elements of doing and learning mathematics that may or may not be at school, e.g., a child working on mathematics alone or doing homework, school mathematics but out of the classroom, or an adult helping/teaching mathematics who may or may not be a teacher. All of these drawings included notions of what these individual students believe doing and learning mathematics is all about, based on their lived experiences of the subject. These drawings referenced all three of Nuthall's worlds of the classroom: they are ostensibly about the public world of doing, learning and teaching mathematics, but they also include elements of the semi-private world of peer interactions, of doing mathematics with others, as well as private experiences of problem solving and doing mathematics on your own (Nuthall, 2007). Many of the drawings also incorporated the language of their mathematics classrooms by using words like equations, strategies, algorithms,"symatrey", probes, solutions, BEDMAS, tessellations as well as the usual words – add, takeaway/subtract, divide, multiply, timestables, measurement, etc. In addition, there were complaints about "weird methods" (David) and value-laden comments like, "Maths is fun and I always treasure it"

(Olivia). These experiences of school mathematics influenced what children believe about the nature of mathematics as well has how they position themselves within the world of mathematics.

Words and concepts like tessellations, BEDMAS, symmetry, infinity and some of the algebra examples seem rather sophisticated for children in Years 5 and 6. I made the assumption that the children who used these terms had come across them in their mathematics classrooms because Tom (Figure 5.23), Michael (Figure 5.13), Sophie (Figure 5.28) and Lucy (Figures 5.19 and 5.26) were in an extremely accelerated mathematics class. Alternatively, children who included these terms and concepts may have adopted them from older sibling, whānau[20] or parents.

Quite a few of the drawings illustrated individual children doing mathematics, often portrayed as the good, well-behaved child sitting at his or her desk "getting on with it" like Harry in Figure 5.35 and Elsa, Figure 5.37.

Figure 5.37: Elsa dealing with "too much hard work"
[Not to be reproduced without permission.]

20 Māori term for extended family

These drawings are examples of children identifying themselves with being good mathematics students by representing behaviours that are associated with availing mathematics beliefs/learning dispositions (Lesh & Zawojewski, 2007; Muis, 2004; Schoenfeld, 1994a).

Another group of drawings include working with a friend, an example of pair work, often a best friend (Govi, Figure 5.6) or 'twin' (Figures 5.38 and 5.39).

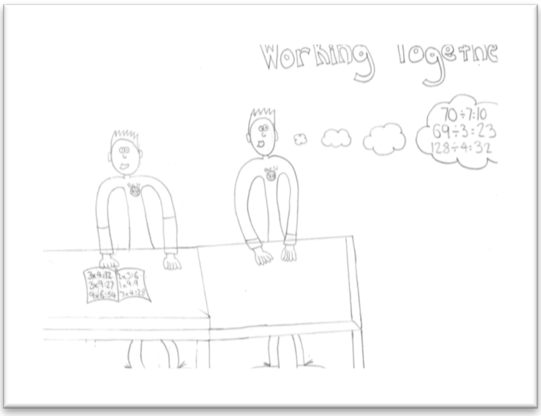


Figure 5.38: Anton's drawing of himself and 'twin' Jordan
[Not to be reproduced without permission.]

Figure 5.39: Bobbi's drawing of herself and 'twin' Cassandra
[Not to be reproduced without permission.]

In both Kikorangi, (apart from Room 11) and Whero, mathematics is an interchange time where children are grouped by 'ability'[21], thus children get to work with friends who may not be in their usual classes.

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21 Often these groupings are based on achievement scores.

There are also drawings that included much larger groups like Steven's 'me and others' (Figure 5.35, p.) or Neo's group of happy students with their teacher (Figure 5.40), Davey's "Maths is fun" story and Fred's classroom.

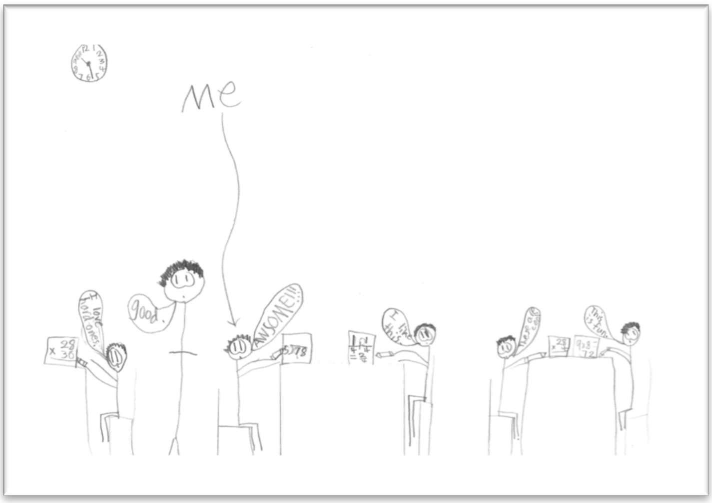

Figure 5.40: Neo's class of cheerful maths students
[Not to be reproduced without permission.]

The "Math is fun" drawing tells the story of our hero, Davey, working hard at his desk in a traditional looking classroom, then going home and working hard at his mathematics (Figure 5.41).

Figure 5.41: Davey's "Math is fun"
[Not to be reproduced without permission.]

In frame three, our hero does very well on his test and goes on to be recognised for his high score at assembly in front of the whole school. Even though I think he is playing with mathematics and calculating the difficult problem in the final frame during playtime, I am not sure about the implications of these frames. I was unable to discuss this drawing with the artist because Davey was not available when I returned to Whero to talk about the drawings with their creators.

Not only are children portrayed in the drawing, but many of the drawings included the teacher(s), on their own, interacting with a student or with multiple students (Figures 5.15, 5.35, 5.40 and 5.41). One example shows a boy at his desk with the caption, "Kid Learning about maths to get smarter", and the teacher at the board "Teacher teaching Kid about Maths" (Bob Smith). The teachers are portrayed in a variety of ways from benign, well liked (Figures 5.15, 5.34, 5.40, 5.42 and 5.43) and helpful, or stressed (Figure 5.44) to quite terrifying (Figures 5.35 and 5.47).

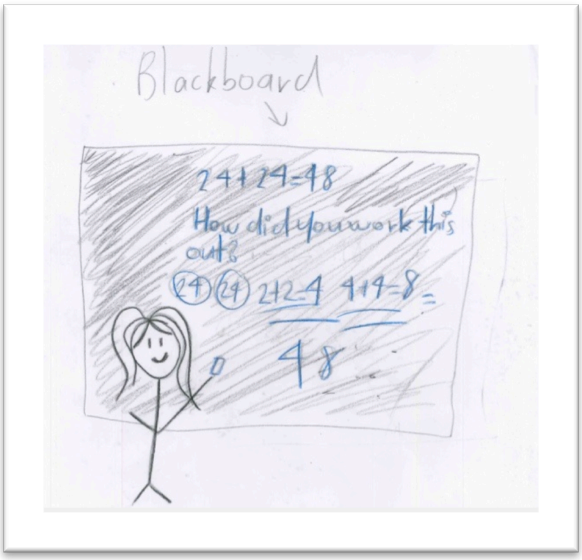

Figure 5.42: Rose's teacher
[Not to be reproduced without permission.]

Ms. K, Rose's teacher, not only asks her students to solve the problem posed on the board but to explain how they reached their solutions. This seemingly simple drawing reveals how mathematics works in Ms. K's classroom. It also suggests that students like Rose know what is expected of them when doing mathematics.

Figure 5.43: Neil's classroom with Mr. Forrest
[Not to be reproduced without permission.]

Neil and another boy sit neatly 'on the mat'[22], as well-behaved students listening to Mr. Forrest who is leading a maths game (Figure 5.43). Mr. Forrest is sporting a moustache[23] and waving a ruler. He is miked up to show he is taking part in the filming and recording of his mathematics lessons. Another interesting detail in this drawing is the 'Black Box' which has the names of non-complying children; however, the first name in the box is Bob which stands for "Bob the ruler", a classroom joke. A much grumpier teacher is Mrs. Umbridge (Figure 5.35) saying "Raaaaaaaaa!". As Harry explained to me, "She yells or frowns all the time."

Fred's mathematics classroom drawing (introduced in Figure 5.3) includes the public world of the teachers and the work they set, his interpretation of the private worlds of the student participants and what they are feeling, as well as his position in the classroom. He uses stick figures to portray the inhabitants of his mathematics classroom and simplified examples of maths problems that are much easier than the problems I noticed his maths group had been given to work

22 A zone of the classroom, usually matless, often at the front or beneath one of the classroom boards from where the teacher instructs, leads discussions, explains tasks, gives out notices, etc.

23 It was Movember when the teacher and some of the fathers were growing moustaches to raise awareness about and money for prostate cancer.

on. Many of the other children's drawings also presented examples of mathematics in terms of simple arithmetic problems rather than the much more advanced work with which they were involved. Like the stick figures and the cartoon-like drawings, this type of depiction of the classroom and the work of mathematics seemed to function as a shorthand or an easily communicated schema of "the maths classroom".

The following examples are also set in the maths classroom, depicting the children's individual experiences with Mr. Forrest's Room 6 mathematics, their feelings about the subject, and their responses to a specific incident. The context of their classroom and their experience were important to understand during the process of reading their drawings.

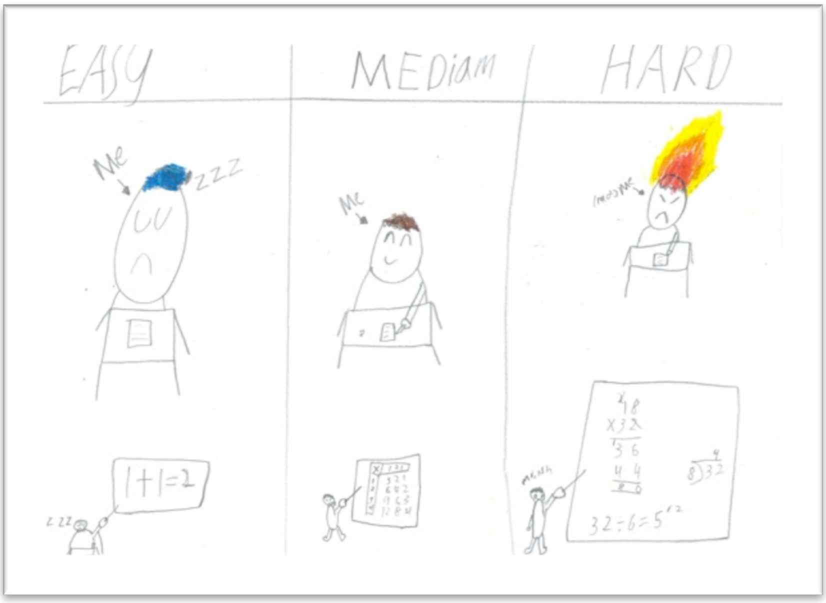

Figure 5.44: Richie in three states during maths classes
[Not to be reproduced without permission.]

Richie's drawing (Figure 5.44) comprises three frames, each with a drawing of himself at a desk/table in his mathematics classroom with his teacher at the board, a combination of the public domain of teacher and private one of Richie's beliefs about mathematics and what he feels. For Richie, mathematics is snooze inducing, just right or the cause of brainburn depending on the level of mathematics he is being presented with. In the easy frame, both the teacher and “me” are asleep as the sum is far too easy; in the “mediam” one, both the teacher and “me” are smiling and getting on with it; however, in the “hard” frame Richie is scowling and suffering from brainburn while the teacher, Mr. Forrest, smiles.

Pink’s, Orange’s and Harry’s drawings include both the group of the friends and their teacher. Pink’s drawing includes the mathematics class relieving teacher, Orange, Harry and himself. Harry’s head is afire and he complains “this is hard”. Both of the other boys are distracted by thoughts of lollies and or in Pink’s case “…jump[ing] on a marshmallow”.

Orange’s drawing captures some of the spark and mischief of his small group working together (Figure 5.45). He positions the group, especially Pink with his horns and trident, as naughty even though ironically he gives two of the others angel attributes. They cannot or do not want to answer the teacher’s simple mathematics question. There is also an indication of stress, at least for Harry who has “spewed”. Orange includes the “Smarty pants” child, the antithesis to the group, who is smiling and has all the answers. Although he uses stick figures he manages to include sufficient detail such as the fashionable brand names on his teachers t-shirt and shoes, the halos and horns, as well as the moustaches, to communicate his experience of the mathematics classroom.

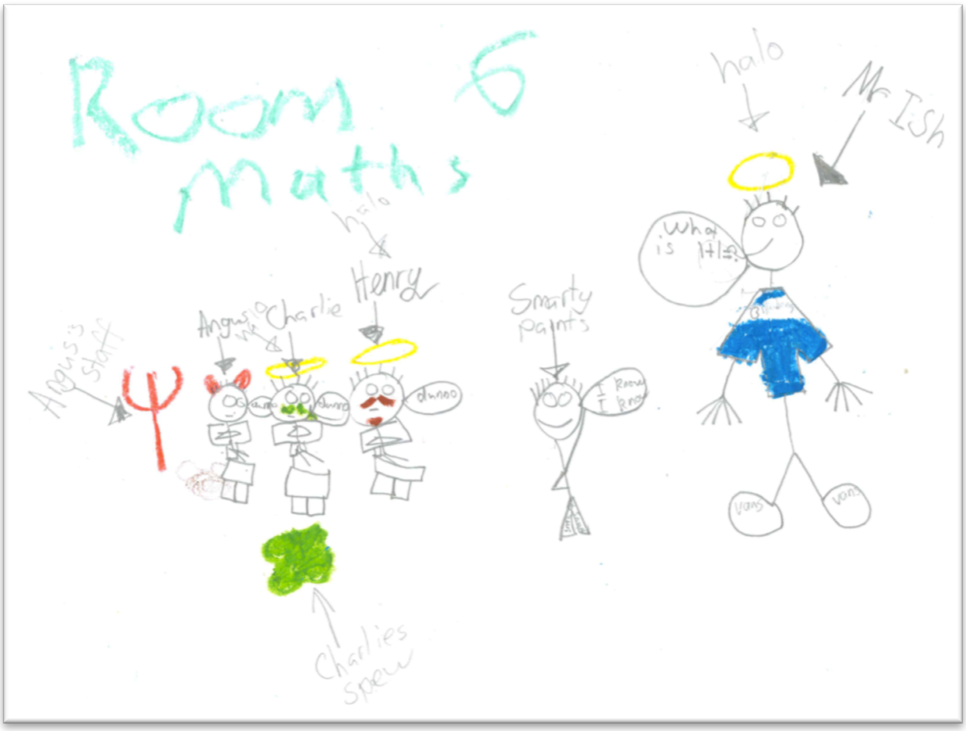

Figure 5.45: Orange
[Not to be reproduced without permission.]

Harry’s drawing presents the extreme stress that he and his friends Orange, Pink and Richie experienced in mathematics (Figure 5.46). It is a much more troubled and troubling drawing than the other examples. All of the children and the teacher are depicted as traumatised: Richie is hanging off a light fixture, Orange has his eyes closed as if he has passed out while calling for his “mummy”, Pink has spewed and has to go home, while our artist, Harry, sits at his desk yelling “ahhhhhhh”; all of the boys have flames shooting from their heads.

Figure 5.46: Harry's horrible experience
[Not to be reproduced without permission.]

In contrast to his drawing, Harry was usually a cheerful, enthusiastic boy. In a conversation with the boys after they completed their drawings, I discovered that they had become extremely stressed the previous day in mathematics when they had an end-of-unit test on fractions. Both their classroom teacher, Mr. Forrest, and I had been present in the classroom on the day of the test; however, we had failed to notice their distress. They all seemed to behave as they normally did without any outward indication of brainburn. This is an example of where accessing the children's semi-private and private worlds through their drawings results in information that would otherwise have been inaccessible. In an interview a year later, Harry could not remember the incident that sparked the brainburn, yet he could remember how the stress of mathematics felt as he commented on his drawing:

> *Harry: My head always used to get hot, like my forehead, and I just imagined my brain going on fire if I did too much... Maths. [laughs]. That's funny."*
> *Cathy: So do you think that's a, not a very happy picture?*
> *Harry: I find it quite funny [laughs]*
> *Cathy: It is, it's a very funny picture.*
> *Harry: Yeah, but... yeah, not too happy.*

All of these boys who seemed fine during a test on fractions drew pictures that included elements of extreme stress such as brainburn, a boy hanging from the light fixture, throwing up and fainting[24]. This particular set of drawings alerted me to the powerful relationship between doing mathematics, their experiences of their mathematics classroom on that day, and feelings associated with mathematics that lasted long after the memory of the incident, at least for Harry, had faded.

Conclusion

The drawings acted as a window into individual children's beliefs associated with mathematics. The children used this medium effectively to communicate a range of beliefs and feelings associated with their understandings of what mathematics is. They also conveyed how they, as individuals, and others belong/do not belong to this world. The majority of the drawings reflected beliefs about mathematics as number in the same way as the written responses to the *Alien Task*; however, the drawings, unlike the written responses, included a much broader range of aspects of mathematics.

During the process of reading and making sense of the drawings, I used both Freeman and Mathison's (2007) "framework for reading images" (Figure 5.1) and Nuthall's three worlds of the classroom (2007) as lenses for exploring the content of the images. Freeman and Mathison's (2009) framework provided an additional set of tools for making sense of the drawings. A literal reading was made of all the drawings while biographical, empathetic, iconic and psychological readings were employed where appropriate. Biographical readings were used to explore student identities and sense of their own and others' positions within the world of mathematics: for example, Ronan (Figure 5.22), Tom (Figure 5.23), Bob (Figure 5.33), Steven (Figure 5.34) and Chloë (Figure 5.36). Empathetic readings were used in reading drawings depicting brainburn (e.g., Figures 5.27, 5.32 and 5.44 and 5.46) and brain as a place where learning is situated (e.g., Figures 5.24, 5.25 and 4.26), as well as in the stereotypical classroom (e.g., Figures 5.3, 5.34, 5.40 and 5.41), stick figures and

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24 These boys were not sitting together when they created their drawings; therefore, I could not assume that they influenced each other with their use of metaphor.

oversimplified mathematics problems (e.g. Figures 5.3, 5.11, 5.17, 5.35, 540, 4.45, and 5.46). Iconic readings were less common but helpful as in the analysis of the value afforded “utility” in Ron’s classroom (Figures 5.14, 5.15) and of other cultural constructs such as “doing school maths”, and the good/bad mathematics student (e.g., Figures 5.35 5.40 and 5.45). Finally, psychological readings were made where images of affective states of mind were included: good feeling, feelings of euphoria, excitement (e.g., Figures 5.4, 5.5, 5.6, 5.22 and 5.26), bad feelings, feelings of stress (Harry, Figure 5.46), despair, agony, tears and frustration (Chloë’s scream, Figure 5.36).

In terms of the Nuthall’s three worlds of the classroom (the public, semi-private and private worlds)(2007), many of the students included their own impressions of the public world of the mathematics classroom inhabited by teachers and students, taking place in recognisable mathematics classrooms and working on recognisable mathematics tasks despite being represented by a discourse which includes a simplified series of easily understood symbols (1+1, $E=mc^2$, etc.), stereotypical (old fashioned) mathematics classrooms and stick figures. Examples of the ongoing peer relationships of the semi-private world were clearly articulated, a world about which the teachers were barely aware. Drawings included children working, competing, and positioning themselves as knowers, and doers. They also illustrated the bright, the naughty, and the well-behaved, diligent students, as well as the inclusion/exclusion of class members. However, of real interest is how the drawings worked as a way to access the private world of children's individual beliefs. Most, if not all of the drawings, give a glimpse into this private area of individual, idiosyncratic mathematics beliefs. The range of these private beliefs was intriguing; even though seven students in Ron's class drew maths as useful, a common discourse in this particular classroom, individual perspective on usefulness included for the future, for now, for adults, for children, for work and for school in general. The range of metaphors portrayed in the images also provided a glimpse into this private world of individual beliefs, metaphors used to depict beliefs such as maths as number, maths as universal, something you do at school, something you do with friends, and/or something you do alone.

The range of beliefs the children chose to illustrate in their drawings resonates with literature about beliefs as discussed in Chapter 4. My analyses of the drawings have identified beliefs about the nature of mathematics (Goldin, 2002; Kloosterman, 2002), about the nature of school mathematics (Grootenboer, 2003; Op 't Eynde et al., 2002; Yackel & Rasmussen, 2002), about self as a mathematic doer and learner (De Corte et al., 2010; Goldin, 2002; Kloosterman, 2002; D McLeod & McLeod, 2002; Op 't Eynde et al., 2002) as well as beliefs about the social contexts, norms and values of the mathematics classroom and about learning and teaching mathematics (De Corte et al., 2010; Kloosterman, 2002; D McLeod & McLeod, 2002; Op 't Eynde et al., 2002; Yackel & Rasmussen, 2002)

Based on my analyses of the drawings, I have reconfigured my initial four categories of *self*, *ability/others*, *learning environment* and *nature of mathematics* into three main themes. Beliefs about the nature of mathematics, how the drawers view themselves and others as belonging/not belonging to the world of mathematics as well as beliefs about the learning, doing and teaching mathematics are all articulated in the drawings (Figure 5.47). In this new configuration, beliefs about *self* and *ability/others* became combined within the theme of identity. These three themes, related to beliefs about mathematics, incorporate various subthemes (Braun & Clarke, 2006). The theme associated with nature of mathematics included the subthemes of ‘maths as number’, ‘maths as number plus other strands’, ‘maths as useful’, ‘maths as everywhere’, ‘maths as problem solving’, and ‘maths as easy/hard, fun/agony’. Subthemes associated with identity are ‘good at maths’, ‘not good at maths’, confused, and stressed. Subthemes incorporated in the teaching/learning/doing mathematics group include working alone, working with friends, teachers (the good and the bad), the language and concepts associated with doing mathematics, and classroom experiences.

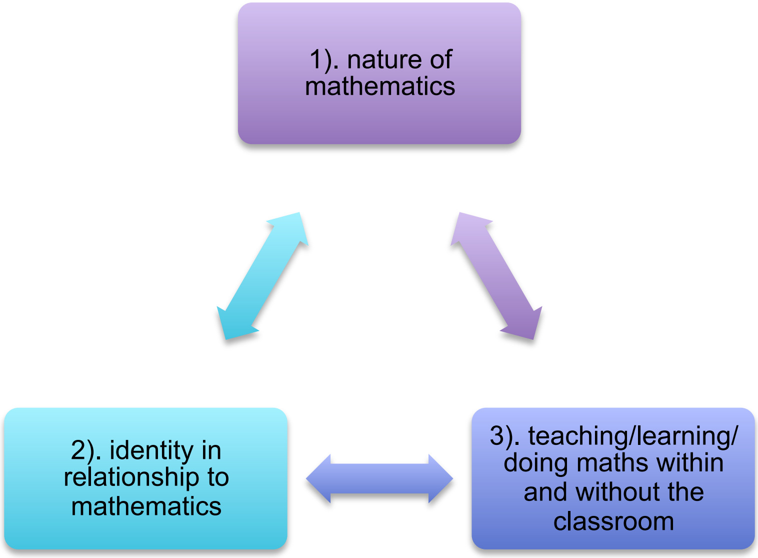

Figure 5.47: Beliefs about mathematics

However, these mathematics beliefs themes seem neither hierarchical nor mutually exclusive. They are interconnected, both influencing and overlapping each other. For instance, Harry's and his friends' stressful experience of doing a fractions test (theme 3) influenced how they identified and positioned themselves and others (theme 2) in their classroom (theme 3) as well as their beliefs that mathematics by its nature can be stressful (theme 1)(Figure 5.44, 5.45 and 5.46). In another example, Ella, who sees mathematics as useful (theme 1), identifies herself as the person getting a problem correct (theme 2) during the process of being taught (theme 3) so that she can find a job working with numbers (Figures 5.15 and 5.16). Her beliefs about the nature of mathematics, as well as her experiences of her mathematics classroom, colour how she identifies herself within this world. In many of the drawings the belief themes overlap and are thus difficult to untangle (Figure 5.48).

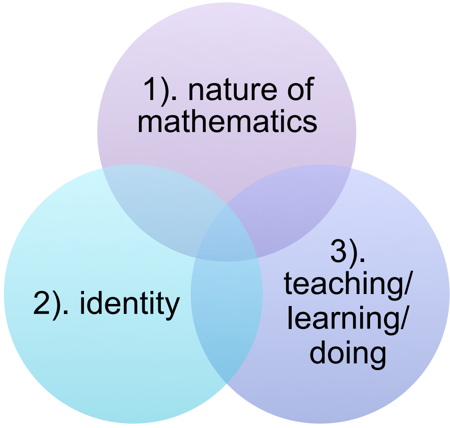

Figure 5.48: Overlapping belief themes

Tom identifies himself as the hero who is capable of leaping hurdles, taming strategies, avoiding equations in a land full of these concepts as well as tumbling shapes and numbers; in this way he combines his notion of the nature of mathematics (theme1) with his identity (theme 2) and the language and work of the mathematics classroom (theme 3) (Figure 5.23). Katia combines her sea metaphor of the nature of mathematics, with her experience of numbers, shapes, sums, fractions, and the language of mathematics (themes 1 and 3) with how easy she finds it (theme 2) (Figure 5.18). Whether these beliefs are viewed as separate and influencing each other or as overlapping, they are anchored in, grounded in, developed from and very much part of the social fabric and experiences of the individual children. These beliefs cannot be removed from their social contexts both within and without the mathematics classroom.

Each child's set of mathematics beliefs is influenced by her/his experience of the classroom and learning mathematics from a series of teachers, influenced by the choreography of the lessons, the layout of the classroom, and the values of the classroom which in turn are affected by the school mathematics program. They are also affected by the community, by family – parents, siblings and extended families – by social class values, by the country they live in which controls the school mathematics curriculum as well as the world-at-large through media such as TV, film, internet, books, games, newspapers, etc., as well as by international comparative testing regimes (e.g., TIMSS & PISA).

Following the iterative cycles of analysis of children's drawings, I embark on further analysis of nine focus children's and their teachers' beliefs about mathematics in Chapter 6: *Narrowing the focus*.

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Chapter 6: Narrowing the focus

In the previous two chapters, I examined an aggregated landscape of student beliefs about mathematics and explored the large gallery of maths beliefs drawings. This chapter presents another analysis frame for investigating mathematics beliefs by focusing on nine students and two teachers in order to illustrate these individuals' beliefs, experiences and stories about mathematics. Although these examples do not represent a full range of beliefs that children and teachers espouse, they demonstrate how some of these beliefs are enacted in the context of the mathematics classroom. These examples work as snapshots of individual students' and teachers' beliefs about mathematics at certain points in time. I was aware of a tension between reporting what I saw as important to communicate while leaving out what I viewed as non-essential, and being true to what the children and teacher said – to what they choose to share. This tension was also present during the process of selecting, grouping and analysing data through the lenses of a specific piece of research as I selected these particular stories and illustrations (Mertens, 2010). The process of narrowing the focus by concentrating on some individuals' beliefs and experiences presented another opportunity to access, explore and interpret beliefs about mathematics.

Children's beliefs about mathematics are affected by their teachers' beliefs, the curriculum as it is presented within the classroom, and the child's prior and present values and beliefs[25]; in turn, these individual personal epistemic beliefs can affect how the children do mathematics, how they engage with tasks, activities, materials, and other students. Their beliefs, therefore, are both shaped by their experience of this and previous mathematics classrooms and affect mathematical experiences because they guide 'being' and 'doing' within the classroom context. Classroom experiences not only influence ways of doing mathematics, beliefs and feelings about mathematics at the time they happen, but well into adulthood as well (Sam & Ernest, 1998). These mathematics beliefs

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25 These values and beliefs are also influenced by their parents'/caregivers' and families' attitudes and beliefs.

may be fluid, changing and reformulating within different contexts (Diversity in Mathematics Education Center for Learning and Teaching, 2007; Franke et al., 2007). Similarly, teachers' beliefs about mathematics are shaped by their own experiences of doing and learning school mathematics, their professional training, the mathematics curriculum, and their experiences of teaching, including interactions with particular students at particular times (Philipp, 2007). To a large extent, it is their classroom experiences of learning mathematics as students that influence teachers' beliefs about the nature of mathematics and how to teach it (Cross, 2009; Stipek, Givvin, Salmon, & Salmon, 2001).

The beliefs data in this chapter are analysed in terms of the frameworks suggested in the Chapters 4 (Figure 4.4) and 5 (Figure 5.48,): how these individuals see and position themselves within the world of mathematics (*self/identity beliefs*); *ability beliefs* about doing and learning mathematics, particularly within their mathematics classes; and beliefs about *the nature of mathematics* or what is the real and proper stuff of maths and how to do it. However, these categories are not as distinct as implied by the four-factor Mathematics Beliefs Framework (MBF) because all of these beliefs affect and overlap with each other, especially when examined within the context of the classroom, i.e., the *learning environment*. In the MBF, *self*, *ability*, *learning environment* and *nature of mathematics* are viewed as separate aspects of mathematics beliefs. In Chapter 5 the mathematics beliefs about *self* and *ability* were combined into an *identity* beliefs category. However, in this chapter, *identity* and *ability* are untangled and reframed. In this chapter, *ability* beliefs are associated with beliefs about what makes people good at mathematics, as well as what sorts of people are good at mathematics.

Identity beliefs are recognised as affecting classroom participation, engagement and motivation (Archer et al., 2010; De Corte et al., 2010; Diversity in Mathematics Education Center for Learning and Teaching, 2007; Franke et al., 2007; Gauntlett, 2007, 2008). Identities are not only constructed by individuals about themselves within the world of mathematics, but they are also assigned by others, teachers, peers and family member (Gauntlett, 2007). Being recognised, by oneself or someone else, as good or bad at mathematics overlaps with beliefs about *ability* and the sorts of people who can/cannot do mathematics (Franke et al., 2007; Philipp, 2007). Also of interest is how these beliefs affect the individuals' behaviours within the context of particular mathematics classrooms with their social norms and ways of doing mathematics (Franke et al., 2007; Yackel & Cobb, 1996; Yackel & Rasmussen, 2002). Focusing on what happens in the classroom is important because, as Philipp points out, it is "while students are learning mathematics, they are also learning lessons about what mathematics is, what value it has, how it is learned, who should learn it, and what engagement in mathematics reasoning entails" (2007, p. 257). It is their classroom experiences of learning, doing and being mathematical that influence children's beliefs about the world of mathematics and who its native inhabitants might be.

Figure 6.1 is a two-dimensional representation of the four belief factors examined from a different perspective. *Self*, *ability* and the *nature of mathematics* beliefs are seen as overlapping and are examined within the beliefs and experiences of the mathematics classroom context. The figure focuses on how these three groups of beliefs play out in specific classrooms; I use this model to examine how nine focus children and their teachers do, learn and teach mathematics. Their mathematics experiences and beliefs are coloured by past experiences of engaging with school mathematics. The empty space in the figure represents other beliefs about learning, knowledge and behaving that are not specific to the domain of mathematics, such as the rules, routines and behaviours of the classroom, being the good/bad student, and the values of the particular school or classroom. Not included in the model is the relationship between these mathematics beliefs and experiences outside the classroom, those coloured by family, peer and communiity beliefs.

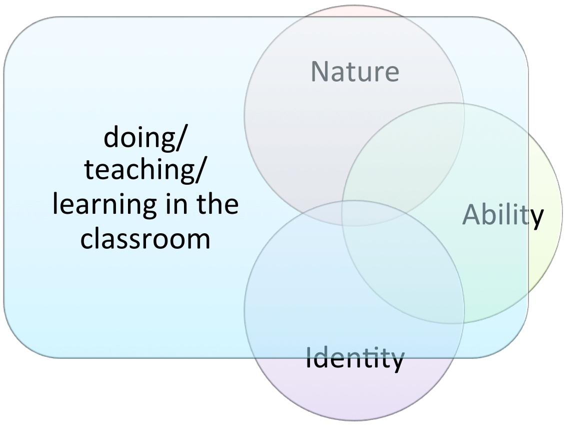

Figure 6.1: Beliefs about mathematics mapped onto the classroom context

This section of the study presented an opportunity to look at and beyond espoused beliefs (Philipp, 2007; Thompson, 1992) by considering a combination of data from written responses on the MBQs (Questions in Appendix D, results summarised in Appendix O), and from classroom observations, recorded materials, interviews, field notes, artifacts, conversations and drawings (Appendix P) (Lester, 2002; McDonough, 2002). These data were collected over an 18-month period, which spanned two school years. Although the focus was on nine children (See Chapter 3, Heading: "Focus students", pp. 66-67) and two teachers, additional children and teachers also contributed to these data: the children by talking to me during my data collection forays, and by interacting with the focus children and/or teacher during my observations and recordings; the teachers by allowing me to observe in their classrooms and to interview them in 2008. During the first year of the study (2007), I collected data from Ron's and Mr. Forrest's classrooms, a combination of MBQ responses, observations, artifacts, drawings and video recordings; the following school year I observed the children in their new mathematics classrooms and interviewed their 'new' teachers. I interviewed Ron and Mr. Forrest as well as all nine children who redid the MBQ and eight of whom drew a second mathematics beliefs drawing. Table 6.1 lists the names of the participants.

Table 6.1: Summary of characters in Chapter 6

Kikorangi **teacher 2007**	**children**	**mathematics teachers 2008**
Ron	George & Sammie	team of 3: Paul, Vern, Sally
	Fred	Ms. K
	Jasmine	Peter
Whero **teacher 2007**	**children**	**mathematics teachers 2008**
Mr. Forrest	Harry	Mrs. Umbridge
	Chloë & Jack	team of 2: Mrs. Hill & Dale
	Caroline	Mr. Lupin
	Lilly	Mrs. McGonagall

Note: the focus children and teachers chose their own pseudonyms. I assigned pseudonyms to the 2008 teachers.

Although all of the focus students and their teachers are included in this chapter, I do not report exactly the same data for each participant. Because of the amount of data and analysis results, I was unable to describe and discuss all of the findings; instead, I have tried to include representative examples to illustrate the essence of the findings (Denzin, 2011).

An introduction to two classrooms

The two teachers, Ron from Kikorangi (a decile 4 school) and Mr. Forrest from Whero[26] (a decile 10 school), were Year 5/6 teachers who taught mathematics interchange classes; the students were grouped by achievement levels for their mathematics classes rather than remaining with their usual peers and teachers. At the beginning of the school year, Mr. Forrest's mathematics class included students from both Years 5 and 6; by mid year the composition changed to only Year 5. Both schools were committed to the Numeracy Development Projects (Ministry of Education, 2010c), often referred to as the Numeracy Project, which was evident in their classroom organisation, resources available as well as the choreography of their mathematics lessons.

26 These two teachers and their schools were introduced in Chapter 3 and mentioned in Chapter 5.

Ron's classroom

The first bell signaling the end of interval has only just begun ringing and most of the members of Ginny/Ron's classes are waiting to start singing.
(from my fieldnotes)

Ginny and Ron team-teach two classes of students who are used to dividing into oft-changing groups and working in different spaces depending on the activity and subject. Every mathematics lesson begins with 10 to 15 minutes of singing[27]. Children then collect their maths gear and slowly wander to their interchange places. They are divided roughly into two with the stronger students following Ron to room 10 while the weaker remain with Ginny in Room 9.

Room 10, the smaller of the two rooms, is bright, airy and somewhat untidy. Although there is colourful student work on the walls, it is messy. The bookcase is untidy and the books look rather dull. There are a lot of signs of Ron and his class's passion for all things Harry Potter with posters, books and paraphernalia on display. The classroom furniture is light, clean, flexible and the correct dimensions for children of this age, and the chairs are comfortable for adults. The classroom also has an old mattress and a sofa, both often in use by those who like working in unusual places and position; however, it means classroom cleanup can be slow as both have to be tidied at the end of the lesson and items such as books, game pieces and pencils seem to vanish into their depths.

The mat area of the classroom is unusual because it is not on the floor; instead, it looks like a conference table which seats at least 20, a change from the norm where the teacher is enthroned while the minions sit at her/his feet. The rest of the tables, which seat between 4-6 students, are grouped in small islands, but, as these can be easily moved, the configurations can be changed to suit the group and/or the activity. There is also a row of carrels along the back wall, popular during speed challenges, for those who wish to work alone.

27 General singing, nothing to do with mathematics

Ron is an earnest, dedicated teacher with whom the students seem to get on very well, as seen from the friendly classroom banter. Although George, one of the focus children, often challenges Ron in mathematics lessons, he indicates his respect with “yeah, Ron’s a good maths teacher”. Ron is in his ninth year of teaching. He is very involved with the student council and choir, plus many other school activities. Like most of the teachers at Kikorangi, he dresses casually, mostly in jeans, and the children call him by his first name. He is exemplary of the school ethos which encourages a discovery approach to learning, children taking responsibility for their actions, choices and learning, restorative justice and being part of the local community. This popular school is in a working class neighbourhood consisting mostly of rental properties, which have a large turnover of residents.

The choreography of Ron’s mathematics lessons

Mondays begin with a 10-minute speed challenge. Students choose where they wish to sit; however, they need something with which to write as well as their exercise books, not always available. The test sheets are distributed, and they begin. George finishes first, around the three-minute mark. The rest of the class gives a combined sigh.

Fred, as usual, arrives late from his extra reading session, which puts him at a disadvantage every week.

George looks around to see how Phisic, his usual rival, is faring. Most of the other students struggle on, with much sighing and looking around. Slowly, the next fastest students limp to the finish line. There is quite a bit of paper shuffling and sighing both from the bored early finishers and from the strugglers.

Finally, time’s up. The rest of the session is spent marking. As usual, George has a perfect score and is cockily challenging everyone, including Ron, to a rematch. There is just enough time to paste tests into books, put them away and tidy the room when the bell goes.

Observation Journal, October 2006

Tuesday, Wednesday and Thursday mathematics classes also start with singing; however, the rest of the lesson follows a completely different pattern to the normal Monday lesson described above. The children are sorted into three ability groups: the Mathematicians (the top group), the Ten Champs (the middle group) and 1+1 Champions (the bottom group). When students arrive in Room 10, they are given information for that day's group tasks, usually written up on the whiteboard at the front of the room. These tasks are usually divided into two, indicating that there will be two activities, for example:

> *1). First time period: Mathematicians on the mat with Ron, Ten Champs work on the following problems, 1+1 Champions games; and*
> *2). Second time period: Mathematicians work on following problems, Ten Champs games, 1+1 Champions on mat with Ron.*

In other words, each group is involved with two mathematics activities a day, while Ron does some direct teaching, discussion or checking with two of the three groups. Sometimes there is time for a combined activity at the end of class, but more commonly time runs out. This rotating group approach is advocated by the Numeracy Professional Development Projects (Ministry of Education, 2008c). In this classroom mathematics homework is optional.

An introduction to four members of Ron's mathematics classroom

All of these students were in the higher of the two interchange classes, and they all enjoy mathematics (e.g., Jasmine, "Maths is cool"; Fred likes it "heaps"). The focus children identified their own ethnicities. George, a high achieving, Korean boy, age 10, Year 6 student is working at Stage 6. Jasmine is a 9-year old Pākehā, Year 5 girl in the middle group (Stage 5). Fred is also 9, in Year 5 and Pākehā, but he is in the top group and functioning at Stage 6. Sammie is a ten-year old Māori girl in the lowest group (Stage 5); however, she scored much higher than the other three students on an asTTle Geometry assessment [28] (Ministry of Education, http://e-asttle.tki.org.nz/ , http://e-asttle.tki.org.nz/About-e-asTTle/assessment_for_learning). Ron organised the groups based on the results from the Numeracy Project Assessment (NumPA) interviews (Ministry of Education, 2008a) and by " casual observation when they are working in class" (Interview).

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28 Frankalina, a classmate, remarked to me that she could not understand what Sammie was doing in the bottom group because "she is so good at maths".

Identities in Ron's mathematics class

Ron enjoys mathematics and sees himself as good at it and capable of teaching all of the primary mathematics curriculum. He says, "I have a good memory for numbers and can usually solve an equation... I've always got on well with numbers and often enjoy a difficult challenge". He has been involved with the Numeracy Project since the early days with the Count Me in Too pilot project (Thomas & Ward, 2001) which he says has helped his confidence in the subject.

George comes across as very confident in class. He looks like a poster boy for the high-achieving Asian student; however, he is a complex mixture of cheeky confidence and diffidence. He knows he is the fastest and most accurate mathematics student in Ron's class. He is always ready to challenge peers and his teacher to compete with him. He seems to need regular proof of his superior skills by constantly renegotiating his position in class by arguing with Ron and challenging him and class members to beat him at speed contests. Yet through his writing, speaking, drawing and actions, he articulates his seemingly contradictory beliefs about mathematics. On mathematics belief factors described and discussed in Chapter 4 (Table 4.2), George has very low scores, both for himself as a doer of mathematics, (*Self* belief =16), about the sort of people who are good at mathematics (*Ability* belief =12), and on the minifactor which combines an I-question with one about boys and Asian students (minifactor = 5) (Appendix O, Table O.1). A detail from his first drawing (2007) is reproduced below (Figure 6.2).

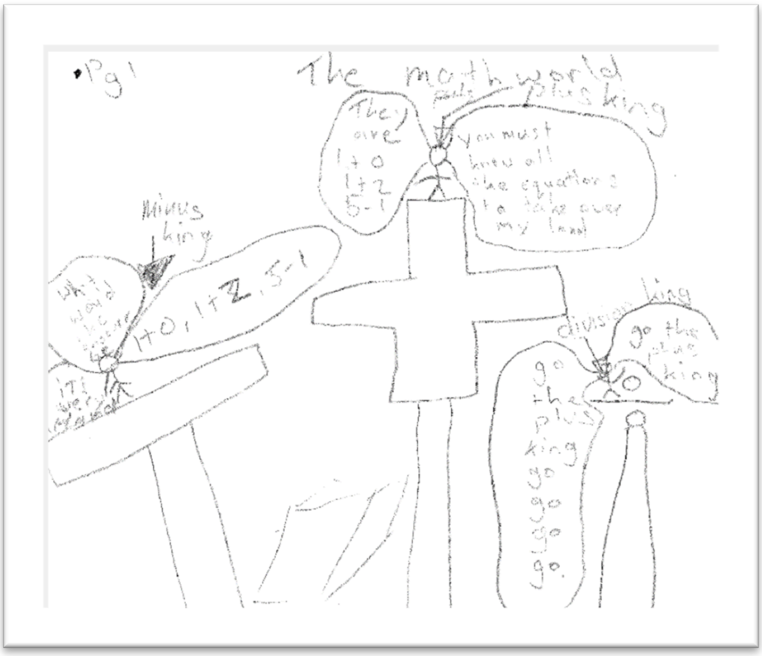

Figure 6.2: George is "math world king", detail
[Not to be reproduced without permission.]

Although he acknowledges that he is the fastest and most accurate maths doer (in his drawing, he is the "maths king"), on the MBQ[29] he disagrees that he can do maths (Q4) or that he is good at it (Q18). He also believes that his family see him as weak even though his teacher sees him as "ok" (Qs 18 and 20). His response on Q22 explains this seeming contradiction: "I'm not good at maths because I haven't learned enough." This is echoed in his response to Q32: he finds maths "easy so far but hard in the future." He recognises that he is good at Year 5/6 mathematics, but he is aware that is not the same as his parents' high expectations, nor the level of mathematics he will need at high school and university.

George also positions himself as being good at certain kinds of mathematics, such as equations, times tables, adding and subtracting, but not at others, for instance, geometry, and he doesn't like fractions although he is ok at them (Interview). He explains that he is better than some students and worse than others. Ron

29 For wording of MBQ questions see Appendix D.

identifies him as "good" and “quite an engaged student”. Paul, one of his Year 7 teachers (2008), comments on George:

> *He’s really good if you give him a textbook and say ‘start here and finish here’. He is very good at that sort of working, but he’s not good at problem-solving and self-management and organising and communicating his ideas with people... Actually that’s not mathematical concepts..., that’s social stuff.*

In this case, his teacher is positioning him as good at certain types of mathematics, at textbook work, not important mathematics, as opposed to the higher valued tasks like problem solving and communicating, behaviours associated with being a “good” student or perhaps one with availing beliefs (Muis, 2004). This interpretation of George’s mathematics identity could possibly be seen as a stereo-type of the Asian child.

Fred identifies himself as good at mathematics because “I like it in my own way” and in relation to other subjects like reading and writing: “See, I’m actually dyslexic and mild short-sighted as well.” Although, like George, his MBQ *Self* score was on the lower side (17), his behaviour in his mathematics classes and what he told me indicated that he was very confident about his mathematics ability. The following year, then in Ms. K’s class, he explains that he was an 11 years-old “working at a 13-year-old’s level”. In his drawings he depicts himself as a holder of knowledge and as someone who enjoys maths (Chapter 5, Figure 5.3). In his second drawing he is also helper, tutor, and monitor of class behavior (Figure 6.3). In the drawing below, he admonishes Keith for swearing and Jim for throwing his koala while responding to Jim’s maths question (“Is 3=2 = 5”). Ms. K sees him as good at mathematics as well as “quite bubbly, he’s got vitality and enthusiasm”.

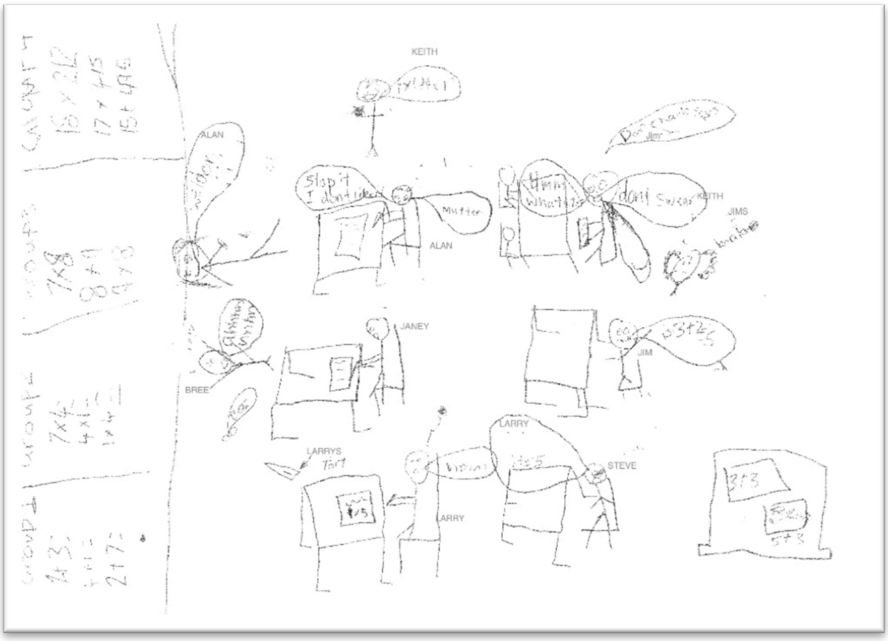

Figure 6.3: Fred's Year 6 maths class
[Not to be reproduced without permission.]

Sammie is a very confident, self-assured student who had the highest score of the Kikorangi focus children on the *Self* belief factor (23). She identifies herself as "ok" at maths because she “thinks and listens” and because “I know all the basic stuff but am still learning the harder stuff”. Vern, another mathematics teacher, positions her as competent but also “God’s gift to teachers. She’s focused, she’s on-task; she’s enthusiastic and motivated.” Like George, Sammie is looking towards the future in learning mathematics (“the harder stuff”).

Jasmine also positions herself as "ok" at times because “I like to do counting” and at others times "good" “because I learn each time”, with a *Self* score of 20. Her teachers refer to her as “a lovely girl” (Ron), “quite quick to pick up new ideas. Quite a bright kid” (Peter) without reference to how good she is at mathematics.

Even though all four of these children in Ron's class are "good" at maths, they do not position themselves as unreservedly talented; instead, they present complex and at times contradictory pictures by explaining, justifying or limiting what they mean by "good" and "ok".

Ability beliefs

Interview with Fred: "There's no magic pill that makes ya good at maths."

Identity beliefs are closely aligned with beliefs about *ability* within the domain of mathematics. In order to identify oneself as good at/not good at mathematics, one needs to have a set of beliefs associated with what it means to be, what sorts of people are, and what makes people good at mathematics.

For Ron, those who are good at mathematics possess a combination of dispositions and skills, a combination of persistence and aptitude:

> *Constant use of skills/concepts, a desire to improve their ability in this area, lots of exposure to mathematical concepts"(Q23);*
> *[t]hose that concentrate, think logically, have a good memory (Q24).*

Discourses associated with gender and ethnicity also surface in an examination of beliefs about who may/may not belong to the world of mathematics. Ron explains his beliefs on ethnicity and mathematics during an interview saying,

> *Well, our Māori and Pacific Island students at school tend to do as well or better than the European counterparts here, but I would have to say the Asian students that we do have are extremely good at computational maths, but the problem-solving or anything like that they seem to fall down at.*

Similar beliefs about Asian students and mathematics were also posited by Paul and Vern, other teachers at this school. On gender, Ron comments that "boys seem slightly better at Maths" than girls, which may explain the composition of his top maths group, the Mathematicians, a large, enthusiastic, exuberant group mostly made up of boys. I observed that one of the very high achieving girl members of the Mathematicians, Rosie Wolf, dressed like a boy and was treated like one. I also noticed that the middle group was predominately female, including many of the 'girlie-girls'[30], and that the lowest group was gender balanced. The other teachers interviewed did not admit to gender-specific *Ability* beliefs; however, the mathematics education literature includes a number of studies where teachers still tend to view mathematics as a male domain as well

30 A term not uncommon in the gender literature (Johnson, 2010; Renold & Ringrose, 2011; Riley & Scarff, 2013; Schmalz & Kerstetter, 2006), as well as in common parlance as the binary opposite of the tomboy.

as over-estimate male students' abilities, which may well affect identity beliefs of their students (Jackson et al., 2010; Mendick, 2005, 2006; Paechter, 2001, 2007; Philipp, 2007).

On the other hand, the children in Ron's class did not mention gender at all as a distinguishing feature. For them, boys and girls were equally good or bad at mathematics. However, Fred commented on ethnicity: "You don't have to be from a certain country to be good at it, you've just gotta…it's just how you wanna do it." (Interview). George talked about Korean students, rather than other ethnicities, describing how hard they worked on maths especially those back in Korea. Sammie also commented on her beliefs about Asian students: "…they work really hard so then like because they have in Korea and stuff they have classes after school that they go to." None of these children mentioned Pākehā, Māori or Pasifika students; it was almost as if they saw only two groups, Asian and non-Asian and had accepted the discourse of Asians being good at mathematics (Chisholm, 2010; Cumming, 2011; Lim, 2013).

For George, Jasmine, and Sammie, practice is what makes people good at maths. Sammie links the notion of practice to enjoyment by mentioning that "some people just really enjoy it so they wanna practise even more". George and Jasmine also mention studying at home, in George's case until very late sometimes. Vern, a teacher, takes a slightly different position to George working at home, commenting that "[h]is mother tutors him *intensively* every day – to the point where we had to complain to her about how tired he was at school". Otherwise, all four children refer to behaviours like "concentration"(Sammie), "[i]f they listen…and who concentrate on their maths more than talking" (Jasmine), and "[j]ust how hard you try" (Fred). Fred also referred to his Mum and teacher helping him be good at maths.

Nature of mathematics

For Ron and these four students in his mathematics class, the usefulness of mathematics is an important element in their conceptions of the *nature of mathematics* as are beliefs about maths as number.

Ron: Maths is part of our daily lives
We use it to measure – Quantities, Time, length
We use it to solve daily problems.
Eg how many? How much? Where? When?
It's about Numbers, groups, shapes, sizes (Q31)

Sammie, Fred and Jasmine include number and basic operations. For Jasmine, "Maths is all different strategies, like fractions and times tables and pluses. Some are easy and some aren't". All three include some elements of utility. Fred, for example, suggests mathematics is for "mony, buying", while Sammie and Jasmine represent measurement in their drawings.

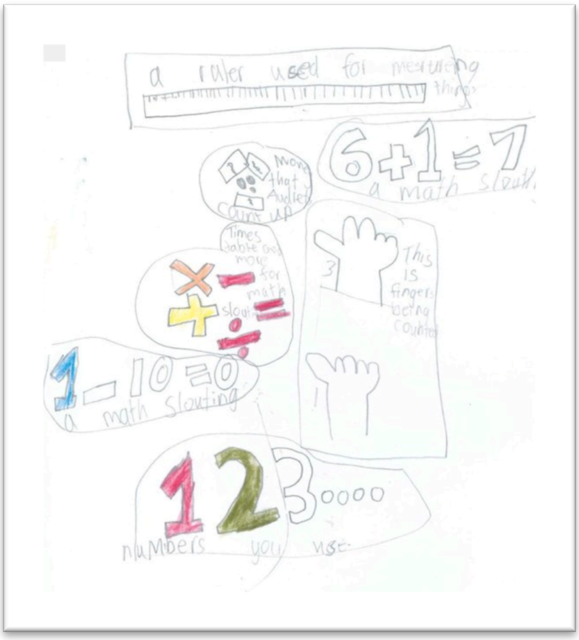

Figure 6.4: Jasmine's maths 'tools'
[Not to be reproduced without permission.]

Jasmine also includes notions of maths 'tools', or things that are useful for doing mathematics, such as counters, rulers and fingers (Figure 6.4). Like Sammie, who refers to brains and thinking, she includes "using your brain to find answers", and Fred mentions "lernin". Sammie, like George, views mathematics as important for the future, for getting a job. As George explains in an interview,

It's not just numbers and words, it's actually preparing for the future, 'cos there's a lot more stuff than I'm doing right now that doesn't include maths, but once I get on, like accountant and all that, all the job, all the *degrees that you'll get will be needing maths, and problem solving and all that...actually preparing for your future.*

George looks not only at the short-term future but towards studying for degrees and professions like being an accountant.

Teaching/learning/doing mathematics

Ron and the four students were enthusiastic and engaged in the doing of mathematics although the two boys, George and Fred, seemed more exuberant than the girls during the lessons I observed. George was busy challenging peers and the teacher during the process of retaining his supremacy in mathematics. Both he and Fred answered questions, worked problems, argued and discussed solutions, with Fred avoiding writing where possible. He preferred doing things in his head. Sammie chose to "get on with" all set and self-selected mathematics activities in a focused way, an example of the well-behaved student doing maths. In Figure 6.5, she depicts herself working on a maths objective of her own choosing, something she enjoys as indicated by the hearts.

Figure 6.5: Sammie cheerfully doing fractions (Drawing 2, 2007)
[Not to be reproduced without permission.]

In contrast, Jasmine, who answered and discussed problems enthusiastically at mat sessions even though Ron failed to acknowledge or notice her contribution, often chatted to friends or flirted with some of the boys at group or free/game activities, a position that contradicts her emphasis on needing to "listen and concentrate" (based on my observations and video recordings). She knows how to do the good, appropriately behaving mathematics student when the teacher is in close proximity or listening. For Sammie and Jasmine doing maths is not just problem-solving or playing games with number(s) but listening, concentrating, using your brain, behaviours or skills that are important for doing school/being a successful student across all subjects, not just mathematics.

George's mathematics world, peopled by competitors, is a place where those who practise get ahead. Fred's, on the other hand, includes stressed teachers, children in tears, confused, bored, behaving badly as well as those "getting on with it", or, in his case, solving problems successfully (video observations and drawings). As an explanation of badly behaving children, Fred gives the example Ms. K's class, (Figure 6.3): "Our teacher doesn't have much control over the class this year, you know." His beliefs about doing and learning mathematicss are coloured in part by these experiences. The boys included images of and symbols associated with numbers as the stuff of maths; however, like the girls, they believe in "maths as useful" similar to their teacher Ron's beliefs about the mathematics.

Even though the children espouse beliefs about problems having multiple ways of reaching solutions, and three of them accept that problems may have more than one solution, they (except for George) defer to the teacher's strategy and solution as the correct one. There may be many ways of solving the problem at hand, but Ron retains the power of deciding which method and solution are acceptable. However, I noticed some contradiction between Ron's espoused beliefs, and George's experience in his classroom. In an interview, Ron claims:

> *The most important thing is to know what type of thing or mathematical equation you need to apply for the certain thing. So if you're faced with a problem: I've got 14 people who need pairs of shoes – how many shoes do I need to get for them – that they know that's multiplication. So being able to work out, what is the problem actually asking me to do. What do I need to do to solve it?*

But solving problems in terms of the Numeracy Project is more complicated than Ron's view for there are a variety of strategies that could be utilised to solve this problem. Ron asked the Mathematicians, the top group, to solve a series of multiplication problems by using the place value strategy. George used a standard multiplication algorithm very quickly and accurately. Ron asked him to “put it another way”. As George explained to me a year later, “I hate place values….'cos I *do* mostly do it the easy way, 'cos that’s what I’ve learnt from my Mum, 'cos I don’t wanna waste too much time”. For George, who knows he is quick and accurate, why wouldn’t he use “the easy way”? His solution to this conundrum was to do the problems using an algorithm then go back if he had to, using the “correct” strategy, the slow way.

> *10 x 1209 = 12090*
> *rather than <10 x 1000 = 10000> + < 10 x 200 = 2000> + < 10 x 00 = 000> + <10 x 9 = 90> = 12090 (from George’s exercise book)*

Ron responded to George’s use of the multiplication algorithm with “and ultimately that’s what they’ll do. They’ll see all these strategies and they’ll choose which one they think works best for them and which fits the question.”
For George, the teacher is not necessarily the authority in mathematics, although he is astute enough to realise he needs to defer to school authority much of the time. He explained his ways of being a good mathematics student:

> *But mostly I write down what the teacher says and do the equation what he says, and then if he get it wrong, then, I disagree. … It’s like that race that I had [laughs] with Ron last [year].*

For him, doing mathematics at home is as important as at school. At home he has to negotiate another mathematics authority, his mother, who insists he does extra work

> *..all the stuff I’ve done so far is…just easy, cos…this year I’ve finished two study guides of Year – oh three, of Year 7, 8 and 9, and yeah, I understand them all, it’s just, I’m weak on some of them and, I’ve got more strength in the others.*

The upshot of his extra work was that he was accepted into high school a year early and placed in one of the top mathematics classes, so in this case his mother's insistence on his extra practice and accurate application of algorithms helped him achieve this goal.

Charles Forrest’s classroom

> *“I absolutely love fractions.” (Remark made at the beginning of a mathematics class with everyone at the mat)*
>
> *“Lemon, that’s absolutely marvelous.” (Mr. Forrest comments on a student’s response)*

Mr. Forrest, in his third year of teaching, is a very high-energy teacher whose classroom is bright and cheerful, full of student work and displays, and appears very well run. His enthusiasm is infectious. There is a light-hearted joking atmosphere with busy-looking, engaged children; however, perhaps I was observing these middleclass children with cultural capital (Bourdieu, 1986)[31] who know how 'to do' school, how to be the well-behaved, enthusiastic, engaged schoolchild rather than actually engaged in learning. His classroom has explicit rules covering behaviour and work with consequences for everything.

Whero is a decile 10 school that is bursting at the seams. Parents try hard to have their children accepted, even going as far as moving house, or using relatives' and friends' addresses. The children mostly come across as confident, cheerful and busy, and they seem to understand the rules. They all wear school uniform and address the teachers and other personnel by title. Children wander around the school at all times, going to pull-out sessions such as music, sport, drama, student council and other pursuits. The children even use the staffroom for activities when needed.

Whero has a school-wide homework policy that the parents demanded. For mathematics homework, they use a commercially published workbook, which does not necessarily match the tasks/activities or emphases of the individual mathematics teachers. The children in Mr. Forrest's class know that if they do not turn in their homework on Monday, barring certain acceptable excuses, they have to stay in during morning interval to complete the work. The same applies if Mr. Forrest thinks they have not been concentrating enough during an activity, or have failed to complete the expected amount of work during class time. I

31 Bourdieu explains cultural capital as a hypothesis that made "it possible to explain the unequal scholastic achievement of children originating from the different social classes by relating academic success, i.e., the specific profits which children from the different classes and class fractions can obtain in the academic market, to the distribution of cultural capital between the classes and class factions" (Bourdieu, 1986, p. 243). In other words, rather than link achievement to ability, achievement is linked to cultural resources (language, ways of talking, ways of behaving and engaging with doing school) inherited from family and social class.

found students, for instance Harry, deciding to finish homework during morning break even though Mr. Forrest had not insisted they do so.

Mr. Forrest's interchange mathematics class (Room 6) is one of two mid-level classes. The school has six interchange Year 5/6 classes. All of them, except the one for the lowest achieving students, have at least 35 class members. The accelerated class also includes a few Year 4 class members. Room 6 mathematics class starts at 9.30a.m. each day and runs until morning interval (10.30). Lessons usually begin on the mat with Mr. Forrest on a chair near a small whiteboard, while most of the children are crowded on the carpeted floor, with a few on chairs at the back. Everyone does the initial activity, usually a maths game that is related to the emphasis of the day. He then discusses the topic, strategy or method that will be the focus of the lesson.

The three maths groups, Graphs, Scales and Clocks, are roughly divided by achievement levels; however, there is a lot of movement between groups, depending on the activity. There is also movement between interchange classes; the children can move up and down classes depending on how they are coping and whether there is space for them. These decisions are made by the mathematics teacher with the head of mathematics.

Groups are assigned activities. Students work together or alone depending on teacher instructions and/or preference. One group usually remains on the mat with Mr. Forrest while the others disperse. After beginning the activity or discussion with the mat group, Mr. Forrest wanders around the class checking on individuals and groups. Some students put up their hands if they require help; others come to find the teacher. Once the activity with the first group is over, a second group moves to the mat, and a different but related activity or discussion tailored to the achievement level of the group begins. Mr. Forrest's organisation of groups and delivering the curriculum to the groups follows the same convention or choreography as Ron's class, which appears to be one suggested by the Numeracy Projects: three groups, two of which received direct instruction from the teacher each day, three different group activities each day, a rotation of mat, etc., activities that translate into each group receiving direct attention as a group at least every other day (Ministry of Education, 2008c)[32].

This classroom is a very busy place with a lot of noise, movement and hard work. Numerous people, students, teachers, other staff, parents and visitors walk in and out of the room at all times which sometimes, but not always, interferes with the concentration and flow of the lesson. The class almost always ends with a game. Finally, Mr. Forrest checks and comments on each child's work as they leave for morning interval[33]; for example,

> *"Do you know where you went wrong with these ones?" (Ellie's exercise book), "Impressive Richie!", and "What's happened Matthew! We'll go over this one again", etc. (Quotations from exercise books).*

Any students having trouble with an activity or problem may go to the mat where Mr. Forrest gives them extra help. At times, he identifies struggling students as he walks around and suggests they come for extra help; other students decide to seek help or clarification of their own accord, another example of students taking responsibility. I observed students from all three groups choosing to go to the mat; no stigma seemed to be attached to seeking help.

An introduction to five members of Mr. Forrest's mathematics classroom

The class was identified as one for mid-achieving students, but in reality it included children from a range of levels. Caroline is a chatty 10 year old student from Malaysia; the other four are 9 year old Pākehā. Harry, an enthusiastic, energetic boy with a naughty twinkle, who likes "practically everything" more than "maths" (Interview), is in the low group with Caroline. Jack, a quiet but bouncy boy, so quiet that his voice does not register on the video recordings, likes maths and is in the middle group. Chloë and Lilly, who are both very articulate and excel in reading and writing, are in the top group. In terms of NumPA Stages, Chloë, Jack and Harry are achieving around 5, Lilly at 6. There

32 An example of this choreography is presented in *Getting started* (Ministry of Education, 2008c, p. 13).

33 He did this checking at every lesson I observed (more than 3 weeks of lessons).

were no data for Caroline, who joined the school after the initial assessments had taken place. However, because of the loose structure of the groups in Mr. Forrest's classroom, these students moved from group to group depending on the activity.

Identities in Charles Forrest's mathematics class

On the MBQs, Mr. Forrest, Caroline, Chloë, Lilly and Jack all identify themselves as competent doers of maths with scores on the *Self* factor of between 19 and 22 (See Appendix O). The anomaly is Harry, who believes he is very bad at maths (*Self* belief score of 9). Although these scores seem to indicate that these individuals see themselves as either good and competent at the subject, or bad in Harry's case, an exploration of how they answer the open questions as well as their interview responses shows a much more complex picture of these beliefs.

Mr. Forrest is confident about his mathematics ability and his identity as a doer and teacher of mathematics. As he explains in an interview, "I *love* the subject, I found it... I love the subject, and did pretty good at it." Unlike many of the other teachers who answered the questionnaires, he enjoyed mathematics all the way through his schooling, even through secondary school, which he views as the result of "several teachers who inspired me when I was at school" (Interview). Most of the children (92%) in his classroom view him as "good" or "excellent" at maths.

Harry hates maths, finds it boring and sees himself as very bad at the subject. He identifies himself as a student who is not comfortable in nor belongs to the world of mathematics, commenting that "I just don't think I'm good at maths", and "I always don't understand" to which he adds that he likes "practically everything" more than maths "and mostly PE, 'cos I"m kind of fit and good at it. And reading, I'm good at reading and writing" (interview). Mr. Forrest sees him as a spunky, "interesting kid.... one of those kids who you really like teaching". Harry is a lively little boy, a risk taker who will break school rules, like riding his bike or skateboard in the playground (Jack's comments), not doing his homework and cheerfully accepting the consequences (based on observation and casual conversation about staying in during Interval even before the teacher insisted he do so). He identifies himself and is identified by others as one of the naughty boys (See drawings in Chapter 5, Figures 5.4, 5.35 and 5.46). Figure 6.6 includes a detail where he draws himself playfighting with a friend during maths class as well as a detail from Orange's drawing with Harry as one of the naughty group.

Figure 6.6: Harry as a naughty boy
[Not to be reproduced without permission.]

Mr. Forrest also commented that although Harry is bright, he seemed to struggle academically. "He'd do anything for sport whatsoever, very much ... and, sort of, his studies came second." In fact Harry enjoyed maths that involved sport, such as activities involving the Rugby World Cup and other class maths games. He also admits he is good at "algorithm's" and "adition" but bad at "Time's table's" and "division". Mrs. Umbridge, his Year 6 teacher views him as a student with "areas of weakness and he has his attitude with him." She also comments that "[h]e does like his old algorithms, old Harry does." Even though Harry and his teachers position him as a student who is not good at/is weak at mathematics, he is achieving in the average band for his age and year (Stages 5 and PATm 7 for Year 5). As Mrs. Umbridge comments, he is "sitting just below average" in the middle mathematics class. In Harry's case, his perceived mathematics identity is at odds with the reality of his achievement. He and his teachers believe he is not good at mathematics even though his performance is average/around the mean, which could have negative consequences for him later in his school mathematics career (Gates, 2001).

Caroline was placed in Room 6 mathematics class because there were no spaces in the lower class. She arrived with no accompanying achievement data; as a result, Mr. Forrest was not "sure of her background" but had observed that she struggled in his class. Her responses on her questionnaire and in her interview were contradictory. In some places she identifies herself as “good” at maths and enjoying it, in others she is "ok" "because I don't know much". At times during mathematics, she failed to realise that she did not understand how to do the tasks or activities set by the teacher[34].

Figure 6.7: Caroline with hearts
[Not to be reproduced without permission.]

34 This did not seem to be a language problem; even though she identified he ethnicity as Malaysian, she spoke and wrote good English.

Perhaps she felt she should like mathematics and be confident about her ability as she explained in an interview when we were discussing her drawing of herself surrounded by "love hearts" (Figure 6.7): "it's you're supposed to like maths so you don't get into trouble..." She described herself in the drawing as "happy, bright and confident" yet said, "I was feeling excited and nervous at the same time" (Interview). The following year she was placed in the lowest mathematics class with Mr. Lupin who calls it “the remedial class”, where she flourished; as he explained, "She is my most able one in my class", "the strongest peer tutor" and the student "making the most progress". Over the year, her mathematics identity changed from being one of the weakest students in Ron's classroom to the strongest in Mr. Lupin's.

Jack identifies himself as a confident mathematics student who is good at "alaorithms, games and problems" and bad at "nothing" (MBQ), and it is "probably my favourite subject apart from sport" (Interview). Like the other five focus students and Mr. Forrest, his responses are more complex than always identifying himself as one thing: he gives himself a 5, the maximum score on being able to do maths, a 4 on being good at maths (Q18: “How good do you think you are at maths?”), and only an "ok" on Q22 (“I am ok at maths because ………..”). In one place on the questionnaire, he responds that he is "ok because I try" suggesting that dispositions or attitudes to maths are as important as being good at the subject. In Year 6, he still sees himself as pretty good at maths but tempers this with "I'm a bit above average" in the second from the top class (of 7 classes). Mr. Forrest positions Jack as "really quite good" but not good at writing about what he is doing, and not necessarily coming up with the “correct” answer although "he had the ability to explain his thinking fairly clearly" (Interview). His two Year 6 teachers differ slightly in the way they identify Jack as a maths student. Mrs. Hill sees him as "slower to pick up concepts, but once he's got them he's fine, methodical". For Mrs. Dale, "He's delightful. ...I find that he's got a very mathematical mind, he's logical..." (Interview). He is identified by one teacher as “slower” and “methodical” while the other believes he has “a mathematical mind” and is “logical”, descriptors containing rather different connotations: the ok, not too swift plodder in contrast to the logical maths thinker.

Chloë presents a picture of a girl who is good at maths turning away from the study of mathematics over the two years(Mendick, 2005, 2006). In Year 5, Chloë has the highest score of all the focus students and teachers on the *Self* factor (22), thus identifying herself as good at maths, particularly at "probes, multiplication and addition" (MBQ). Like Jack, she labels herself "ok" on Q22, in this case "because I try hard to learn".

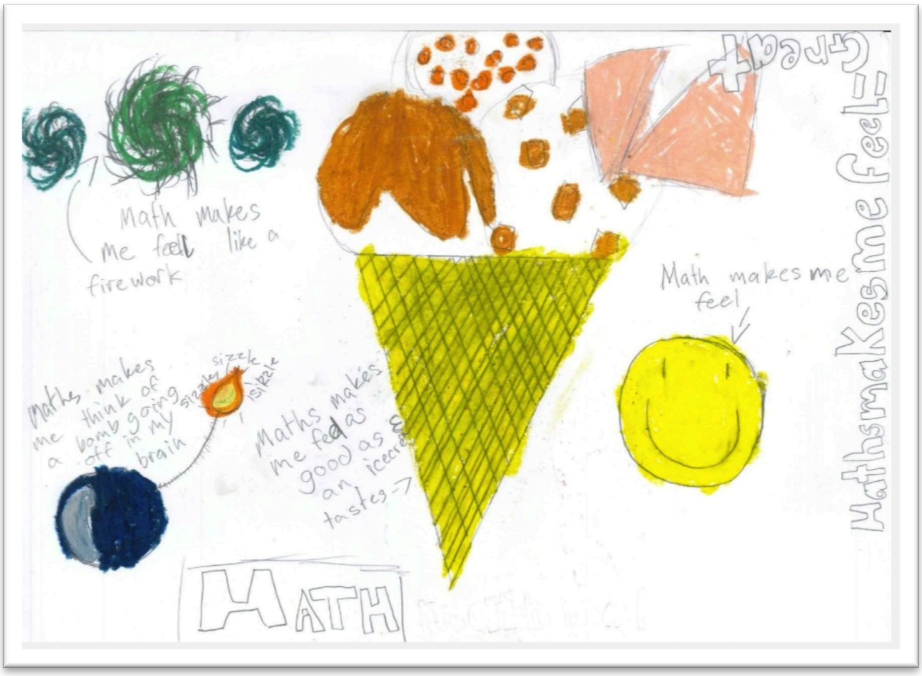

Figure 6.8: Chloë in Year 5
[Not to be reproduced without permission.]

Her Year 5 drawing is full of positive comments about the way maths makes her feel like a firework, like icecream, like a bomb fizzling and about to explode, a picture of someone who enjoys mathematics (Figure 6.8). Mr. Forrest sees Chloë as "very, very capable.... good work habits". By the following year she finds maths boring, "not fun", commenting that she is "[p]retty good. Just not quite as competent as I'd like to be..." (Interview). Her Year 6 drawing (Chapter 5, Figure 5.36) includes her screaming as she rails against the notion of having to get things right and being tested, having to complete a “giant nine-paged book” (Detail in Figure 6.9 below).

Figure 6.9: Chloë in Year 6, detail
[Not to be reproduced without permission.]

When discussing being placed in a top mathematics class for Year 6, she claims, "You're not necessarily good to get in that class, just what happens in a test". She seems to realise that successful test taking is not necessarily an indication of being good at maths. She also likes reading, writing, sport and drama more than maths. Despite Chloë's ambivalence about mathematics and her ability, Mrs. Hill, one of her Year 6 teachers, positions her as "an able mathematician, she picks up concepts quickly" and is part of the gifted programme.

Lilly sits comfortably in the top group in Room 6 mathematics and is confident about her ability to do maths. However, she does not position herself as someone unambivalently good at mathematics despite Mr. Forrest's comments on her work, "Awesome, yet again". When asked to explain her success in his class, she comments:

> *Lilly: Well, I got most things right and got the hang of things quite quickly."*
> *Cathy: And so you find it quite easy?*

Lilly: Not really, but I seem to get it right [laughs] somehow. Probably by accident.

At other times, she writes about being ok at maths "because I enjoy doing it" (Year 5) and "because I use logic" (Year 6), in neither case identifying herself as a good mathematician. In Year 6, she was placed in the accelerated class having been identified by others as extremely capable, if not gifted mathematically; yet although she acknowledges that she is "probably quite good because I'm in Mrs. McGonagall's class", she goes on to explain that "I'm doing quite well because Mrs. McGonagall is a really good teacher". Lilly describes mathematics as "not something I'm wildly excited about, but it's not horrible". She says, "Maths. Quite enjoyable but there are just more enjoyable things in life." She identifies herself as a non-maths person:

Maths isn't really my strongest point. I like doing writing and reading. I enjoy them more because maths gets a bit boring....

Lilly is an example of a student who is very good at mathematics, explaining her success in terms of good teaching, luck and enjoyment rather than her ability or effort. Lilly, like Chloë, is a very bright girl turning away from mathematics.

Ability beliefs

For Mr. Forrest, the students who are good at mathematics are "[o]nes who want to learn, have intrinsic motivation" (Interview) combined with good number knowledge and support from their parents (Interview). The notion of family support for successful mathematics students is also mentioned by Caroline, Chloë, Lilly as well as Mr. Lupin (Caroline's Year 6 teacher) and Mrs. Hill. Chloë includes "the way you are taught" (MBQ) as an explanation for being good at maths, which is echoed by Lilly in her comments that her good teacher contributed to her mathematics prowess (see previous section). Attitudes or character traits as explanations for successful students are mentioned by Caroline who claims "believing in yourself" is important as is "practising at home every day", "listening well and learning their timestables" (Interview). Harry also mentions practising. Chloë speaks about trying hard, but also about "confident people, who actually think they're good at maths". Harry explains that "some people like it" (Interview). Lilly expands this idea by linking liking maths with success, "I think if you enjoy doing maths, you are often very good at it."

A genetic explanation for mathematics *ability* was suggested by Harry and Jack, who refer to brains, and for Harry "smart people" who say "smart stuff". Both Chloë and Mrs. Hill, her Year 6 teacher, mention being "born good at maths" which the latter tempers with "but practice, absolute practice can make you better at maths" (Interview). Mrs. Dale (her other Year 6 teacher) mentions the notion of the "mathematical mind". Mrs. Umbridge refers to maths ability running in families that may be read as a genetic reason but could also refer to family support and attitude. In addition, for her, successful students "are analytical, who can think through a process" (Interview).

Gender was not specifically identified as a contributing factor to mathematics ability and achievement apart from Harry (MBQ), who based on his experiences of doing /learning maths, identifies boys as bad and girls as good at maths, and Jack who views boys as slightly better than girls. On the other hand, most of this group of students and teachers mention Asian students as better but not any other ethnicity.

> *Mr. Forrest: The majority of the Asian students we have here at school are exceptionally good at maths" identifying them as coming from Korea, China and Japan, which he explains in terms of different education systems and being "pushed ... from an early age (Interview).*

He tempers this by explaining that they are less successful with the Numeracy Projects,

> *...the Korean and the Asian students we get...find it really quite bizarre why they need to ...work out something a different way. It's something that's quite hard dealing with for those students (Interview).*

Caroline, a weak student, sees Asian students as weak and Pākehā strong; her beliefs are probably based on her experiences as an Asian child doing poorly at mathematics. Jack identifies Korean students as good at basic facts. Chloë: "I've seen *so* many people that are really good at math and for some reason they come from that part of the world. And I'm not trying to be racist" (Interview). Lilly:

> *"They might have been taught maths by their parents as immigrant people[?] but just because you're Chinese or Australian doesn't mean*

> *you're not any better or worse than you. People think Asian people are good at maths ... it's not really because of where you come from; it's because of how you think about things" (Interview).*

Mrs. Umbridge's comment on Asian students is similar to both Mr. Forrest's and Lilly's:

> *We probably tend to think that Asian students... are perhaps -- we see often them as being good at maths, yet I think a lot of it is because of the extra tuition and the repetition and the rote learning and those sorts of things. They tend not to be as dominant, or don't tend to shine as much in the Numeracy Project because they are unwilling to verbalise what they do (Interview).*

Although most of these comments position Asian students as very successful doers of mathematics, it is at certain kinds of mathematics and as a result of education, attitude and practice rather than a genetic predisposition.

Some of the focus children spoke about how one identifies those who are good at maths, like Harry's and Chloë's "smart people" or Jack's reference to "inside kids" [children who like to be indoors] who enjoy it. He also mentions the way you can tell is by which group or class they are in: in other words, being labeled by placement. For Lilly it's

> *the person who puts their hand up when they're asked questions and they get it right. They're the one who's always telling everyone that they're wrong (Interview).*

All of these comments by the focus children refer either to observable behaviour or how children are identified and labeled as the result of testing.

The children did not refer to those not good at maths; however, some of the teachers articulated their beliefs in terms of a variety of discourse. Mrs. Umbridge's explanation included attitude, for example, "I can't do it. I give up" as well as not able to verbalise more than one strategy/ways of solving a problem. Mr. Forrest sees those who struggle as perhaps struggling with the Numeracy Project:

> *Could be overloading with ways to work out certain problems... They find it too complicated. ... I assume from what I've seen, it's just some of these, some ...number of strategies are great, but other kids just find it*

> *far too difficult to see why they need three or four different ways to work out one problem when they can just do it one sure-fire way (Interview).*

For Mr. Lupin, children from lower socio-economic backgrounds and those with special leaning needs or who struggle with reading and language are inclined to be weak at mathematics. These teachers suggest student attributes such as SES, background, attitudes, and weak literacy skills as elements that contribute to poor mathematics skills. Mr. Forrest also views the NDP ways of doing maths as disadvantageous for some students while Mrs. Umbridge suggests almost the opposite, viewing those who cannot or will not follow the NDP ways of using strategies rather than standard algorithms as weak at mathematics.

The reasons for being good at mathematics as presented by this group of children and their teachers include attitude to the subject, as well as attitude to learning in general, such as believing you can do it and practising. They also refer to analytical minds, brains. Parents and teachers/teaching are also mentioned. On the whole, Asian children are viewed as good at maths, in particular at basic facts but less good at coping with the uncertainty of multiple ways of doing things/explaining things inherent in the Numeracy Projects. For Mr. Forrest, weaker students also struggle with multiple strategies.

Nature of mathematics

The beliefs about the *nature of mathematics* that these children and their teachers articulate are inextricably connected to and affected by their beliefs and experiences of themselves as doers of mathematics (De Corte et al., 2010; Franke et al., 2007). Like their counterparts at Kikorangi, they view mathematics in terms of number and basic operations. Although none of them, apart from Jack who writes about the importance of "counting money", includes notions of utility in their questionnaire or drawing responses, some refer to the usefulness of maths in the interviews. Mr. Forrest writes of "problems/solutions with numbers" and the importance of "different strategies and algorithms". Caroline also believes maths is about problem solving to which she adds beliefs about the relationship between maths and the brain:

Maths is like plussing and dividing and trying to find the answer to it" (Questionnaire)
It's kind of like training your brain to think, yeah, so you don't get like so lazy you don't do anythink (Interview).

Harry sees counting as important, Jack basic facts because "you don't really need decimals everyday life, do you?", Chloë basic operations and Lilly "place value, measurement/volume/weight".

Mr. Forrest does not limit mathematics to number and the classroom:

Numbers, people… it's every part of our lives really. Maths I think it's really important, it's all around us, going to the supermarket, or at home, cooking dinner, or looking at your watch -- it's everywhere (Interview).

Chloë also goes beyond the classroom saying,

You use maths everywhere in everyday life. Sometimes its easy and sometimes it is complicated. Overall math makes everything easier (MBQ).

In addition, she believes that maths is

a way of making everything fair. Like fractions – there's one cake, four people, you'd have to make each piece a quarter of the cake to make it fair, so that everyone gets the same amount.

Lilly, unlike the other focus children, is prepared to question the usually accepted notion of mathematics as a closed, finite universe governed by rules that are always so (Franke et al., 2007; Lampert, 1990). An example of this is her conversation with Morpeus, another student in Room 6:

Lilly: Why is 10 after 9 and not 8? Why is 9 = 9 and not 18? Why is 2+2 = 4 and not 22?
Morpeus: 1 2 3 4 5 6 7 8 9 10. It just is.
Lilly: But why?
Morpeus: I'm not Einstein

She is prepared to question convention and have fun with the commonly acknowledged truth. Morpeus will not even discuss her questions because the number system "just is", and he is not the brilliant authority, Einstein, to whom he defers. As I listened to this exchange, I wondered how many primary mathematics teachers would be aware of mathematical conversations like this, and if they were, would they be prepared to engage with Lilly's questions that imply a philosophical challenge to the accepted notion of what mathematics is about.

Figure 6.10: Lilly's beliefs about maths
[Not to be reproduced without permission.]

[Transcript: Maths. Quite enjoyable but there are just more enjoyable things in life. Homework is a stressful procedure. Slightly scary. I'm not sure whether "slightly scary" belongs to homework or to maths.*]*

After completing this drawing (Figure 6.10), Lilly explained it to me: "The numbers all seemed to swirl around in my head until I tell them to stop".

The Whero children not only include number but also affective elements in their articulations of their beliefs about the *nature of mathematics*. In Lilly's swirling drawing, she illustrates maths homework as stressful and maths/homework as "slightly scary", and she writes that "algorithms are stressful but fun", a notion included in her MBQ. Jack also includes fun, and Chloë's Year 5 drawing includes how positive maths makes one feel. On the other hand, Harry describes maths as boring, "HARD AS!!!", "… very annoying and inpossible". An 'inpossible' problem is illustrated in Figure 6.11 below. In his Year 5 drawing (Chapter 5, Figure 5.46), he depicts stressed children with maths-induced brainburn.

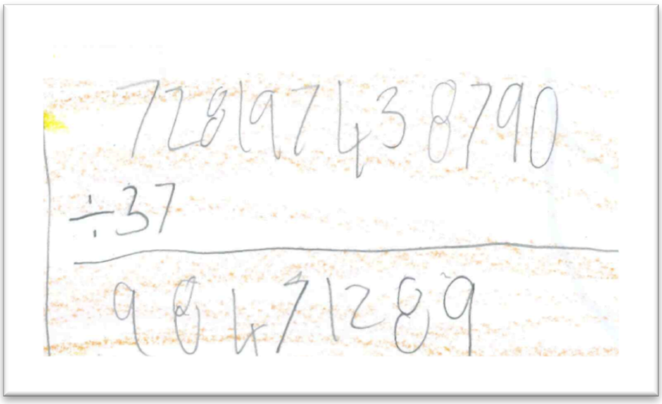

Figure 6.11: Harry's maths problem, detail
[Not to be reproduced without permission.]

Teaching/learning/doing matematichs

Room 6 mathematics has ways of doing, teaching, learning that seem to be created, negotiated and accepted by both Mr. Forrest and the members of the class (Cobb et al., 1992a; De Corte et al., 2010; Franke et al., 2007; Lampert, 1990; Yackel & Cobb, 1996). These norms, however, are interpreted differently by different class members. On the whole, Mr. Forrest sets the activities, the pace, the rules of behaviour such as the rotation of the groups, the expectations of completing sufficient work, underlining and dating written work, etc. He marks the tests, homework and checks class work, thus is the decider of acceptable mathematics performance and answers. Jack on looking back at his experiences in Room 6 comments, “You realise how strict sort of Mr. Forrest is, but then he was a really fun teacher.” However, Mr. Forrest’s behaviour and ways of being a mathematics teacher are not always read in the way he expects or intends them to be. For instance, Luke explains the reason he believes Mr. Forrest is not all that good at maths:

> *When we ask him a difficult problem, he does it very, very slowly on the board… like we would do it… so he can’t be that good.*

For Luke, explaining things slowly on the board is an indication of a lack of ability rather than a teacher trying to reach the achievement levels of his students by using words and concepts the way the children would.

Caroline speaks about her experience of Mr. Forrest breaking the rules or what she views as the accepted behaviour in the mathematics classroom when he pounces on her, a non-volunteer, and asks a question:

> *I was like, I would be like … astonished, that like I was surprised that I got picked and I didn't put my hand up. Well it's—do you get the feeling when someone asks you a question that you don't know.*

Chloë gets away with being a chatterbox; she is allowed to talk to her friends during mathematics classes when some of the other children are reprimanded for the same behaviour because "I do my work *and* talk. Because I'm a girl, I can multi-task." She uses a gender discourse to explain her special status; however, an alternative explanation could be her membership as a clever student in the high group or her social standing within the class.

The social status of the various members of groups within the mathematics class also had an effect on which of the children were listened to and learnt from (Franke et al., 2007; Nuthall, 2007). Caroline and her friend Annabel, both of whom struggle with double-digit multiplication, were working at a table with Jack, the lone boy, plus Chloë and her friend Bertha. They were all cheerfully sharing resources, books, rulers, erasers, and pens. Chloë and Bertha were comparing answers and methods while chatting. Caroline and Annabel were checking answers with each other. Caroline asked for the answer to 7 x 8. Jack gave her the correct answer which she disregarded. The high status, 'good at maths' girls, Chloë and Bertha ignored her repeated question. Finally, Caroline turned around to the table behind her and asked Ellie, another student in the top group, who gave her the answer. Caroline failed to listen to Jack who not only knew the answer to her timestable question but had a good grasp of how to do the problems. In this case, she chose not to learn from her more experienced peer perhaps because of his gender or perhaps he held lower status in the class, either as a result of his membership in the middle group or because of some lack of social standing. Perhaps Caroline deferred to the high status girls in the group in order to identify with the children with more social clout, the 'in', bright girls who in this case ignored her until she went outside the group (Nuthall, 2007).

This is an example of social interactions and barriers, often unobserved by the teacher, that affect the doing and learning of mathematics.

Mr. Forrest, Harry, Jack, Chloë and Lilly share the belief that there are multiple ways of solving mathematics problems (Ministry of Education, 2008a) but have different ways of acknowledging correct solutions. For some children, it may be the teacher's answers, answers on the board or at the back of the book, but others have gone beyond just relying on the teacher and/or textbook as the authority for the correct solution. Jack and Chloë use calculators to check their work. Lilly finds errors in the printed answers at the back of the textbooks and is prepared to challenge this so-called authority. Harry explains his strategy:

> *I write my times tables in the back of my book, and then I work to try and work out the answer, and then I check it with the back of my book (Interview).*

They talk of answers looking right and of understanding how to do things. Both Chloë and Jack talk about estimating as she explains,

> *Well you estimate it, and you then get a reasonably same answer once you've done the question – you sort of know you're on the right track.*

These children also spoke and wrote about coping strategies they used when coming across difficult maths problems. For Jack, it is using an "alaorithm", Lilly tries different maths strategies, Harry asks for help, Caroline talks of trying "my best", and Chloë of doing "my best to figure out the question". These behaviours match the identity beliefs these students espouse: the confident Jack, Lilly and Chloë rely on themselves to solve the problem; Caroline relies on the notion of trying hard; and Harry, who lacks confidence in his ability, gets help from others.

Despite Mr. Forrest's beliefs to the contrary, three of these focus children, along with 57% of the rest of the class, believe that mathematics problems may have more than one solution. From the activities and discussions, the ways of doing mathematics in Room 6, I observed examples of how the children may have reached this conclusion. From their experiences of doing mathematics at school, they saw how certain solutions could be presented differently such as 0.25, 25%

and 1/4. They were also required to estimate answers to problems and then work them out 'properly' thus getting two different solutions:

10) Est: 18000
6027
x 3
18081 (Chloë's maths exercise book)

In discussions, they heard different children describe solving problems in different ways and coming up with different solutions. The teacher would acknowledge each response in a positive manner as he was eliciting ideas. The children may have read and understood these teacher responses as the acceptance of a variety of 'right' answers. As I revisited this notion of multiple solutions to a mathematics problem, I thought of Bill Barton's cautionary fraction story, "A Story of Four Parts" in *The language of mathematics* where the children present three very convincing non-traditional solutions to a problem and one unconvincing but 'correct' one (Barton, 2009).

Concluding comments

Multiple portraits emerged from examining the beliefs about *identity*, *ability* and the *nature of mathematics* of these nine focus children and their teachers as they wrote, spoke, drew and did mathematics. This closer examination of mathematics beliefs, this narrowing of my focus, afforded me the opportunity to focus on a variety of expressions of these beliefs by a small number of individuals (McDonough, 2002, 2004). What the children believe about the *nature of mathematics* was coloured not only by their experiences of doing school mathematics but also by the classroom norms and cultural beliefs about the nature of mathematics as well as whose responses, questions and solutions were accepted in their classrooms (De Corte et al., 2010; Franke et al., 2007; Lampert, 1990).

The children's and their teachers' experiences of doing, learning and teaching school mathematics was directly influenced by the Numeracy Projects (NDP)(Ministry of Education, 2010c), the de facto mathematics curriculum, in that all of these children view mathematics in terms of number and accept that there are multiple strategies or ways of solving problems. The Number Framework, which underpins the project and how it is articulated in the classroom, differentiates between *strategy*, "the mental processes students use to estimate answers and solve operational problems with numbers", and "*knowledge*, the key items … that students need to learn" (Ministry of Education, 2008d, p. 1). Students are divided into groups depending on the stages they are operating at, based on their NumPA interviews (Ministry of Education, 2008a). Each group is assigned work appropriate to its level. A clear example of this system is included in Fred's drawing in which he illustrates his understandings of the different levels (Figure 6.3). On the board in the front of the classroom is work listed under Groups 1-4 with problems ranging from single-digit addition for Group1, to single-digit 4 x multiplication (Group 2), more difficult 8 x multiplication (Group 3) and difficult multi-digit multiplication for the highest group: these simple examples were used to illustrate the differences in achievement level even though the class was actually working on fractions, more "difficult stuff", at the time of the drawing. The children at both schools use the language of the Numeracy Projects, for example, strategy, solution, maths objectives and algorithms, which suggests that they are comfortable with language and routines employed by this mathematics curriculum.

Not only were the children comfortable with the language and choreography associated with the NDP, they referenced some of the same cultural images and metaphors in their portrayals of their mathematics beliefs (See Chapter 5, "Multiple readings of the data", "The third reading", "The mathematics classroom" and "Conclusion")(Johnson, 1993; Picker & Berry, 2000; Wetton & McWhirter, 1998; Wright, 2010). Harry, Fred, George, Jack and Sammie used stick figures. Almost all of the children included over-simplified examples of maths problems; some included traditional looking classrooms and desks quite unlike their actual classrooms. Some of the girls used hearts or a smiley face to indicate things they liked. These elements were easily understood by other children and worked as a short-hand mechanism for communicating their ideas; however, they could also be viewed as an essential component of visual communication.

In addition, some differences between the responses of the two classrooms emerged. In the *Number Framework*, teachers are instructed in bold: "**Students should not be exposed to standard written algorithms until they use part-whole mental strategies"**(around Stages 5/6) (Ministry of Education, 2008d, p. 14). Ron believes that this way of teaching mathematics by using the NDP and focusing on number has "given me the structure where I feel quite secure with it, and actually I've improved my own maths through having to teach in these different ways" (Interview). The children in Ron's class at Kikorangi showed a more utilitarian view of the world of mathematics, a world that included fewer affective elements in either their drawings or their responses on the *Alien Task* than those in Charles Forrest's room (Appendix P, Table P.2). Except for George, Ron's students accepted the pro-strategy, anti-algorithm approach favoured by the NDP. However, Charles, unlike Ron, was more critical of the Numeracy Projects as he explained:

> *They say 80% of the time should be Numeracy and 20% on other strands, and it's not enough. And I've read a couple of wee research papers ... it's pretty much stated that kids are coming to high school far, you know, just under-prepared with other strands, that they're really, really struggling. (Interview)*

These differences in acceptance of the Numeracy Project could explain some of the differences in the children's responses (Cross, 2009). The children in Mr. Forrest's class are comfortable using strategies as well as standard algorithms and textbooks. On the other hand, George is the only one of the Whero children who believes the use of standard algorithms is the fastest, most accurate way of solving the majority of mathematics problems. This anti-algorithm stance was also noticeable in Mrs. Umbridges's critical comments about Harry's use of algorithms as if it were a bad thing rather than a tool.

Most of the children recognise the teacher as authority although George at Kikorangi and Lilly at Whero are happy to challenge authority about what constitutes correctness, and in Lilly's case the *nature of mathematics*. In addition, Chloë, Jack and Harry are confident enough to accept their own solutions as right rather than merely relying on their teacher all the time. An additional difference was that the girls and boys in Ron's class were equally enthusiastic about doing and being able to do mathematics; whereas, Mr. Forrest's class was more varied with Jack and Caroline liking it, Harry hating it and Lilly and Chloë becoming disaffected.

In this chapter, I have narrowed the focus of this study by reporting on the mathematics beliefs of nine children and their teachers. The analysis was based on data collected from a combination of MBQs, observations, drawings, interviews and classroom work spanning two school years. By changing the perspective and looking at beliefs within the classroom context, I was able to explore how these individual students and their teachers constructed and explained their beliefs about the nature of mathematics, mathematics identities, ability and about how doing maths works in their classrooms. In addition, my analysis of my observations of the children and teachers being mathematical in their classrooms assisted me in making sense of what the children and teachers said.

In the final chapter: *Stepping back (…….and beyond)*, I revisit what I learnt from the children and teachers who participated in this exploration of mathematics beliefs. Each focus from the broad landscape of beliefs (the MBQ findings, Chapter 4), through the gallery of maths beliefs drawings (Chapter 5) and finally to this narrowing focus on a small group of children and their teachers afforded me a different perspective on the complex range of beliefs about the nature of the world of mathematics and the identities of those who do or do not belong to this world.

Chapter 7: Stepping back (….. and beyond)

The researcher as wanderer/wonderer.

The wonder of mathematics

From much of what I've read and most of my classroom observations, mathematics seems to be presented in terms of its utility or usefulness, a rather mundane, plodding, boring approach to a curriculum subject. Even though the Numeracy Project includes a lot of games and encourages teachers to present games to the students, the underlying philosophy doesn't seem to be one of learning maths for the wonder and joy of the subject. There seems to be an emphasis on showing primary age students how useful and relevant the subject is. However, in this approach much of the magic is lost. Even many of the students view it as the gateway to success at secondary school, university or employment.

Why is there a focus on the utilitarian, on the prosaic? Other primary subjects don't seem to follow this pattern. PE, art and music are viewed by children as fun, satisfying but not useful, and many students identify them as their favourite subjects. What about doing maths for the fun of it, the challenge, playing it like a game, solving challenging puzzles, exploring, discovering and manipulating numbers and patterns and finding the beauty in them? What about figuring something out for the hell of it? Discussing the aesthetics of the subject? Even English is delivered in terms of developing a love of reading and literature, not in terms of a qualification for the world of work.

The interesting thing is that I found some evidence of alternative views of mathematics in my research. There was Mr. Forrest, an infectiously enthusiastic teacher who said, "I love fractions." And many of his class loved fractions as well. Mrs. McGonagall, who took a very accelerated group encouraged her class to love the subject, argue it, live it, reinvent it, play with it, and, as she was totally unconcerned with managing the behaviour of her class, it was a very noisy, exciting place to be as was reflected in some of the most interesting, thought-provoking drawings I collected. I also have video footage of Chloë singing as she works, Jack balancing his cardboard pyramid on his head, George challenging his teacher in speed and accuracy, Jasmine jumping up and down with delight when she solved a problem, and Fred running around the class explaining how to do a very complicated problem... the sheer joy of being able to work out how to do it. And then there are the few students who, despite their teachers' approaches managed to develop universal rather than utilitarian beliefs about maths... the drawings of maths as everywhere not just the adult world of work but in the sky, the ocean, the volcano.

Journal entry, January 21, 2010

I use the metaphor stepping back because it includes notions of both time and space. I go back to the beginning of my exploration of beliefs and re-examine the temporal journey. But I also step back in theoretical space, out of the frame, a zooming out. I change the focus and perspective in order to view a more complex picture once more. In my quest for understanding children's beliefs about mathematics, metaphors play an essential role as a tool for researching, theorising, and making sense of data. Throughout this study, I have crossed many borders by combining methods, methodologies and ways of looking at the evidence provided by children that have resulted in an interesting and satisfying research journey. In this final chapter, I reflect on this study of personal epistemological beliefs about mathematics, on the routes I selected and those I decided to exclude, and on the perspectives I used during the process of accessing and making sense of children's and their teachers' mathematics beliefs. In addition, I reflect on what I learnt from the children and some of their teachers. By stepping back, I not only look at what I discovered during this exploration but at the view ahead, at what still needs to be explored and considered (the stepping beyond).

The aim of this research was to explore children's personal epistemological beliefs about the world of mathematics, about the nature of the world and about how they position themselves and *others* within this world. Rather than to prove anything or give definitive answers to the research questions in response to the concerns about mathematics achievement, the aim of this study was to further discussion about mathematics beliefs and the effects they may have on children's performance and interests in the field. This study of children's mathematics beliefs endeavoured to explore the range of beliefs that a group of children espoused and enacted within the context of school mathematics. I was interested in what the children believed, and what they and their teachers were willing to share with me. The research was framed within the New Zealand context and prevailing discourses, both here and abroad, concerning achievement, gender and ethnicity and mathematics. Data were collected from a number of sources including the children's Maths Beliefs Questionnaires (MBQs), their drawings, observations, video-recordings, and interviews as well as from a number of teachers in order to develop a picture of the mathematics beliefs.

Routes

This section describes my journey as a researcher where I reflect on the routes I followed. The first route I chose was through the landscape of mathematics beliefs held by Year 5 and 6 children by employing a maths beliefs questionnaire (MBQ) as my guide and tool for accessing a large number of participants' beliefs about mathematics. In the spirit of an exploration, I used statistics to analyse patterns in the data and to generate a framework for understanding this landscape. Some of the issues I encountered while following this route were associated with the large size of the sample, which, as a consequence of resource constraints, affected my capacity to analyse all of the questions for all of the participants. Because of the size of the sample and the volume of data generated on this route, I could have stopped data collection at this point and spent the rest of the journey analysing the MBQ responses in order to answer the research questions. However, the potential limitations inherent in this tool, such as the wording of questions that may lead to certain types of answers, issues with answering in a way to please the teacher (Johnson & Turner, 2003; Mertens, 2010), problems with the way the children answered the *Alien Task* and some of the children's reluctance to write, led me to explore other paths and ways of accessing beliefs (Lester, 2002; McDonough, 2002, 2004).

I found a barrier across the initial route during data entry and pre-analysis of the MBQ that resulted in a new path and a change of direction. The problem I perceived was the children's very narrow beliefs about the nature of mathematics in response to the *Alien Task* as reflected in Spud Murphy's response: "'It's nummmmmbbers' and dots and x's and +'s and %'s and . and – and that's it" (Whero, Ron's class). For the detour, I chose to include a drawing task as a way of accessing what the children at the focus schools believed about mathematics. Although, like the MBQ, this was a one off task for all of the children apart from the focus children, the task was easy to implement, and it was well received by the children. Most importantly, the drawings delivered extremely rich and varied data about mathematics beliefs. Initially, I had difficulty locating methods for analysing visual materials, but once I developed efficient procedures by applying multiple analysis 'frames', the results were fascinating. The most serious limitation of using this visual task was that because of time constraints, I was unable to discuss the drawings with all of the artists, which meant I was unable to have the children check my analyses of their drawings. In future, when using this approach with a large sample of participants, I would employ a *draw-and-write* (*tell*) procedure (Backett-Milburn & McKie, 1999; Wetton & McWhirter, 1998; Wright, 2007a, 2010) in order to have accompanying, complementary text to help in the analysis of all of the images. At this point in the journey, I had enough data to make the detour the main focus of this study; however, I decided to return to my original plan that included the study of the focus classrooms and students because I remained curious whether these additional contexts would provide me with a greater understanding of the beliefs I was exploring.

A dead end appeared midway through the data collection process as I was beginning to collect data from the two focus classrooms. My original plan was to follow Graham Nuthall's methodology in the design of both my classroom data collection and analysis of the data (Nuthall, 1999, 2001, 2007). Initially, I intended to link the children's espoused and enacted beliefs to learning or not learning certain mathematics ideas. However, at the point where I was beginning the pre-recording, familiarisation phase with the recording equipment at Whero, the teacher decided he did not want the children to take pre- and post-tests. Because I had to drop this element from one of the focus schools, I decided to drop it from the second school as well. A second problem, logistical rather than methodological, arose with technology. The Nuthall classroom video-recording equipment was designed to be used by a small group of researchers with a technician. Although I had help from Michael McKinnon[35] with setting up and taking down the cameras and bank of recorders, I was on my own checking the system every day, running the computer, recorders, cameras and microphones, taking field notes during recording as well as answering children's questions. I had trouble with the ageing cameras and microphones: one camera stopped

35 The technician for the School of Educational Studies and Child Development at the time

working during the second week at Kikorangi, and at Whero, one of the microphones did not pick up soft-spoken Jack at all. As a result of these problems, rather than use the classroom recordings as the central data collection and analysis mechanism for this study, I used them as cues for discussion during interviews (See Appendix H), as well as to support or explain findings from other data sources.

The final route included an examination of the espoused and enacted mathematics beliefs of a group of nine focus children and their teachers within their classroom context over two years. The tools employed to access these beliefs were a combination of MBQs, drawings, observations, recordings (both video and audio), classroom work and individual interviews. Once again, the amount of data generated was extensive which led to problems of not being able to analyse all of these data. Finally, I limited the analyses of these data by focusing on a combination of responses on the MBQ, drawings and interview while relegating the remaining data to supporting roles of confirming, contradicting or supplementing conclusions based on the focus data.

An example of how the routes worked together to create a picture of one of the focus children's (in this case, Fred's) beliefs about himself as an inhabitant of the world of mathematics, based on data collected in two consecutive years, follows:

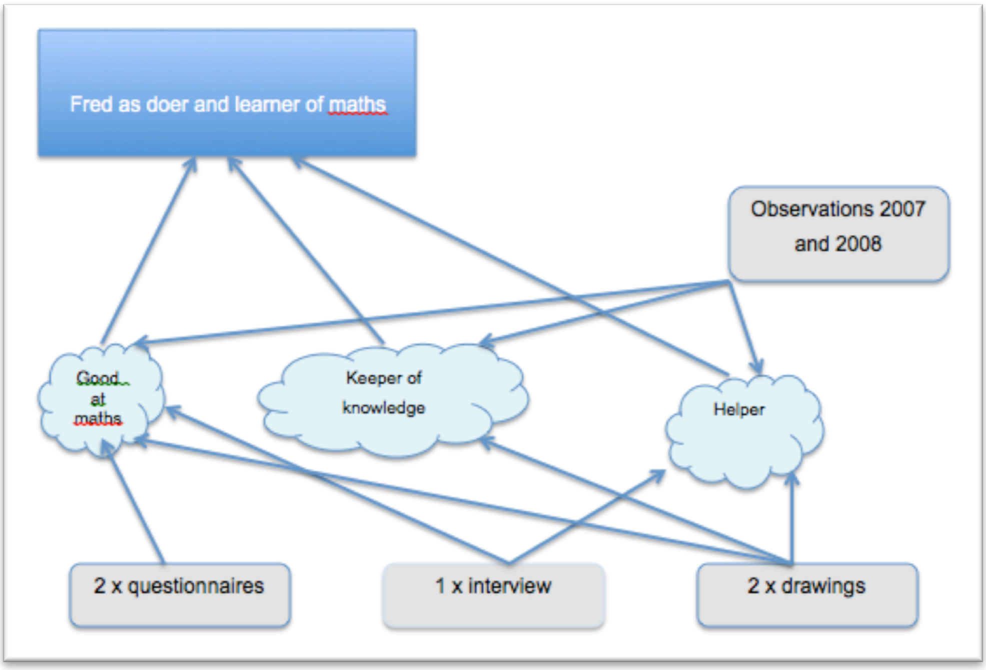

Figure 7.1: Data contributions to a construction of Fred's maths identities

In his questionnaires, Fred identifies himself as "good at maths", a position he expands on in his two drawings which include notions of himself as a "keeper of knowledge" and a classroom "helper" both by being good at mathematics and by trying to moderate other's classroom behaviours (See Figures 5.3, 6.3 and 7.6, later in this chapter). In his interview, he speaks both of being good at maths and at helping in the classroom. During their interviews, his teachers confirm his "good at maths" identity. From the various observations of Fred in his Year 5 and 6 maths classrooms, I saw him doing and being mathematical, as well as teaching concepts to other children, which corroborate his "good at maths" identity (Chapter 6). These sources of data about Fred and mathematics work together to create a detailed picture of Fred's identity, his self-beliefs, his notions of the mathematics classroom and his beliefs about the nature of mathematics.

Each one of the routes followed through this exploration of personal epistemological beliefs about mathematics was constrained by a number of confounding elements. One of the most noticeable is the lack of agreement about definitions of beliefs and how best to access them. No one best method of discovering and/or analysing beliefs is accepted by the majority of researchers in this field (Hofer & Pintrich, 2002; Törner, 2002). The central problem is how to access things that, in essence, are unseeable and to a certain degree unknowable, or only 'knowable' to the extent that individuals are willing to share what they believe. My way of dealing with this lack of agreement was to posit a number of acceptable definitions (Chapters 1 and 4) and to employ a number of complementary methods of data collection—MBQ, drawings, observations, recordings and interviews —and methods of analysis realising that each of these methods is limited (Cohen et al., 2007; Hofer, 2002; Lester, 2002). Another way I approached this issue was through placing the study in a sociocultural context and by using Nuthall's three worlds of the classroom as a way of making sense of what I was seeing and hearing. Other constraining elements were that the study was limited by the age of the participants (8-11), by the stage of their academic careers (Years 5 and 6) and place (in and around a city in New Zealand). If the children had been older and had experienced more years of school mathematics, they may have given me a different picture of beliefs about mathematics, or they may have been able to articulate their beliefs more easily. If the children had come from a variety of places in New Zealand or beyond, they may have communicated a different range of beliefs about mathematics. A final confounding element was the volume of data collected, some of which will only be analysed as part of some future journey or subsequent exploration of mathematics beliefs.

However, if I were to choose one method, out of the combination of methods of data collection and analyses included in this study, as the most informative, easy to use and capable of engendering the greatest breadth and depth of interesting data from a large number of participants, it would be the drawing task: "Draw what maths or doing maths means to you".

Perspectives

During the process of collecting, analysing and making sense of the children's beliefs about mathematics, I explored a number of perspectives, each of which suggested a different model that helped me understand what I was looking at and what was being communicated. I began this exploration by looking at the importance of mathematics as a gatekeeper subject for further study, professional careers, middleclass aspirations, as well as exploring the context of mathematics education, the discourses with their accompanying implications/barriers that work as a mechanism for ex- or including individuals in the world of mathematics. Discourses about gender, ethnicity and international rankings all position certain individuals as indigenous inhabitants of this world rather than mere visitors or aliens.

After framing the study, the first perspective was a very broad one where the landscape of mathematics beliefs of 823 Year 5 and 6 children were scrutinized. This perspective led to the development of a four-factor model of mathematics beliefs based on an analysis of the patterns of answering from the MBQ data. The factors included beliefs about *Self*, *Ability*, *Learning environment* and the *Nature of mathematics*. These factors were affected by school and student characteristics: the school characteristic that accounted for the most difference in patterns of responding was decile, a measure of social economic status; while gender, ethnicity and achievement were some of the personal characteristics that influenced responses. The four factors are illustrated in Figure 7.2 below (Introduced in Chapter 4, Figure 4.4).

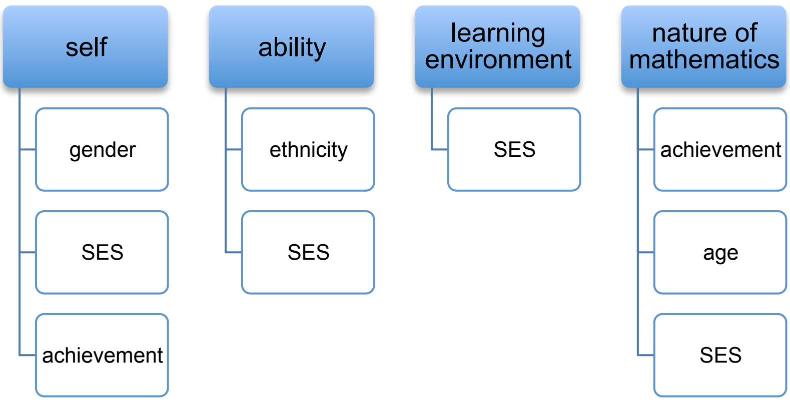

Figure 7.2: The Four-Factor Framework of student mathematics beliefs

This framework was also useful as an initial means of categorising and making sense of the data that were collected from different perspectives, for instance, the drawings and interviews.

After viewing the extensive landscape, my focus zoomed closer in order to examine the regions occupied by Kikorangi and Whero, the focus schools. In this frame, I explored an extensive gallery of 182 mathematics beliefs drawings, initially by using the Four-Factor Framework. My subsequent readings of the drawings and of the beliefs I extracted from them suggested a model in which three themes of mathematics beliefs influence and interact with each other (Figure 7.3, Introduced in Chapter 5, Figure 5.47). In this model, *identity* includes the beliefs about *self* and *ability* that were separate factors in Figure 7.2. The *identity* theme suggested a way of interpreting the data about *self* and *others* as they were illustrated in the drawings. The thirds theme was associated the *learning environment* (Figure 7.2). An advantage of this model was that it reflected a relationship between the groups of beliefs that was not hierarchical; on the other hand, the model still suggested more separation between the themes than was evident from the drawing data.

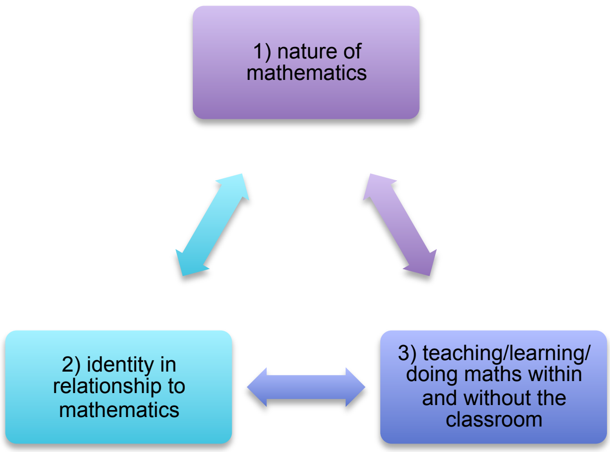

Figure 7.3: Model based on the drawings (Chapter 5, Figure 5.47)

In Chapter 6, the perspective was narrower yet, focusing on nine children, their mathematics classrooms and teachers. This perspective allowed me to concentrate on how mathematics beliefs play out in specific classrooms, as well as how individuals position themselves and are positioned by others as inhabitants of this world of mathematics. From this focus, I found that the three themes were so intertwined that they were better represented by three overlapping circles than by three distinct themes (Introduced in Chapter 6, Figure 6.1).

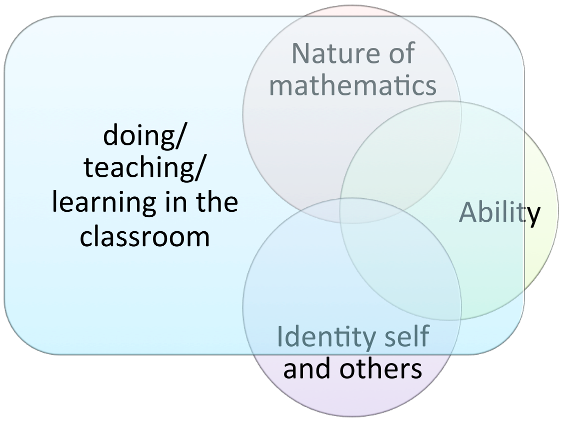

Figure 7.4: Beliefs about mathematics mapped onto the classroom context

In this model, the overlapping beliefs related to *identity* (about themselves and others), *ability* and the *nature of mathematics* are considered within the context of the mathematics classroom with its accompanying routines and myths about doing, teaching and learning mathematics. The beliefs about *ability* in this model are a different set of ability beliefs from those included in Figure 7.2, where *ability* was viewed in terms of groups who were good at/not good at mathematics. The groups in 7.2 were identified by gender and ethnicity. In Figure 7.3, *ability* is included within the theme of *identity*; in other words, the focus was on how and to what extent individuals identify *themselves* and *others* as inhabitants of the world of mathematics, as well as how they are positioned by *others* as good/not good at mathematics. In Figure 7.4, however, *ability* is viewed from a different perspective in terms of behaviours, dispositions and genetic qualities that influence "[w]hat … makes people good at maths?" or that explain "[w]hat sorts of students are good at maths?" (MBQ, see Appendix D). A problem I have with these evolving models of mathematics beliefs is that I am unable to come up with suitable terms as descriptors that would clarify my three different, yet related, conceptions of *ability*:

Which groups of people are by their membership of the group, 'able doers' of mathematics? (Beliefs about the relationship between gender and ethnicity and being good at mathematics)

Which individuals identify themselves or are identified by others as good at mathematics?

What is it that explains being good at mathematics?

Despite the problem associated with the term, *ability*, Figure 7.4 proved helpful for understanding the overlapping nature of these themes within the complex context of mathematics classrooms.

An ongoing influence on my approach to accessing, understanding and interpreting the complexities of classroom data was Nuthall's conception of the three interacting worlds of the classroom, the public world of the teacher, the semi-private world of peer interactions, and the private world of individual children (2007) (See Chapter 3). My final framework, Figure 7.5, reflects how the three overlapping themes of mathematics belief from Figure 7.4 were mapped onto Nuthall's three worlds of the classroom. This 'frame' provided an essential perspective during the process of making sense of the drawings and the focus children/classroom data because it provided a more sensitive lens for the identification and interpretation of these data.

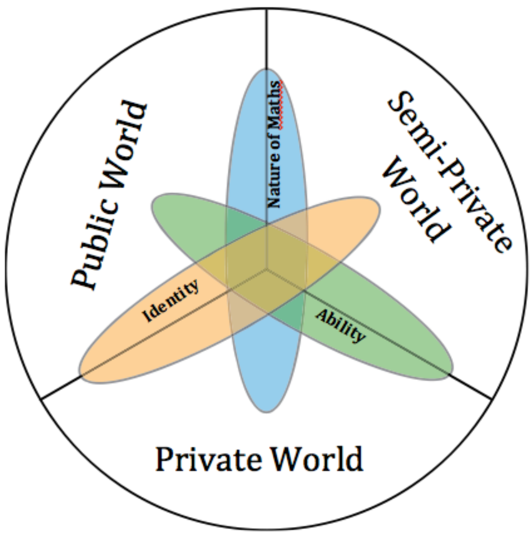

Figure 7.5: Identity, ability and nature of mathematics beliefs in the Three Worlds of the Classroom

These evolving perspectives started with the Four-Factor framework of *Self, Ability, Learning environment* and *Nature of mathematics* (Figure 7.2) and morphed through the three mathematics belief themes of 1) *nature*, 2) *identity* and 3) *teaching/learning/doing* (Figure 7.3) to the three overlapping mathematics beliefs of *nature, ability and identity* within the classroom context (Figure 7.4), and finally within Nuthall's three worlds of the classroom (Figure 7.5). Through the application of this series of perspectives to the data, the landscape of mathematics beliefs unfolded and sorted itself into comprehensible elements.

Trustworthiness

Every route investigated, every perspective explored, every "angle of repose", suggested different, enticing glimpses into beliefs about the world of mathematics (Richardson & St. Pierre, 2005, p. 963). By using multiple, complementary methods of accessing and multiple perspectives for analysing beliefs, I have tried to present a detailed picture of a range of mathematics beliefs held by a particular group of children and teachers located in a particular place at a particular point in time (Denzin & Lincoln, 2011a; Lincoln & Guba, 1985; Mertens, 2010; Richardson & St. Pierre, 2005; Tashakkori & Teddlie, 2008; Willis et al., 2007). I have, where possible, checked my analyses of these pictures with their creators and with colleagues. I have positioned myself, biases and all, within the research by presenting my background and excerpts from my research journals in a variety of places. Finally, I have attempted to present my findings with sufficient detail so that the reader may find the information interesting and applicable to her or his context (Creswell, 2007; Holliday & Primary English Teaching Association (Australia), 2004; Sandelowski, 2004; Schoenfeld, 2007). Based on the multiple routes and perspectives explored, I summarise what I discovered about the children's mathematics beliefs.

Beliefs about mathematics

My analysis of the data showed that the beliefs about mathematics, which the children communicated, seemed to be influenced by gender, ethnicity, achievement levels as well as by school decile; however, even though these elements affected beliefs, this study did not intend to establish a causal relationship between them and beliefs. I found that mathematics beliefs were also influenced by the mathematics curricula, a combination of the New Zealand Curriculum (Ministry of Education, 1993), the NZ Mathematics Curriculum (Ministry of Education, 1992), the Numeracy Development Projects (Ministry of Education, 2008d, 2010c), and perhaps the 2007 New Zealand Curriculum (Ministry of Education, 2007b), as well as how these curricula were enacted locally. In addition, the participants' experiences of doing mathematics and being mathematical affected their beliefs about mathematics. In the following sections, I summarise, discuss and develop some of the main themes covered in Chapters 4, 5, and 6 in response to the research questions. In this discussion, I include an element of wondering by suggesting additional or alternative explanations for some of the findings.

The world of mathematics

The complex world of mathematics as portrayed by the children includes some very clearly articulated commonalities as well as a large range of different conceptions. Common to most of the participants is the notion that mathematics is equated with number ('maths is number' theme), and the skills associated with number, and that the ways of doing mathematics as part of the Numeracy Projects (NDP) are privileged. Differences were more obvious in beliefs about mathematics that is more than number, in metaphors for describing the nature of mathematics, beliefs about authority/status quo, and affective elements. The ways the children (and the teachers) expressed their beliefs about the world of mathematics in response to questions about "what is maths?" seemed to pertain to three different but related categories restated as questions:

What is mathematics?
How does mathematics work?
How does maths make me feel?

What is mathematics?

The following discussion summarises some of the prevalent conceptions and metaphors the children used in communicating their beliefs in answer to the "what is maths" questions: maths as number, maths as number plus other strands, maths as useful, maths as everywhere (and other places), and maths as problem solving.

For most of the participants in this study (children and adult), mathematics is equated with numeracy and number even though at the time of this study the curriculum also included the strands of measurement, geometry, algebra and statistics (Ministry of Education, 1992). For some, numbers themselves are important as are counting, the symbols associated with the basic operations of addition, subtraction, multiplication and division, and most importantly, times-tables. Other children saw number in terms of "easy stuff", adding, subtracting, multiplying and some times dividing, and "hard stuff", especially long division, decimals, percents and fractions. The finding that mathematics is equated with number is similar to conclusions reached by other mathematics researchers in New Zealand primary schools (Grootenboer, 2003; Walls, 2009; Young-Loveridge et al., 2006). Most of the teachers who took part in this study also believed that maths is number, and that basic facts, knowing the times-tables, and understanding number is extremely important; like the children they espoused a narrow view of this world. However, alternative explanations of this pattern of teacher responding may be that their beliefs were coloured by the curriculum, in this case the NDP, or by their beliefs about 'school maths' as opposed to 'real' or post-school mathematics, or they felt obliged to reflect what they perceive the Ministry of Education's position to be. By 2007, most schools and teachers had seen the revised New Zealand curriculum either in its draft or final form, even though it was not implemented until February 2010. As a result, teachers may have been aware of the change in the mathematics curriculum which was amended from five to three strands incorporating 1) Number and Algebra, 2) Geometry and Measurement, and 3) Statistics. Figure 7.6, illustrates the emphasis placed on the Number and Algebra strand for Level Three, roughly, Years 5 and 6, although some children will be functioning at levels below or above this (Ministry of Education, 2007b).

Figure 7.6: Venn diagram from New Zealand Curriculum, Level Three Mathematics and Statistics (Ministry of Education, 2007b)

In Chapter 5, I wrote of my concerns about the children's very narrow conception of maths as number. The two explanations that I considered were that the participants were influenced by the focus on numeracy as a result of the NDP, and/or that this is what the majority of children of this age believe. However, a third explanation of this pattern suggested itself on a return to my observation notes and teacher interviews. At Whero, one of the classes had two mathematics teachers, Mrs. Hill who took the class for numeracy three times a week, and Mrs. Dale who took them for the other strands, once a week. The children viewed Mrs. Hill as their “real” maths teacher and number as the real stuff of maths while the rest was more fun maths and thus didn't count as the real stuff. In the same vein, when talking to the children in one of the bi-lingual classes at School 17 about the other classes they were taking, they mentioned geometry as a non-maths class. All of this class had described maths as number taught by their usual mathematics teacher. Because number is valued and equated with “real” maths, anything that is not number was not “real” maths. Geometry was taught by someone else and was viewed as a different subject in its own right, thus not seen as maths. Another explanation of the ‘maths as number’ belief could be that many children believed that “maths is hard”, and because they saw “number as hard”, they equated number with maths. On the other hand, geometry, which is not part of the NDP, does not have to follow the NDP way of doing mathematics, such as sorting children into ability-groups and following the choreography of the lesson, is different, perhaps more fun, thus maths is not geometry or the other mathematics strands. There seems to be an implicit hierarchy of mathematics knowing that privileges certain sorts of mathematics, in this case, number, as more important and more real than others, for example, geometry.

Even though some children saw the non-number strands as non-maths, many others presented a broader picture of mathematics in their drawings as well as in discussion. In these cases, references were made to number as well as to geometry, algebra and measurement. The geometry images included two- and three-dimensional shapes such as triangles, pyramids, rectangles, squares, cubes, circles, cylinders and polygons, but there were also references to the areas, perimeters of some of these shapes, as well as to Pythagoras' theorem in one case. Algebra was illustrated through the inclusion of letters in problems as well as formulae, especially $E = mc^2$ indicating that quite a few of the participants were aware of more than just the number strand. The children that referenced geometry and/or algebra seemed to have a broader or more sophisticated perspective than those who did not. It is unclear whether children who included the additional mathematics strands did so because they had more experiences of doing algebra and geometry than those who did not include the other mathematics strands. This may not be the case for two reasons: there was no consistency within mathematics ability groups of including these elements; and in Ron's class at Kikorangi, very few of the children had any geometric shapes even though the class had just completed a geometry unit when I started collecting data in their mathematics classroom. Despite their recent experience of geometry, the majority of drawings from Ron's class illustrate the privileged position of number over any other strand. Another explanation for the inclusion of other strands may be from outside the world of these mathematics classrooms, from siblings, from other family members, or from mathematics experiences in previous years. The inclusion of these strands in their drawings, thus, would seem to be idiosyncratic responses.

Another common metaphor was 'maths as useful' (Young-Loveridge et al., 2006) which included the measurement strand as can be seen in Jasmine's, Ella's and Spud Murphy's drawings (Figures 6.4, 5.14 and 5.15/16) and geometry to a lesser degree. The notion of utility was illustrated with tape measures, rulers, scale drawings, maps, money, dates and calendars as well as references to adult work out of school. However, some of the children also believed mathematics was useful for doing well in future school mathematics classes and, in George's case, university study. Although 19% of the children at the focus schools referred to the utility of maths, a greater percentage of children at Kikorangi than at Whero referenced this metaphor. The teachers at Kikorangi also used a utility metaphor more than those at Whero, which may explain the difference between schools. Perhaps finding a job is a more obvious issue for the lower decile school, where the teachers may believe they have to sell the importance of mathematics, or engagement in the subject in terms of the future. Another reason might have been that, by emphasising the connections between school mathematics and the children's interests and concerns based on the teachers' knowledge of their student population, they were trying to motivate the children to engage in doing and learning mathematics.

I am not arguing against the linking of mathematics to children's interests and concerns, nor against showing children that having an understanding of the subject and the tools it uses can affect their present and future worlds. However, what stood out when exploring the 'maths as useful' metaphor was that it seemed a very adult, contrived conception of usefulness that was being discussed. No one seemed to be asking the children what they as 9 and 10 year olds find useful or intriguing about mathematics. Even the practical story problems that teachers posed, problems that were supposed to appeal to the young were either fairly mindless, "I have $10.07 and want to know how many 3 cent lollies I can buy with it" when no lollies are that cheap, and we don't have one or two cent pieces anymore; or the measure up your bedroom for a new carpet type which do not interest most children of this age, especially those who do not have their own bedrooms, and whose families could not afford new carpet. The literature includes examples of teacher and textbook notions of practical, 'real-world' problems that are neither real nor interesting and relevant to many students (Anthony & Walshaw, 2007; Boaler, 1993; Ollerton, 2006). There was a hint of the beliefs about usefulness that related to the children's real interests, in some of the MBQs where children wrote about using maths in sport, especially for scoring, in music and dance, and in Chloë's beliefs that maths is about fairness and sharing. Mathematics *is* both useful and important; however, notions of usefulness need to be connected to children's lives in ways that are meaningful to those children and their communities rather than teachers', education officials' and curriculum writers' ideas about what are suitable contextual problems for children.

If one views 'maths as number' and 'maths as useful' as a narrow set of mathematics beliefs, then the small group of children who believe maths is everywhere or in all of life inhabit a much broader mathematics world. There is an element of universality in these images of neverendingness, of everywhere and of life (Zach, Katia, Lucy and others). Other children use geographic metaphors depicting mathematics as a place (Mathsland, Numberland, a shop, a club or a prison) with its own language and ways of doing things. When I first looked at these more expansive, more universal metaphors of mathematics beliefs, I made the assumption that they belonged to more advanced, more sophisticated mathematics students. In the literature, a broader set of beliefs are associated with the more sophisticated (Perry, 1970; Schommer-Aikins, 2002) or availing beliefs (Muis, 2004) of more advanced students. I was, however, incorrect in this case because the students came from a mixture of very advanced, ordinary and very weak mathematics students, and from a range of classes and teachers. This forced me to reconsider my notions of sophisticated doers of mathematics. Perhaps this set of children had developed broader, more availing beliefs based on connections they made between mathematics and other elements of their lives beyond this study, for example, family and community influences and values, or based on their personal experiences of reading, thinking and making connections outside of the classroom.

A metaphor both children and teachers used when discussing mathematics was that 'maths is problem solving' (Walls, 2009; Young-Loveridge et al., 2006), a theme which includes notions of "figuring" or working things out (Ronan), manipulating numbers, solving puzzles, challenges (George) and being heroes in their own adventures (Tom). Some of the children like Sammie and Miriama link problem solving to thinking and using your brain to work things out, as well as for gaining more knowledge and developing brain capacity. Lucy includes multiple drawings of brains filled with interesting mathematics ideas and ways of doing things. For other children like Lyle, Joshua, Harry and his friends, problem solving is the source of brains igniting or exploding. The 'maths is problem solving' metaphor is interesting in that it was applied in so many ways with different feelings associated to the different depictions, some neutral as in working things out, using your brain, to the positive 'aha' moments (Lucy and Ronan), as well as Harry's negative beliefs.

In summary, although the children and teachers communicated a variety of beliefs in response to the "what is maths" questions, the most dominant belief equated mathematics with number. This privileging of number as "real" mathematics seemed to reflect the NDP and Ministry of Education's emphasis on number (Ministry of Education, 2007b, 2008d, 2010c) Nevertheless, many of the children also referenced other strands, such as geometry, algebra and measurement, as well as metaphors associated with 'maths as useful', 'maths as a place', 'maths as everywhere', as well as 'maths as problem solving' – with its own related metaphors of challenges, adventures, brains, learning and burning.

How does mathematics 'work'?

For most of the participants in this study, the doing, learning and teaching of mathematics was inextricably woven into beliefs about "what maths is". Their experiences of doing mathematics seemed to be influenced by the values inherent in the NDP; for instance, strategies are superior to algorithms, you need to know multiple ways of solving problems, children are separated by achievement levels which offer different access and opportunities to groups at different strategy stages, to name a few (Ministry of Education, 2008a, 2008c, 2008d; Walls, 2004).

The dance of the mathematics lesson follows a more or less standardised choreography which was familiar to both children and teachers: 1) begin on the mat with a teacher-led activity; 2) break into ability groups, usually three, and follow the prescribed steps as set by the teacher—one group receiving direct instruction, the other two engaged in ability appropriate tasks; 3) change activities—another group with teacher, remaining groups at activities; 4) a finale of sorts, a wrap up, a game, handing in work, etc. In this dance, not all children have direct access to their teachers during any single mathematics class as only two groups receive direct instruction. This is the way you 'do' maths. Inherent in this dance is the practice of ability sorting and allocating children of differing achievement levels different mathematical experiences, a practice which few children (notable exceptions Chloë and Lilly), and no teachers challenged (Walls, 2009). Examples of these different experiences are illustrated by Rosie (Chapter 5) and Fred (Chapter 6) and show groups at different Numeracy stages being set different sorts of mathematics problems thus being offered different opportunities to learn important concepts in what Walls calls "an apartheid" system (2004, p. 37). This dance of the NDP mathematics lesson perpetuates a lack of equity in that certain children are publicly excluded from certain mathematics experiences, and their exclusion from these activities may reinforce their and *other's* beliefs about their mathematics abilities.

However, there is also an awareness of classroom and mathematics values that are important for doing mathematics. The children reported that you need to listen, use your brain, concentrate and follow the rules. There are issues around what it means to be the good, well-behaved student who is willing to put up your hand if you know the answer or have a question, and only talk when you are instructed to answer a question or work with others. These values are common to other subjects as well. Caroline is taken aback by her teacher who she sees as "breaking the rules" when he asks her to answer a question even though she has not put up her hand. The ways of doing that are peculiar to the children's experiences of doing mathematics are understanding the dance, being willing to explain how you worked out a problem, being able to share multiple strategies for solving problems, and deferring to the teacher as the source of correct solutions (Herbel-Eisenmann, 2009). Interestingly, although the children recognised the teacher as the mathematics authority, some like George and Lilly were willing to challenge this. Even Chloë, Jack and Harry did not feel they always needed to defer to the teacher for the correct answer (Herbel-Eisenmann, 2009; Pimm, 2009).

Many of the children include their experiences of 'working' in the classroom. They tell of working with others in pairs and groups, collaboratively as in Govi's drawing, just together in Orange's one, or working alone. Britt's not only draws doing mathematics alone, but the isolation of having to work it out herself because "no one can help me..." They also communicate experiences of working with teachers, some helpful, others not. Harry illustrates his experiences of "getting on with it" alone and under the nose of his teacher who shouts all the time.

The children were familiar with the language and tools associated with doing mathematics. They talked, wrote and drew using words like strategies, algorithms, equations, solutions, problem solving, symmetry, probes, estimation calculations, BEDMAS, maths games, etc., as well as referencing the usual mathematics symbols (+, -, x, ÷, %, =, etc) and operations (add, subtract, multiply, divide, etc.). Most of the children and teachers believed that there are multiple ways of solving mathematics problems, a position reflecting the emphasis put on multiple strategies in the NDP. However, some of the participants challenged this emphasis on multiple strategies. David writes of "weird methods", and Harry likes using algorithms. George solves problems by first using an algorithm, then adding various other strategies if forced to by his teacher. He knows that even though he is extremely fast and accurate, in order to be a "good" maths student you need to buy into the dance of the NDP.

On the other hand, a large number of children believe that problems have multiple answers, an interesting difference from the teachers' belief (Chapter 6, Teaching/learning/doing mathematics). This mismatch between the children and their teachers warrants more unpacking. If the children believe that a problem has more than one answer, then this belief might explain why certain children are confused about mathematics and find it "inpossible" (Harry), "weird", or burst into tears when a new strategy is introduced. The children were also concerned with “getting it right” (Ella and Chloë) when problem solving and taking tests; but if there are multiple answers, then the notion of ‘rightness’ seems arbitrary which may explain Chloë’s screams of frustration.

The children’s beliefs about how mathematics ‘works’ are closely linked to their experiences of ‘doing’ mathematics at school, to the ways the NDP is enacted in their classrooms, as well as to their beliefs about the nature of mathematics. In addition, these experiences of ‘doing’ mathematics influence what the children feel about mathematics as a subject, or about specific sorts of mathematics (e.g., fractions, test taking), ways of working (alone or in groups) or approaches (strategies versus algorithms).

How does maths make me feel?

This third category, though briefer, relates to the children’s emotional responses to mathematics. As I explained in Chapter 1, I view feelings about mathematics together with attitudes, interests and opinions as part of mathematics beliefs. The children portrayed a complex range of feelings about mathematics through their drawings, and to a lesser degree through their MBQ responses, interviews and classroom behaviours. These responses were closely linked to beliefs about the nature of this world, the experiences the children had within this world as well as how they viewed themselves within this world. At the positive end of the spectrum are the children who view maths as fun, exciting, challenging and awesome: for example, Tom taming a strategy, Fred knowing the answer to a very hard problem, Chloë’s fireworks, Lucy’s invigorated brain, etc. Despite mathematics’ usual portrayal as a cold, hard, exclusionary world, the majority of the participants believed that maths is fun and liked it, a position often attributed to those who are good at mathematics (Caygill & Kirkham, 2008). In this study, Māori students as a group were the more positive about mathematics than Pākehā students (Chapter 5). A prevailing view in the New Zealand context is that Māori students are not usually thought of as good at mathematics (Caygill & Kirkham, 2008; Collins, 2010; Rees, 2012); however, like any other ethnic group, their achievement levels range from excellent to weak.

A group of students illustrated how mathematics engendered a range of beliefs, both positive and negative depending on the context, the activity or task: for instance, enjoying maths games, but not tests (Richie); liking geometry and BEDMAS but disliking difficult division, decimals, percents and fractions (Sophie). Then there are the children who find mathematics distressing, "very annoying and inpossible" (Harry, Chapter 6), stressful, alienating, horrible, black and red (Hazel), a prison (Luke), brain burn inducing and boring, screams of frustration (Chloë), black and white (Hamish). Some of these affective responses to maths raise interesting questions:

How does a very successful student move from finding mathematics extremely exciting and fun to making her scream with frustration by the following year (Chloë)?

If one accepts that better students are more positive about mathematics than other students, then what does this say about the responses from the Māori students in this study?

These questions warrant further exploration.

The feelings engendered by mathematics, feelings of fun, and excitement to horror, anxiety and boredom were not only related to the children's beliefs about the nature of mathematics and about 'doing' school mathematics, but also related to their identities within the world of mathematics.

Identity beliefs

Identity is something that is fluid, involving a continual process of negotiation, of positioning one's self and being positioned by others depending on the context (Archer et al., 2010; Franke et al., 2007; Gauntlett, 2008). Another major group of beliefs explored in this study were the beliefs about who belongs and who is excluded from this world of mathematics: who are the rightful inhabitants as opposed to the foreigners and sometime residents. These are the beliefs associated with self-beliefs about doing mathematics, beliefs about others and mathematics, as well as beliefs about mathematics ability. They are influenced by beliefs about the nature of mathematics as well as feelings about mathematics. Also of interest was the extent to which the prevailing achievement discourses around gender and ethnicity might influence these beliefs, what was acknowledged and what left unsaid.

The beliefs about who is good or not good at mathematics are closely linked to practices of sorting and measuring children inherent in the ways of doing primary school mathematics experienced by the participants. For many schools that adopted a mathematics interchange system, children were sorted at the beginning of the year based solely on assessment results, the Numeracy Diagnostic Interview (Ministry of Education, 2006a) and for some children the PAT Mathematics assessment (Darr, 2006; Reid, 1993). The same assessment tools were used to divide children into ability groups within their mathematics classes as well. As a result of this practice, the children constantly measured themselves against their peers by referring to which classes or groups they were in. For instance, Jack and Chloë accept they must be 'ok' at maths because they are in one of the most advanced classes; however, Chloë also questions this practice of sorting based on a single test which she believes does not really identify those good at mathematics. This comparing and measuring of themselves and others against visible groupings of students, which may be based on inadequate assessment, can be especially problematic when children are miss-sorted or have strengths that are ignored when only a narrow skills-set is assessed (Walls, 2009).

In describing their beliefs about themselves and mathematics, that is their self-beliefs about mathematics, the participants included a range of identities from being very good to being awful at maths with some children adopting more than one identity depending on the context. Many children linked being good, ok or bad to the activity they were involved in; they were good at maths games, adding, subtracting and multiplying but bad at some of the "harder stuff" like fractions, for example (Hamish, Richie, George, Harry, Sophie, Jasmine and Sammie). Other children positioned themselves as always good (Fred, Neo, Ziro, Steven). Although all the groups involved were more positive than negative about their mathematics abilities, boys came across as more self-confident than girls (Carr et al., 2008; Crooks et al., 2010; Flockton et al., 2006; Vanayan et al., 1997). Boys were more likely just to accept that they were inherently good at maths whereas some of the girls associated being good at maths with good teaching, family help or luck, a difference in "loci of control" (Dweck, 2002; Schunk, 2008; Wigfield & Eccles, 2002).

Some of the responses from the focus children could be interpreted as challenging the notion of identity fluidity. George, Fred, Harry, Jasmine and Lilly all present relatively consistent mathematics identities over the two school years. Perhaps these fixed identities are based on similar experiences of mathematics over this period of time, or their beliefs about mathematics did not change, thus they saw themselves as remaining in the same context, in the same roles (Boaler, 2000; Boaler, Wiliam, & Brown, 2000). On the other hand, Lilly, Chloë, Harry and Fred indicated that they had different identities in non-mathematics contexts: Lilly and Chloë are extremely good at English (both reading and writing), Harry excels at sport and P.E., and Fred positions himself as poor at reading, "I'm dyslexic. You know what dyslexic is?"

Apart from how the children are sorted in mathematics classes, other characteristics of indigenous inhabitants of this world are the children who know how to do school mathematics, by answering what is asked of them, providing multiple strategies, understanding what counts as an explanation of your chosen method, the well-behaved—the turn takers, the listeners and concentrators—the persistent, the logical thinkers, people who practice and who work hard. These behaviours are recognised by teachers and children with cultural capital as valued practices (Bourdieu, 1986) (See Chapter 6, Mr. Forrest's class). In addition, some teachers and children believe that certain people are genetically disposed to being good at maths (e.g., Harry, Jack, Mrs. Dale and Mrs. Umbridge).

Unpacking beliefs about which groups, which *others*, are included or excluded from this world of mathematics proved more complex. Only one teacher, Mr. Lupin, mentioned socioeconomic status as a factor that influences how well children do at mathematics. On the whole, the children did not communicate strong beliefs about gender and mathematics, for most of them girls and boys were equally good or bad at maths. Most teachers claimed that gender was not a contributing factor; one noticeable exception was Ron who had more boys in his top group and thought boys were more able. Like the children, the teachers claimed there was no difference between Māori and Pasifika students and other ethnicities at their schools, implying that the differences between the nationally reported achievement levels and their schools happen at *other* schools. One explanation of this position is that both children and teachers are correct, based on their experiences of different ethnic groups within individual schools, where children usually come from the same or similar socioeconomic backgrounds. From the teachers' responses and practices, it was not clear whether some of them actually believed there were no differences between groups or whether they thought it would be politically incorrect to voice such a position. This line of speculation is, in part, influenced by "off the record" information and conversations with teachers who were unwilling to put their views in writing or have them recorded despite my assurances that they and their schools' identities would be protected. Even though they claimed not to sort Māori and Pasifika students into weaker groups, I observed examples of this practice. An example of a disconnect between an espoused belief and a classroom practice was the placement of Sammie, a Māori girl, in the bottom group even though she had the highest score in the class on one of the achievement measures; other children in the class commented on this anomaly (See Chapter 6).

Most of the teachers and quite a few children were aware of the prevailing discourse of Asians being good at mathematics. The teachers—Ron, Paul, Vern, Mr. Forrest and Mrs. Umbridge—claimed that Asian children are good at certain sorts of mathematics, such as rote learning, speed and accuracy, and working hard, but not good at applying various strategies and being creative. Yet the higher achievement groups had more Asian children in proportion to others, which perhaps suggests that the tests used to sort children into their mathematics groups favour the skills that Asian children are good at, or that, despite claims to the contrary, Asian children at these schools are more advanced at mathematics than their non-Asian peers. Some of the children also question the 'Asian as good' discourse, for instance Lilly, Chloë and Fred, who although acknowledging this discourse, question it: "It's not really where you come from, it's because of how you think of things" (Lilly).

What is happening here? The teachers are saying they value thinking, flexibility and creative ways of doing mathematics, but may be using another skill set to sort children. Or are Asian children really better in all areas of mathematics? Similarly, the Kikorangi and Whero teachers' claim is that Māori and Pasifika students are just as capable as any other group of students. Then why do they do less well and are more likely to be sorted into lower groups? Are the teachers ignoring certain children's potential and achievement levels, or are we testing a very narrow set of skills which favour Asian, Pākehā, middleclass students, those with cultural capital? This type of sorting seems at odds with the New Zealand Mathematics Curriculum document (the curriculum in use at the time of this study) that encourages a move away from "[t]raditional time-constrained pencil and paper test" which "have proved unreliable indicators of Māori achievement in the past" (Ministry of Education, 1992, p. 13). Children with cultural capital are more likely to behave in ways that count in the classroom by playing the well-behaved student who knows the language and the ways of doing assessment, and thus teachers may perceive these behaviours as indicative of ability. The current New Zealand Curriculum includes the five key competencies of

> "thinking
> using language, symbols and texts
> managing self
> relating to others
> participating and contributing" (Ministry of Education, 2007b, pp. 12-13)

and stresses the importance of ongoing assessment , and that "[a]nalysis and interpretation often take place in the mind of the teacher, who then uses the insights gained to shape their actions as they continue to work with their students" (Ministry of Education, 2007b, p. 39). Although the language and the focus have changed between the two curriculum documents, the five key competencies still favour the child who knows how 'to do' school, the child with cultural capital. Teachers are being asked to judge their charges in terms of competencies that favour children who are well-behaved, sit still, answer when asked a question, who complete their work neatly, who can use the language of the classroom or domain, who know how to work co-operatively in pairs and groups, who have the communication skills to show what they are thinking, and that allow them to participate and contribute—competencies that children with cultural capital are more likely to have inculcated from home than children from *other* social classes or cultures.

Attached to beliefs about who is or is not good at mathematics, who does or does not feel comfortable in this world are a rich range of idiosyncratic mathematics identities often presented as metaphors. Fred positions himself as good at maths, 'a holder of knowledge' and a moderator of maths classroom behavior. George, the maths king, is always challenging classmates and teachers in order to retain his position. There is successful Ella. There is Tom, the hero of his adventure in Mathsland, Ronan the problem solver. There is well-behaved Neil. There is Lucy whose brain is enlivened by exciting maths concepts, and Miriama whose brain links maths to thinking, learning and acquiring knowledge. Then there are the naughty boys, the badly behaved, Harry, Orange, Richie and Pink who are stressed, heads on fire, or fighting, who claim not to know answers or how to do things, or perhaps don't want to. There are the bored, the confused, the tearful, the violent, the imprisoned as well as the delighted and entranced. There are the "dumb bums" and the "smarty pants". There are the children like Steven and Ziro who see themselves as part of this world while they exclude *others* who are not allowed in. There is Chloë who finds maths exciting and explosive in Year 5 but by Year 6 is screaming with frustration. There is Jack, identified as so-so by Mrs. Hill and as a talented mathematician by Mrs. Dale at the same time. These vibrant mathematics identities, and others, populate the children's worlds of mathematics.

What is of concern is not those who self–identify or who are positioned by others as good at mathematics for they are enthusiastic and engaged, but rather those who are marginalized, sorted, or place themselves at odds with this world. They are the children who do not believe they belong so why bother to engage, who find it too hard or too boring (Boaler, 2010; Boaler & Wiliam, 2001; Boaler et al., 2000). In particular, I am concerned about those who are excluded from interesting and exciting mathematics experiences by their teachers or the school system of interchange sorting, and those who are excluding themselves. Students like Lilly and Chloë who are very talented at mathematics are turning away from and may join the ranks of those missing in action (Gladstone, in print). Other children missing in or, rather, from action, are Māori and Pasifika non achievers, working class boys, girls who believe mathematics isn't for them, and those perceived to have disabilities that deprive them from inclusion (Bartell, 2012; Boaler, 2002; Boaler et al., 2000; Collins, 2013; Gladstone, in print; Gutstein & Peterson, 2006; Mendick, 2006; Ollerton, 2001, 2006; Snook & O'Neill, 2010).

However, the dichotomy between beliefs about the nature of mathematics and the identities of those inhabitants of this world is misleading because they are interconnected and both are influenced by experiences of doing mathematics. An example of this merging of beliefs about the nature of mathematics and about mathematics identities is illustrated in Fred's maths classroom drawing which includes his experiences within the world of "school maths", how it works, how he identifies himself as a native of this world while some of his classmates are positioned as foreigners. It describes the public world of the teachers and the work they set, Fred's interpretation of the private worlds of the student participants and what they are feeling, as well as his position in the classroom (Figure 7.6). He divides the participants into teachers, "freands", "random kides" and "me". He positions himself as good at maths and the holder of knowledge. One of his friends is asking him for help. Some of the "random kides" look very stressed, in tears, puzzled, wanting to leave or saying, "my brans exploding" [my brain is exploding]. Ron, one of the teachers, is in tears and indicates that the problems are too hard for him. Fred positions himself as both an 'insider', as someone good at mathematics and comfortable in this world, but also as an 'outsider' as different from his peers and teacher who are shown as stressed and struggling with mathematics.

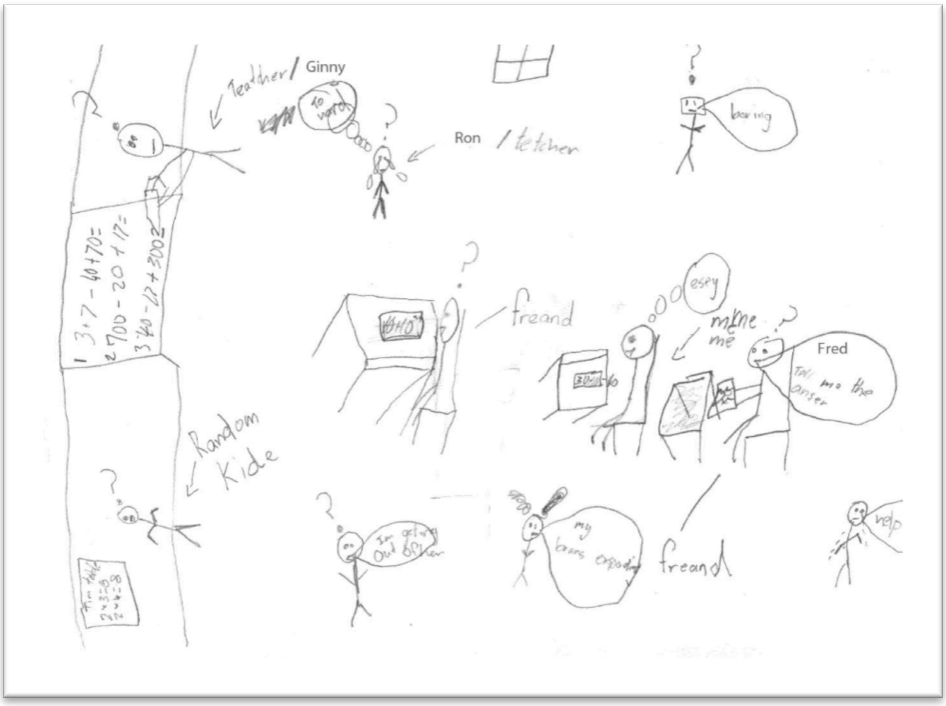

Figure 7.7: Fred's classroom
[Not to be reproduced without permission.]

Fred's drawing represents his idea or notion of the 'maths classroom' that is rather different from my experience as an observer in his classroom. His drawing is of a traditional, rather old-fashioned mathematics classroom, whereas his actual classroom has children sitting in groups at large tables, on a comfortable couch, a mattress or playing maths games on the floor. In addition, the hard problems he includes are much easier than the problems his maths group were working on (fractions and decimals rather than addition, subtraction and multiplication of whole numbers), thus this social construction of "hard" mathematics is a metaphor for his experiences and beliefs about mathematics. Other examples of this conceit are illustrated in Chapters 5 (The nature of mathematics) and Chapter 6 (Fred's Year 6 drawing, figure 6.3). Nevertheless, Fred's drawing tells a story of mathematics being a hard subject for many people although he finds it easy.

In summary, by following a variety of routes and applying a number of different perspectives, I glimpsed a rich and complex landscape of mathematics beliefs. These overlapping beliefs included beliefs about the world of mathematics (what it is, how it works, and the feelings it engenders), beliefs about membership in this world (mathematics identities, who is good and who is not) as well as beliefs about mathematics ability (beliefs about gender and ethnicity, about selves and *others*, beliefs about characteristics and behaviours associated with being good at mathematics).

Stepping beyond

Even though this study produced a rich, complex picture of the children's personal epistemic beliefs about mathematics, there are still so many avenues that deserve further exploration, study and consideration. In the following section, I present some of the implications of this research, outline areas for further research and propose a challenge to go beyond neoliberal mathematics achievement discourses.

Implications

In the following paragraphs, I discuss several implications of this study in terms of the importance of exploring beliefs about the world of mathematics and about who is positioned as able to function well within this world. Although I focus predominately on the implication for teachers, this study also has implications for children, for curriculum, for policy makers and for researchers in the areas of mathematics, beliefs as well as those interested in using drawings as a data source.

Children's beliefs about mathematics affect their attitudes and performance as well as the decisions they make about mathematics. However, a compelling reason for being aware of students' beliefs is explained by Lester: "It is the internal knowledge of an individual that matters because it is this kind of knowledge that directs her or his actions and subsequent learning " (Lester, 2002). In order to improve performance and interest in mathematics, teachers need to engage with children's beliefs and not assume they are universal and 'correct'. As educators, we not only need to develop availing mathematics beliefs but confront some of the ways of 'doing' mathematics that seem to contribute to the problem of believing mathematics is not for you.

Before considering these implications, I believe it is important to address the terminology used by teachers in this study as well as by the literature about achievement (Boaler et al., 2000; Butterworth, 1999; Chambers, 2008; Cotton, 2001; Marks, 2013; Ministry of Education, 2006a). There seems to be some confusion between the terms, *ability* and *achievement* which are used interchangeably in schools despite their different meanings. Even the NDP, *Getting started* refers to "diversity of student ability" in reference to children's achievement levels (Ministry of Education, 2008c, p. 14). *Ability* and its binary opposite *disability* imply innate aptitude and talent or the lack thereof while *achievement* implies a level attained or accomplished. What is usually referred to as mathematics ability in the context of school mathematics is actually the achievement level reached by children. The identity connotations associated with *ability*, through chance and thus no fault of your own, and *achievement*, over which you, teachers and curriculum have some power, are very different. Perhaps, rather than use the term *mathematics ability*, *mathematics capabilities*, a term suggesting potential as well as the availability of opportunities, is preferable (Nussbaum, 2011). Perhaps we could follow the example set by Te Whāriki, the New Zealand early childhood curriculum, with its reference to children as "capable people and competent learners" (Ministry of Education, 1996, p. 30). I acknowledge that Anthony and Walshaw do seem to adopt a similar stance in their introduction to the Mathematics BES : "all students irrespective of age have the *capacity* [my italics] to become powerful mathematical learners" (2007, p. 1).

From my perspective, it is important for teachers to be aware of and willing to articulate their own beliefs about mathematics in order to understand their attitudes to the curriculum, to teaching mathematics and to positioning certain children as capable mathematicians. In order to do this, both beliefs about the nature of mathematics and about the identities of those positioned by others and by themselves as successful and/or unsuccessful doers, teachers and learners of mathematics need to be explored.

Only once teachers are aware of their own beliefs will they be able to recognise and gain an appreciation of differences between the various sets of beliefs that lead to understanding or misunderstanding aspects of the curriculum. By encouraging children to voice their beliefs about mathematics, teachers will be able to compare their own beliefs with those of the children. The one particularly perplexing difference between children's and teachers' beliefs that surfaced in this study was that many children believed that mathematics problems had many solutions. Perhaps if teachers were more aware of their students' beliefs, they would understand why certain students are more engaged and motivated about learning mathematics than others (Carpenter et al., 2000; Crooks & Flockton, 2002; Edwards, 2000).

Comparing beliefs about mathematics can also work as an inspiration for conversations and discussions between teachers, teacher and student(s), or between children about differences and similarities of approach to curriculum, to teaching and learning this subject, and about who is being positioned as being good/not good at mathematics.

It would also be helpful if teachers, schools and the curriculum could help children develop availing beliefs about mathematics, the sorts of beliefs that assist children in learning mathematics (Muis, 2004). Children who hold epistemological beliefs about the world of mathematics and their place in it, which include that

- there are many ways to solve mathematics problems,
- mathematics is more than just number,
- mathematics is relevant and connected to their lives, their interests, and other areas of study,
- mathematics is fun, interesting, important and challenging
- they belong to the world of mathematics,
- they are capable of doing it, learning it, and/or
- hard work and practice helps one become good at mathematics,

are likely to be motivated to continue to engage with mathematics and to persist when mathematics becomes difficult. On the other hand, those who hold epistemological beliefs about the world of mathematics and their place in it, which include a narrow conception of mathematics and that

- there is only one correct way to solve a problem and the teacher knows it,
- solutions to problems and ways of solving them are arbitrary and don't make sense,
- mathematics is boring, cold and frustrating,
- it has no relevance,
- it is impossibly difficult
- they don't belong to the world of mathematics,
- they have been excluded from being good at mathematics, and/or
- one has to be born good at mathematics to be successful

are less likely to be engaged in availing mathematics behaviours (De Corte et al., 2002; Leder et al., 2002b; Muis, 2004; Muis & Foy, 2010; Perry, 1970; Schoenfeld, 1985; Schoenfeld, 1994a; Schommer-Aikins, 2002).

Teachers could utilise a variety of methods in order to access their students' beliefs about mathematics. Methods such as the implementation of a drawing task combined with classroom discussion, and/or small group discussion would fit into the usual routines of the classroom and would supply interesting and useful information about mathematics beliefs. Nevertheless, there needs to be an awareness that the reason for exploring children's beliefs is not to judge and use the children's beliefs as yet another mechanism for ranking and sorting children, but as a means for understanding differences between individual children, and between children and teachers, a means for improving all children's experiences of doing and being successful at mathematics.

In addition to the importance of exploring teachers' and children's beliefs about mathematics, based on this study, I suggest that teachers, school leaders and those responsible for curriculum and assessment policies re-consider the tradition of mathematics interchange and *ability* grouping for mathematics that takes place in New Zealand primary school classrooms. We need to be willing to examine what mathematics skills we consider important, and how we are assessing these mathematics skills in order to sort children for mathematics lessons, or whether we should be sorting them at all (Boaler, 2010; Chambers, 2008; Cotton, 2001; Walls, 2004, 2009). In particular, we need to be aware of the implications of these practices in terms of their influence on mathematics beliefs.

The findings from this research into children's beliefs about mathematics, especially beliefs about who can or cannot do mathematics has implications for those children who position themselves or are positioned by others as not belonging to the world of mathematics. My discussion of this issue is included in next two sections.

Some ideas for further exploration

This study has focused on a broad conception of mathematics beliefs that include the nature of mathematics, how mathematics works and who belongs to the world of mathematics. A more in-depth, micro exploration of specific aspects of mathematics beliefs warrants further study, such as beliefs about fractions, long-division, the other strands or beliefs about testing, for example, multiplication tables. Beliefs about each of these areas, strands or tasks were referred to by children and/or teachers involved in this study, but, because of time constraints, I was unable to follow these paths. For instance, many of the children wrote or drew about fractions being difficult, an area that teachers also mentioned, and an area assigned a whole section in the Mathematics BES (Anthony et al., 2007). If beliefs about fractions were studied, it may be possible to discover if certain children have more or less availing beliefs about fractions, as well as identify what availing beliefs for dealing with fractions might be. In addition, beliefs associated with the National Standards, implemented since this study took place (Hattie, 2011; Ministry of Education, 2009a, 2010b), deserve investigation because the effects of the standards on children' s beliefs about the nature of mathematics and on children's mathematical identities, and ultimately the effects on how they do mathematics, have not been explored.

Other areas of interest that merit further exploration are studies that compare different age groups, as well as longitudinal studies that follow children from primary through high school years, as well as international comparative studies of mathematics beliefs. It would be especially important to follow groups of children who are in the process of becoming disaffected with or disengaged from mathematics (Walls, 2003, 2009), very capable girls like Lilly and Chloë (Boaler, 1997, 2002, 2010; Boaler & Wiliam, 2001; Fennema & Hart, 1994; Fennema & Leder, 1990; Mendick, 2005, 2006), and those who form New Zealand's under-achievement tail, who start believing that the study of mathematics is not for them or not for people like them. To date, the only New Zealand longitudinal study of children's mathematics lives and developing mathematics identities is Fiona Walls' exploration of children's mathematics subjectivities which followed ten children from Years 3 through 5, and later through their secondary mathematics experiences (2003, 2009). The time for many students when they begin to believe that mathematics is not for them seems to happen between the end of primary or intermediate school and the first two of years of secondary (Malloy, 2008); therefore, following students through these years and studying their changing beliefs about mathematics and about themselves as learners of mathematics may explain these changes and perhaps identify elements that influence these changes in belief that result in students disengaging from and often giving up on mathematics (Cobb, 2007; Niss, 2007).

All of the teachers I surveyed and interviewed during this study claimed that Māori and Pasifika students at their schools did as well as Pākehā students. The long brown tail of under-achievement was not present at their schools even though the prevailing New Zealand achievement discourse and results on national and international tests suggests otherwise. Are Māori and Pasifika students much weaker at mathematics than other ethnic groups? Is this a socio-economic rather than ethnicity issue? Or do the specific skills assessed and methods of assessment disadvantage certain populations (Harris, 2007)? These questions deserve further exploration because more Māori and Pasifika are leaving school with fewer and worse mathematics NCEA results than their Pākehā and Asian peers (Haines, 2008; New Zealand Qualification Authority, 2010; O'Callaghan, 2013). One way to untangle the SES versus ethnicity explanations of achievement would be to study Māori and Pasifika children's mathematics achievements across a range socioeconomic backgrounds by looking at their mathematics performance at schools with high, medium and low decile ratings. Although NEMP studies undertake this approach to studying achievement, by reporting results in terms of school decile as one category and ethnicity as another, rather than looking at ethnicity within decile, they do not make the relationship between ethnicity and SES clear (Crooks et al., 2010; Flockton et al., 2006). Looking at the values of the mathematics skills being assessed and culturally empowering methods of assessment are complex and require an ethno- or indigenous- mathematics approach (Barton, 2009; Begg, 2001). This sort of approach requires a willingness to listen to and collaborate with Māori and Pasifika participants and leaders about the values of different kinds of important mathematics knowledge and skills, as well as to explore collaborative and narrative methods of assessment which use language in ways that children from a variety of backgrounds understand (Hāwera & Taylor, 2011).

From a personal perspective, I am also interested in continuing to use and refine the use of drawings as a method of accessing children's beliefs. Based on the findings from this study, I am interested in continuing to explore ways of accessing mathematics beliefs that could be employed in a variety of places and population groups beyond this context. In 2010, I conducted a related study to discover whether the drawing task and the analysis methods I used in the New Zealand context would be appropriate in another country and another language. I trialled the task in Ecuador at a school where both Spanish and English were taught. I spent a month at Island School, observing mathematics classes, talking to teachers and reading curriculum documents. Two small classes, equivalent to Year 5 and Year 6, were involved with drawing their mathematics beliefs. One major difference between the studies is that I used a *draw-and-write* protocol (Backett-Milburn & McKie, 1999) with the Island School children and thus had verbal texts from the 20 Ecuadorian children. I employed the same analysis frames as with the New Zealand study: first the four-factor framework (Figure 7.2), then the three themes from Figure 7.3. The drawing task and analyses frames worked as well in both contexts producing a rich source of data which I could use to compare beliefs and experiences between two very different cultures with different classroom experiences and different ways of doing mathematics (Solomon, 2012b). Like the New Zealand children, the Ecuadorian children included themselves and others, the mathematics classroom with teachers, doing mathematics, and elements of the nature of mathematics. (See Appendix Q for some examples from the Island School children's drawings.) They also referenced some of the same metaphors: the brain (Tod[36], Figure Q.3) and personified mathematics symbols (Melisa from Island and Ari from Whero, Figures Q.4 and Q.5). One difference between countries was the content of the mathematics curriculum; the children at Island School were doing "harder" mathematics, square and cubed roots, but in a teacher-centred class where classroom practice did not include group work, and where everyone was given the same tasks irrespective of achievement level. Another difference was the language the children used to describe math, for instance Jami's " Math is beautiful and incredible".

Finally, I would like to suggest that teachers investigate their students' epistemological beliefs about mathematics by using a drawing task with accompanying verbal text (*draw-and-write* protocol). In order to analyse these drawings, they could employ the Figure 7.3 analysis frame which teases out beliefs about the nature of mathematics, about mathematics identities and about the doing of school mathematics.

Beyond the prevailing discourses: Reframing mathematics with social justice

> Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. … The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding
> (National Council of Teachers of Mathematics, 2000, p. 3).

36 Pseudonyms were chosen by the children.

These words are part of the NCTM (USA) *Principles and standards for school mathematics,* a wonderful ideal yet to be realised in the USA and most other countries. Of especial importance is the ideal that *all* have access to important, powerful mathematics concepts and excellent teaching. From a social justice perspective, it is the right of *all* children to have access to this vision of the world of mathematics. “All children” includes those from all cultural and ethnic groups, all social and socioeconomic classes, and all abilities (Banks, 2006). Although social justice can be thought of in a variety of ways (Wager & Stinson, 2012), Planas and Civil’s definition of social justice as “equal access to opportunities to participate in the social construction of reality and … access to opportunities to improve living conditions of individuals and groups” (2009, p. 392) as well as the acceptance of the equal status of all groups (Stinson & Wager, 2012) is pertinent to this discussion. For researchers and activists like Moses and Cobb, access to important mathematics ideas, especially for minority and disadvantaged students, is a civil right (Malloy, 2008; Moses & Cobb Jr., 2001). Any notion of teaching and learning mathematics in terms of social justice, or the right of all children to learn mathematics and be mathematical in their own worlds (National Council of Teachers of Mathematics, 2000; UNICEF, 1989; Wager & Stinson, 2012) makes no sense if children either believe they are excluded, or are actually excluded by others from this world (Malloy, 2008; Powell, 2012). If one believes as Chloë does that mathematics is “a way of making everything fair”, then issues associated with equity in access to mathematics, as well as what and how mathematics is taught to primary age children need consideration (Cotton, 2001; Gates, 2001; Gutstein & Peterson, 2006; Peterson, 2012).

In a neoliberal world where education is viewed as a commodity rather than a common good, countries, schools and children are measured, sorted and compared (Grace & Thrupp, 2010; Walls, 2009). By using group identifiers such as ethnicity, class and gender as explanations for lack of achievement, certain children are positioned as *other* which absolves schools, education systems, teachers and society from responsibility for the failure of these groups. In New Zealand, we do not sort and separate primary school children for school subjects like art, social studies and science, but it is acceptable to do so for mathematics as encouraged by the NDP. This process of dividing primary age children into achievement-based mathematics classes and groups acts as a mechanism of exclusion from full participation because children who are identified as ‘less able’ are offered different mathematics experience than their ‘more able’ peers. It is based on perceived cultural and/or gender deficits (Banks, 2006) although schools and programs claim it is a way of teaching children “where they are at” (Ministry of Education, 2008c; Walls, 2004, 2009). Sorting children into homogenous interchange classes or homogenous groups within classes can be viewed as an organisational tool to simplify the teachers’ lesson preparation and management of classroom mathematics behaviour. This point may seem contradictory because I am suggesting that planning two or three mini lessons is less work than planning a lesson for a heterogeneous class where every class member can engage in a meaningful way. Even though I am arguing for an end to ability grouping or segregating children based on perceived ability or lack thereof, I realise that we need to change more than mathematics classroom practices: we need to change the way we, the public at large—teachers, educators, politicians, parents and students, etc.—think about *ability, capability* and *achievement*, a much more difficult task (Marks, 2013).

Although there is a hierarchy of skills that are necessary for children to acquire in order to move on to more difficult mathematics concepts, I would like to suggest that confining groups to one or to a narrow band of achievement levels does not guarantee the acquisition of necessary skills (Boaler, 2010; Boaler & Wiliam, 2001). Despite claims of moving students between groups as they develop the necessary skills, in most cases children do not seem to change groups: even, as Walls discovered, in cases where there were potential opportunities to change groups as in changing from the number strand to another, such as geometry (2009). Instead, those who are placed in the lowest groups seem to remain in the lowest groups throughout their schooling (Boaler, 2010; Boaler et al., 2000). Even more troubling than being stuck in a low-achieving group is being deprived of opportunities to engage with more interesting and advanced mathematics and mathematical ideas, a practice Greer and Mukhopadhyay label “intellectual child abuse” (cited by English et al., 2008,

p. 864). These students are often restricted to repeating, year after year, mathematics content they have already covered (Rousseau Anderson & Tate, 2008). In addition Nuthall discovered in a range of subjects that the 'new' material teachers think they are teaching is already (fifty percent of it) known by the children(2007).

This sorting of children illustrates how an aspect of mathematics continues to be used as a gatekeeper subject, a form of border control, which maintains the status quo; it identifies certain children, often those with cultural capital, as 'good at maths', a way of protecting the professional classes and thus keeping out the *other*, often those from marginalised communities. This sorting by social class, often masked as an ethnic and cultural sorting based on perceived ability (Bishop & Forgasz, 2007; Boaler & Wiliam, 2001; Ollerton, 2006), and deficit discourses of *other* (Gonzalez, 2012; Koestler, 2012) is reflected in achievement results and school league tables as reported in the New Zealand media (Napier, 2013; O'Callaghan, 2013). Mathematics seems to be viewed as a discipline not suited to 'less able' people, which could mean those who belong to certain groups are viewed as less able mathematically, as are those who are measured and found wanting. The *other* children may be excluded from engagement with more advanced mathematics and empowering mathematics ideas while at school, and then excluded from higher education and higher paid jobs. This exclusion is contrary to the notion of children's rights to an education that prepares them for a productive and fulfilling life in the 21st Century. One of the purposes of sorting is to filter certain individuals towards important things while trying to make sure that the rest of the population has a high enough level of skill to be good, compliant workers in a globalised world (Tate, 2006; Walls, 2004), as well as avoiding embarrassing their schools and country in comparative achievement tests. A limitation of this approach of focusing on skills suitable for a globalised world which values comparative achievement testing is that, because it is impossible to foresee what mathematics skills may be of value and interest in the future, schools are concentrating on skills of the past rather than making sure that all children have the critical mathematics skills they might need for any potential future in a fast-changing world (Gutstein & Peterson, 2006; Romberg, 1994).

I am not arguing against the learning of number and number skills but rather for a broader perspective on primary school mathematics. If we could accept that all children have the right to and are capable of learning mathematics (Leonard & Evans, 2012; Malloy, 2008), not just about number, as Moses and Cobb found to be the case through the Algebra Project (Moses & Cobb Jr., 2001), and my experiences with Cynthia and Daphne (Chapter 1), then we can begin to view mathematics through a social justice lens. By doing so, we could teach mathematics in a way that provides children with " opportunities and self-empowerment for them to understand and use mathematics in their world" (Stinson & Wager, 2012, p. 10). This sort of mathematics curriculum would recognise other knowledges and ways of doing mathematics, those associated with a range of different cultural practices (Gutstein & Peterson, 2006). One way to engage all children in being mathematical might be to consider teaching mathematics across the curriculum as a connected rather than isolated subject, as a subject accessible to all, that is interesting and engaging, but above all important (Peterson, 2012). Another way might be to adopt the philosophy present in primary school literacy teaching that aims to develop a life-long love of reading and literature, a disposition to be a reader. Thus children would be encouraged to develop a life-long interest in mathematics and being mathematical (Cobb, 2007).

The discussion in this section has been critical towards what I perceive as a narrow approach to mathematics and a lack of equity inherent in the ways of doing classroom mathematics. However, I also observed teachers and children being mathematical and engaging with unusual ideas as well as making connections to other parts of their lives and interests. These observations often took place during unplanned excursions where children and teachers were happy to be diverted from the intended choreography of the lesson. The following extract from my observations in Mrs. McGonagall's mathematics class illustrates how some children are engaged in disrupting a textbook created task and turning it into an interesting discussion of important mathematical ideas.

Important mathematical ideas in Mrs. McGonagall's classroom:

The students were supposed to be taking a pre-test before the beginning of a statistics and probability unit. A couple of minutes in, they start objecting to one of the "stupid" questions which ran something like: if you went around kissing frogs, and some turned into princes and/or princesses, what would the ratio of princes to princesses be?
Student A: We can't answer this.
Mrs. McG: Why?
Student A: We need to know the population statistics for where we are doing the kissing.
Student B: What's it in New Zealand?
More discussion
Student C: Now in China they have many more princes.
Student D: They do infanticide?
Student E: That's not the right word.
And they went on to discuss the one child policy, aborting girl foetuses, the problems associated with too many males in the population. No one completed the test yet every child in the classroom was riveted by the discussion.
(The age range in this class was between 8 and 11, Years 4 through 6.)

Based on Observation notes, 30 July 2007

In order to change the focus of primary mathematics to include more equitable and interesting approaches, what we mean by important mathematics skills need to considered. As part of his definition of literacy, D'Ambrosio classifies the number skills often associated with the narrow perspective mathematics:

> the critical capacity of processing information, such as the use of written and spoken language, of signs and gestures, of codes and numbers. Nowadays, reading must also include competency of numeracy, of interpretation of graphs and tables, and of other means of informing the individual. (2012, p. 211).

The New Zealand Curriculum takes a similar approach in the description of the second of the key competencies:

> Using language, symbols, and texts is about working with and making meaning of the codes in which knowledge is expressed. Languages and symbols are systems for representing and communicating information, experiences, and ideas. People use languages and symbols to produce

> texts of all kinds: written, oral/aural, and visual; informative and imaginative; informal and formal; mathematical, scientific, and technological
> (Ministry of Education, 2007b, p. 12).

However, D'Ambrosio goes beyond this perspective and coined the term "mathemacy" to define the sorts of powerful mathematics skills children need to develop, "the critical capability of inferring, proposing hypotheses, and drawing conclusions from data" (2012, p. 211). Children need to develop mathematical reasoning, generalising, thinking mathematically, solving real-world problems, understanding connections between mathematics and the rest of their worlds (Langrall, Mooney, Nisbet, & Jones, 2008; Perry & Dockett, 2008) and the understanding that mathematics works as a language of communication (Ollerton, 2006; Peterson, 2012). Frankenstein emphasises the importance of critical mathematics and the notion of "reading the world with mathematics" (2006, p. 19) thereby developing the skills and understanding necessary to recognise links between mathematics, knowledge and politics (2012). Children also need to develop the skills of using mathematics to argue a position and to challenge the status quo. Mathematics can arm children with the skills they need to understand their world, so that they can take part in their worlds in a meaningful way as full participants in society (D'Ambrosio, 2012; Gutstein & Peterson, 2006; Koestler, 2012; Walls, 2009).

Concluding thoughts

Mathematics is both powerful and empowering. It is essential for understanding our world. It can be used to pose problems and to develop solutions. It can be both wonderful and terrible. Terrible, in that it can be used to create weapons and systems for misleading people, for destroying people, places and environments as much as to create solutions to local and world problem. We need a vision of mathematics that values and accepts all children as capable inhabitants of this world where teachers and curricula recognise all have a right to learn to use mathematics critically. Perhaps in this world, people will believe that they are capable of being mathematically competent and thus will be willing to be active citizens of this world.

Finally, by returning to the initial question of this study, a question central to any exploration of epistemological beliefs about mathematics, I include a response from a mathematics educator and responses from three children who took part in this study:

What is mathematics?

Mathematics is beautiful, intriguing, elegant, logical, amazing and mind-blowing: a language and a set of systems and structures used to make sense of and describe the physical and natural world. It is a set of tools and processes used in decision-making, a discipline upon which questions are formulated and problems are solved. It is used to model environmental conditions and applied to make sense of social phenomena.
(Ollerton, 2006, p. 18)

Maths is a veary fun and easy topic where you add subtract divide or multiply and play maths games. you can work the problems given to you out with algirithims and other strategys.
(Richie, Mr. Forrest's class, Whero)

Maths is like a Productionline. you add two things (numbers) in a certain way (+ - ÷ x) and you get a new number (1+1=2).
(Jaden, Ms. K's class, Kikorangi)

Maths is something you need in your life and you can't live without it you can use maths for anything. In math there are x timestables, ÷ division, - mius, + plus and numbers like thise 0123456789 these are the only numbers
(George, Ron's class, Kikorangi)

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Appendices

Appendix A: The Focus Schools

The Focus Schools database is an imbedded subset of the City Schools one and includes data from both Kikorangi and Whero. These two schools are not necessarily representative of the whole landscape of classroom mathematics beliefs; instead, they are the schools that were used as sites for observations, video-recordings, interviews, and the drawing task, and as such are schools about which much more background information was available.

Table A.1: Background information for the focus schools

	Kikorangi		Whero	
Number	56		179	
School type	Full Primary		Contributing Primary	
Decile	4		10	
Gender	Female	50%	Female	46%
	Male	50%	Male	53%
			missing	1%
Ethnicity	Asian	7%	Asian	11.7%
	Māori	20%	Māori	8.9%
	Pākehā	73%	Pākehā	77.7%
			Pasifika	1.7%
Year	Year 5	46%	Year 5	52.5%
	Year 6	54%	Year 6	47.5%

Both of the focus schools are state primary schools: Kikorangi is a decile 4, full primary school while Whero is a large decile 10 contributing primary. For Kikorangi, the participants were equally split by gender and slightly more Year 6 students took part than Year 5. More Year 5 than Year 6 students and more boys than girls took part at Whero (See Table A.1). The ethnic breakdown was also different at the two schools; even though both schools were over 70% Pākehā, Kikorangi had 20% Māori students to Whero’s 8.9%.

Appendix B: Teacher sample

Initially, the teacher sample included 71 teachers, at least one teacher from each of the seventeen schools, which participated in the student sample. The teachers at Kikorangi and Whero Schools became the nested sub-sample, and the two focus teachers, Ron and Charles, were members of this group.

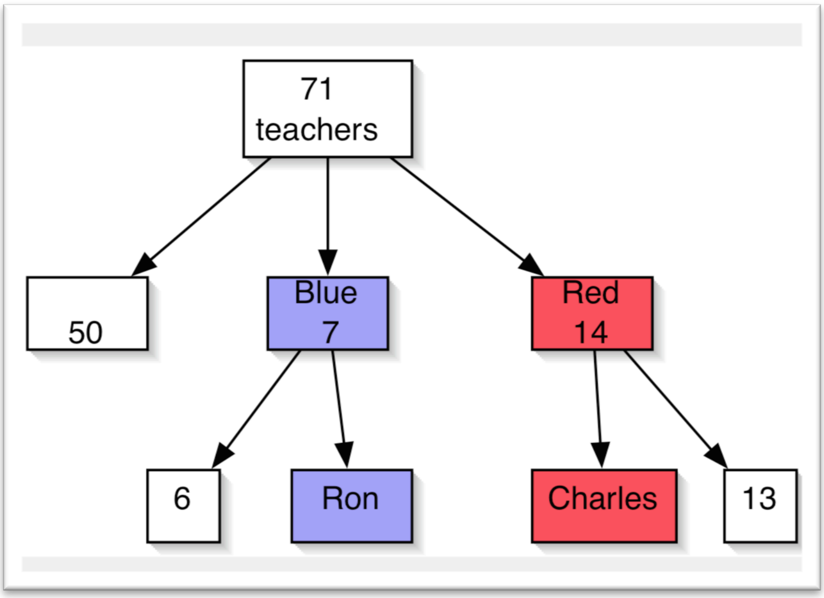


Figure B1: Teacher Sample

Even though the teacher sample is a convenience one and is thus unsuitable for drawing general transferable conclusions, the data from this sample are useful in that they provide additional perspectives or depth to attempting to unpack beliefs about mathematics that help develop a picture of the variety of beliefs around the teaching and learning of the subject. This information can also be used to confirm or contradict the information gathered from students at the same set of schools (Mertens, 2010; Yin, 2006). Furthermore, the data from this sample is worth considering in terms of what is known about the effects of teachers' beliefs on how they teach, present materials, choose tasks, interact with students, etc. which in turn affect the beliefs of the students in their classes (De Corte et al., 2010).

Apart from one school where the head of mathematics required all the teachers in her syndicate to answer the Teacher Maths Beliefs Questionnaire, the sample was a voluntary or convenience one. Each one of the 17 schools was sent or given information about the research project. I visited each staffroom and addressed the staff, describing the project, fielding questions and asking for teacher volunteers. There were 71 teachers, at least one from each of the 17 schools; 63 of these teachers completed the Maths Beliefs Questionnaire in 2007. However, in 2008 I returned to the two focus schools and observed eight of the focus students in their new maths classrooms. Their eight teachers, three from Year 7/8 and five from Year

5/6 completed the questionnaire before I entered their maths classes to observe; they also allowed me to interview them.

The following table includes a breakdown of teacher participants by school, gender and the year/s they teach.

Table B.1: Teacher Sample Characteristics

School #	decile[37]	gender	ethnicity[38]	classes[39]
1	low	f = 3	P = 3	j = 1 m =2
2	middle	f = 4	P = 4	m = 4
3	high	m = 1	P = 1	m =1
4	high	f = 4	P = 3 ? = 1	j = 1 m = 2 I = 1
Kikorangi	middle	f = 4 m = 3	P = 6 ? = 1	m = 4 i = 3
6	middle	f = 5 m = 1	P = 6	j = 1 m = 2 i = 1 o = 2
7	high	f = 1 m = 1	M = 1 P = 1	m = 1 o = 1
8	high	f = 3	P = 3	m = 1 i = 1 o = 1
9	middle	f = 1 m = 1	P = 2	m = 2
10	middle	f = 2	P = 2	m = 2
Whero	high	f = 10 m = 4	P = 13 ? = 1	j = 4 m = 8 o = 2
12	low	f = 2	A = 1 P = 1	j = 1 m = 1
13	middle	f = 5 m = 2	P = 6 ? = 1	j = 1 m = 3 o = 3
14	high	f = 2 m = 2	P = 3 ? = 1	j = 1 m = 2 o = 1
15	low	f = 3, m = 1	P = 3 ? = 1	m/i = 4
16	high	f = 4	P = 4	j = 1 m = 3
17	low	f = 2	M = 1, P = 1	m = 2
Totals	**l= 4, m = 6, h =7**	**f = 53, m = 16, ? = 2**	**A=1, M=2, P=62, ?=6**	**j =11, m=40, m/i=4, i=6, o =10**

.

37 decile 1-3=low, 4-7=middle, 8-10= high decile groups (Flockton et al., 2006).

38 A = Asian, M = Maori, P = Pakeha, ? = undisclosed.

39 j = juniors, new entrants – year 2; m= middle years, 3-6; i = intermediate years; 7-8; m/i = combined classes, years 4-8, o = other, e.g.,2-6 or 1-9, reliever, principal, etc.

Appendix C: Example of student consent forms

A study of primary students' and teachers' maths beliefs and how they affect the learning and teaching of maths

Observation Consent Form for Students

I have read or heard the information about the project.

I have talked to my parents/caregivers and teacher about it.

I am happy to be videoed in maths class.

I understand that I can change my mind about taking part in this project and no-one will mind.

I know that if I have any questions I can ask Cathy, Mr F or my parents about it.

Child's full name (please print) ______________________________

Signature ____

Date ___________________________

Alias ___________________________

Please give this form to Mr F or Cathy

Appendix D: Student and Teacher Maths Beliefs Questionnaires

D.1: Student Maths Beliefs Questionnaire

Maths Beliefs Questionnaire

School: Name:
Year: Age:
Maths teacher: Gender: m f

Please read each question carefully and answer it honestly.
There are no right and wrong answers.

1. Circle the group or groups you belong to.
Asian................ Maori Pakeha/European Pasifika Other...............(fill in)

Please put a cross in the boxes that match what you think

2. Only some people can do maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

3. Everyone can learn maths
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

4. I can do maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

5. Boys are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

6. Girls are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

7. Asian students are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree ☐ Don't know

8. Maori students are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree ☐ Don't know

9. Pacific Island students are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree ☐ Don't know

10. Pakeha/European students are good at maths.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree ☐ Don't know

11. There is only one way to **work out** a maths problem.
Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

Go to the next page

12. There is only one **answer** to a maths problem.

Strongly agree | 5 | 4 | 3 | 2 | 1 | Strongly disagree

13. How much do you like doing maths at school?

Heaps | 5 | 4 | 3 | 2 | 1 | Not at all

14. What kinds of maths activities do you like doing best?

15. List some interesting maths activities you do in your own time.

16. How do you use maths out of school?

17. How often do you do very interesting things in maths at school?

Heaps | 5 | 4 | 3 | 2 | 1 | Never

18. How good do you think you are at maths?

Excellent | 5 | 4 | 3 | 2 | 1 | Suck

19. How good does your teacher think you are at maths?

Excellent | 5 | 4 | 3 | 2 | 1 | Suck ☐ Don't know

20. How good does your family think you are at maths?

Excellent | 5 | 4 | 3 | 2 | 1 | Suck ☐ Don't know

21. How good do you think your teacher is at maths?

Excellent | 5 | 4 | 3 | 2 | 1 | Suck

22. Answer **one** of the following questions:

I'm good at maths because ....

OR

I'm ok at maths because...............................

OR

I'm not good at maths because..................

Go to the next page

23. What is important to know in maths?

24. Is anyone in your family very good at maths?
Yes No Not sure

If you answered **yes**, who?

25. Is anyone in your family very bad at maths?
Yes No Not sure

If you answered **yes**, who?

26. What do you think makes people good at maths?

27. What do you do if you have a very hard activity in maths?

28. How do you feel when you solve a very difficult maths problem?

29. How do you feel when you have trouble with a maths problem?

30. List **three** maths activities you are good at:

31. List **some** maths activities you are not good at:

Go to the next page

32. Do you find maths hard, easy or in-between?

33. If you met an alien who had never done maths, how would you describe maths and what it is about?

PLEASE CHECK THAT YOU HAVE ANSWERED EVERY QUESTION.

Thanks for your time!

D.2: Teacher Maths Beliefs Questionnaire

School: Name:
Years of teaching: Age: 20s 30s 40s 50s 60+
Classes you teach: Gender: m f
Ethnicity:

Maths Beliefs Teacher Questionnaire

Please put a cross in the boxes that match what you think

1. Everyone can do maths.

Strongly agree	5	4	3	2	1	strongly disagree

2. Only some people can do maths.

Strongly agree	5	4	3	2	1	strongly disagree

3. Everyone can learn maths

Strongly agree	5	4	3	2	1	strongly disagree

4. I can do maths.

Strongly agree	5	4	3	2	1	strongly disagree

5. Boys are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

6. Girls are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

7. Asian students are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

8. Maori students are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

9. Pacific Island students are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

10. Pakeha/European students are good at maths.

Strongly agree	5	4	3	2	1	strongly disagree

11. There is only one way to **work out** a maths problem.

Strongly agree	5	4	3	2	1	strongly disagree

12. There is only one **answer** to a maths problem.

Strongly agree	5	4	3	2	1	strongly disagree

13. How much do you like teaching maths at school?

A great deal	5	4	3	2	1	not at all

14. What kinds of maths activities do you like teaching best?

15. List some interesting maths activities you do in your own time?

16. How do you use maths out of school?

17. How good do you think you are at maths?

Excellent	5	4	3	2	1	awful

18. How good does your family think you are at maths?

Excellent	5	4	3	2	1	awful

19. How good do your students think you are at maths?

Excellent	5	4	3	2	1	awful

20. Answer **one** of the following questions:

I'm good at maths because ....

OR
I'm ok at maths because..............................

OR
I'm not good at maths because..................

21. What is important to know in maths?

22. Is anyone in your family very good at maths?
Yes No Not sure

If you answered **yes**, who?

23. Is anyone in your family very bad at maths?
Yes No Not sure
If you answered **yes**, who?

24. What do you think makes people good at maths?

25. What sorts of students are good at maths?

26. What do you do if you have to teach a maths concept you find difficult?

27. How do you feel when you solve a very difficult maths problem?

28. How do you feel when you have trouble with a maths problem?

29. I fell confident teaching the following maths concepts/activities:

30. I don't feel confident teaching the following maths concepts/activities:

31. Overall, how difficult do you find maths as a subject? Why?

32. How would you describe what maths is about?

Appendix E: Question types and origins

Student Questions	Teacher Questions	Question types	Origins
2	1	Likert, 5-point	
3	2	Likert, 5-point	
4	3	Likert, 5-point	common classroom activity
5	4	Likert, 5-point	(Barlow & Reddish2006)
6	5	Likert, 5-point	
7	6	Likert, 6-point	
8	7	Likert, 6-point	
9	8	Likert, 6-point	
10	9	Likert, 6-point	
11	10	Likert, 5-point	(Ministry of Education, 2010c; Seaman et al., 2005)
12	11	Likert, 5-point	(Vanayan1997)
13	12	Likert, 5-point	NEMP (Flockton et al., 2006)
14	13	open	NEMP (Flockton et al., 2006), Vanayan (1997)
15	14	open	NEMP (Flockton et al., 2006)
16	15	open	
17	--	Likert, 5-point	Alison Gilmore
18	16	Likert, 5-point	NEMP (Flockton et al., 2006), Jinks & Morgan(1999) (Vanayan et al., 1997)
19	18	Likert, 6-point	NEMP (Flockton et al., 2006)
20	17	Likert, 6-point	NEMP (Flockton et al., 2006)
21	--	Likert, 5-point	
22	19	closed /open	NEMP (Flockton et al., 2006), Jinks & Morgan(1999) (Vanayan et al., 1997)
23	20	open	NEMP (Flockton et al., 2006)
24	21	closed/open	suggested by conversations with adult students
25	22	closed/open	suggested by conversations with adult students
26	23	open	(Barlow & Reddish, 2006) House (2006)
--	24	open	
27	25	open	NEMP (Flockton et al., 2006) & Turner, Meyer, Midgley & Patrick (2003)
28	26	open	Turner et al (2003)
29	27	open	Turner et al (2003)
30	28	open	common classroom activity
31	29	open	common classroom activity
32	30 open	closed, 3-point	House (2006)
33	31	open	AlienTask. Stodolsky, Salk, & Glaessner (1991). Young-Loveridge, Taylor, Sharma & Hawera (2006)

Appendix F: Maths Beliefs Student Questionnaire: Version One

1.	Everyone can do maths	True	False	Not sure
2.	Only some people can do maths.	True	False	Not sure
3.	I can do maths.	True	False	Not sure
4.	Boys are good at maths.	True	False	Not sure
5.	Girls are good at maths.	Ture	False	Not sure
6.	Asian students are good at maths.	True	False	Not sure
7.	Maori students are good at maths.	True	False	Not sure
8.	Pasific Island students are good at maths.	True	False	Not sure
9.	Pakeha students are good at maths.	True	False	Not sure
10.	There is only one way to work out a maths problem.	True	False	Not sure
11.	There is only one answer to a maths maths problem.	True	False	Not sure
12.	I am good at maths.	True	False	Not sure

13, If you answered True: I'm good at maths because……………………..

If you answered False: I'm not good at maths because …………………

14. How much do you like doing maths at school?
15. What kinds of maths activities do you like doing?
16. What kinds of maths activities do you do out of school?
17. How good do you think you are at maths?
18. How good does your teacher think you are at maths?
19. How good does your family think you are at maths?
20. What do you think makes people good at maths?
21. Is anyone in your family very good at maths? Yes No Not sure

    If you answered yes, who?
22. Is anyone in your family very bad at maths? Yes no Not sure

    If you answered yes, who?
23. What do you do if you have a very hard activity in maths?
24. How does it feel when you solve a very difficult maths problem?
25. How do you feel when you have trouble with a maths problem?
26. What is important to know in maths?

27. I am good at doing the following maths activities:

28. I’m not good at doing the following maths activities:

29. Do you find maths hard, easy or in-between?

30. If you met an alien who had never studied maths, how would you describe maths and what it is about?

Appendix G: Student Interview questions and procedures

The focus students and their 2008 maths teachers have agreed to be interviewed. All the students have redone the Maths Beliefs Questionnaire, and I am waiting for some of these from the teachers. All of these teachers will also have completed a consent form by the time I interview them.

Focus Students

1. Intro: Before I begin interviewing the students, I'll briefly recap the study and what I am researching. I will also check that they feel comfortable with and have given their permission to the interview.

Show them how the digital voice recorder works.

Explain what we are going to be doing today: talking about their beliefs, looking at a video of them in maths class, discussing it, talking about the answers they gave on the survey and looking at the pictures they drew.

2. Establishing rapport questions:

Can you remember when I was recording in Mr X's room?
Who is your maths teacher this year?
Which group are you in?
How are you doing this year?
How much do you like it?
Any differences from last year easy/hard, fun/not fun?
Why?

2. Video clip:

"I am going to show you a video clip and ask you about it."

Do you remember this? No response: try to get them to describe what they see and talk around what seems to be happening.

Yes response: talk about the memory, and what they notice now. Can you remember what you were thinking, feeling? Do you still feel that way now? etc.

Try to ask open questions in response to their answers or descriptions.

3 Questionnaire responses: Ask them to explain any unusual or unexpected responses. Perhaps chat about differences between the way they answered this year and last.

I will definitely go over the following questions trying to tease out more information:

How good do you think you are at maths? Why do you think this?
What sorts of people are good at maths? Try to link this to the individual responses as well as to things like gender and ethnicity.
What makes people good at maths?
How do you know when someone is good at maths?
What is important to know in maths?
Some question about how do you know when you are right or on the right track

What exactly is maths?

4.Drawings: Brief discussion of and ask to explain any unusual features.

5 Conclusion: Try to bring them back to how they are doing in maths this year. Encourage them to ask me any questions.

Thank them for taking part and give them my email address so that if they have any additional information they'd like to give me they can.

If at all possible, I like to be able to show them the interview transcripts so they have the option of making changes; however, I'm not sure it will be possible because of time restrictions and approaching holidays.

Appendix H: Interview questions and video cues for focus teachers

H.1: Focus Teachers

Name:
Alias :

Introduction

Beliefs about maths and how these beliefs may affect what goes on in the classroom.

3 sorts of beliefs:
1) What is maths? The world of maths. The nature of maths. Who belongs/ doesn't belong to this world? Why?
2) Beliefs about self and the relationship to maths. How good are you? How do you relate to this world/subject as a teacher or learner?
3)Self-efficacy beliefs. What good at doing/ teacher?

Focus students:
Look at video clips of them in class, same ones I showed the students.
What going on? Specific reactions, observations of students , yourself or class

Going to be asking questions and recording you responses. OK?

General Questions
1) In your experience, what sorts of students are good at maths?

2) What makes them good at maths?

3) Have you noticed any particular group or type of students who tend to be good/bad at maths?

4) Why do you think some students find maths harder than others do?

5) How can you tell how well a student is doing in maths?

6) What is most important to know in maths?

7) The Numeracy Project. Did you teach before it? Have you noticed changes in achievement, understanding, belief, attitudes?

Video Clips
Any specific or general comments on the segments

Other Questions

1) How would you describe maths?

2) How good do you think you are at doing
teaching maths?

3) What sorts of things are you most confident about?

(what is your relationship to the world of maths)

4) How are things going in maths this year? Any changes to your beliefs or how you teach the subject?
content, enjoyment, attitude, etc?

H.2: Videos cues for Ron:

1) **George** date: 14/11 disc: 4

20.40 ---- > 22.38

38.41 ----- > 43.11

Questions / comments

2) **Sammie** date: 13/11 disc: 3 &4

4.40 ---- > 5.11 6 x 7

6.03 5 x 7 = 35

+ 1 more 7

= 42 apples

Questions / comments

3) **Jasmine** date: 13/11 disc: 4 & 2

22.36 shows set up, what asked to do

using 5 x table

28 start again

Questions / comments

4) **Fred** date: 14/11 disc: 1 & 4
28.20 ---- > 29+ 16 x 5

39.01 ---- > Goerge to front

Questions/ comments

H.3: Vidoes cues for Charles Forrest:

1) **Caroline** date: 26/11 disc: 3 & 4

21.00 ---------->31.43

22.6 thanks

compensation strategies

tidy #s 20 x 9

24.44 ------ > 25.35 put a zero

20

x9

31.43 ---- > 34 Do #2 by self back, etc

Questions/ comments

2) **Chloë** date: 27/11 disc: 1 & 3

33 ---------> 35.36 estimation

39.27 ---------> 40.26

Questions/ comments

3) **Jack** date: 28/11 disc: 3 & 4

00.20 ----------> 1

04.30----------> 5.28

39.00----------> 49.38 5+4=9

Questions/ comments

4) **Harry** date: 29/11 disc: 2 & 4

16.24----------> 17 46 x32 =

STOP

26.37----------> 28.38 Is this right?

48 put.........

Questions/ comment

Appendix I: Interview questions for new teachers

New Teachers

This will probably be a short, rather informal interviews covering questions like:

In your experience, what sorts of students are good at maths?

What makes them good at maths?

Have you noticed any particular group or type of students who tend to be good/bad at maths?

Why do you think some students find maths much harder than others do?

How can one tell if a student is good/bad at maths?

What is most important to know in maths?

Briefly how would you describe maths?

How many maths groups do you have?

What range of levels?

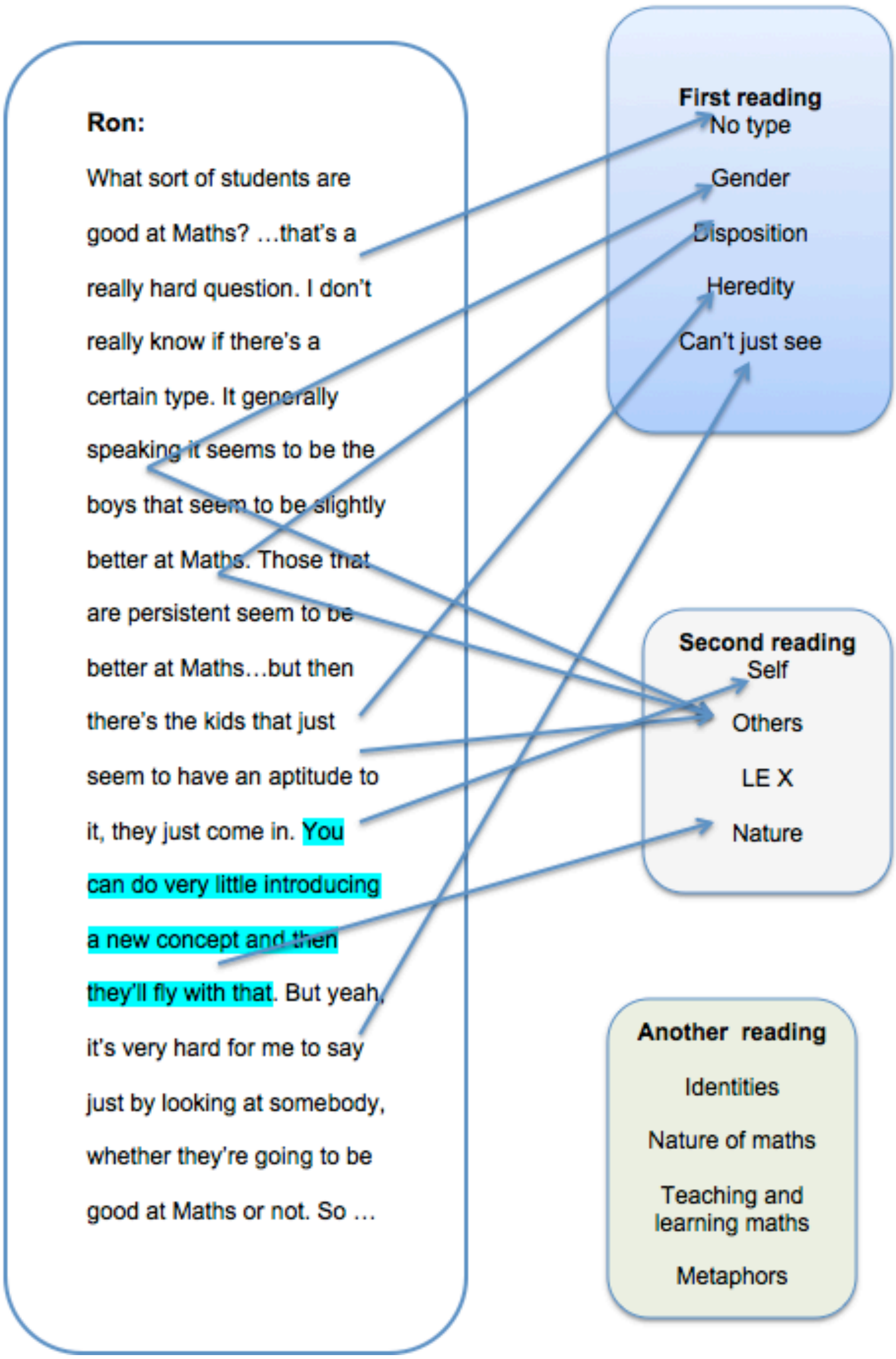
How is x doing?

What sort of student is x?

What group is s/he in? What level working at?

Question about the numeracy project...

Appendix J: Example of interview analyses



Appendix K: Analysis of achievement data

The students' responses on the Maths Beliefs Questionnaire indicate that maths achievement accounts significant difference in means for the self-identity factor in line with the results from TIMSS (Caygill & Kirkham, 2008). However, before even considering this relationship any further, it is important to unpack achievement results by looking at the variables that may affect achievement, such gender, ethnicity and school decile (Caygill & Kirkham, 2008; Crooks et al., 2010; Flockton et al., 2006; Smith & Hung, 2008).

Table K.1: Mean, standard deviation, t-values and significance for achievement differentiated by gender

		Number	Mean (SD)	*t*-value	p	Cohn's *d*
PAT	Female	194	1.90 (.64)	-2.64	.004*	.29
	Male	205	2.08 (.69)			
NumPA	Female	352	1.95 (.35)	-1.45	.148	.10
	Male	350	1.99 (.42)			

*significance at p<.05

In this study, boys had significantly higher scores than girls on the PAT (Mathematics) (*M* = 2.08 and 1.90 respectively) (*t*=.64, df=397, p<.01, d=.29)
but not on the NumPA (Table K.1).

Table K.2: ANOVA of ethnicity and achievement

		M (SD)	*f*	df	p	*post hoc*
PAT	Asian	2.21 (.69)	4.11	3, 395	.007*	ns
	Māori	1.82 (.68)				
	Pākehā	2.03 (.66)				
	Pasifika	1.64 (.50)				
NumPA	Asian	2.20 (.51)	6.82	3, 130.002	.0001*	A & M
	Māori	1.90 (.38)				A & Pas
	Pākehā	1.98 (.37)				
	Pasifika	1.79 (.42)				

* significance at p<.05

Results from the ANOVAs, summarised in Table K.2, indicated that ethnicity was a significant factor on both the PAT: mathematics (*F* (3, 395) = 4.11, p<.01) and NumPA (F (3, 130.002) = 6.82, p<.001) achievement results. Even though the Scheffé test showed p>.05 between the different groups on the PAT results, the effect sizes were worth considering with Asian students scoring considerably higher than both Māori and

Pasifika (Cohen’s d for Asian and Māori = .57, for Asian and Pasifika = .96 and Pākehā and Pasifika = .67) (Table K.3). On the NumPA, Asian students also received higher scores than the other ethnic groups scoring significantly higher than both Māori and Pasifika students (Asian M =2.20, Māori M =1.90 and Pasifika M = 1.79). Cohen’s d indicates a moderate to high effect size between Asian and Māori means (.67), Asian and Pākehā (.50), Asian and Pasifika (.88) as well as between Pākehā and Pasifika (.48).

Table K.3: Cohen's d ethnicity and achievement

		PAT	NumPA
Ethnicity	A & M	.57	.67
	A & Pak	.27	.50
	A & Pas	.96	.88
	M & Pak	.31	.21
	M & Pas	.31	.28
	Pak & Pas	.67	.48
SES	L & M	.03	.19
	L & H	.91	.76
	M & H	1	.64

School decile rank, an indicator of SES, also accounted for differences in the means on both the PAT and NumPA. High decile schools had significantly higher mean scores than did either low or middle schools (Table K.4) on the PAT (F (2, 398) = 48.35, p<.001) and the NumPA (F (2, 498.338) = 38.85, p<.001). The *post hoc* tests showed the significant differences between low and high, and middle and high decile schools on both tests. The effect size was substantial between these scores on both measures with d = .91 and 1 for the PAT and .79 and .64 for the NumPA.

Table K.4: ANOVA of SES and achievement

		M (SD)	*f*	df	p	*Post hoc*
PAT	Low	1.67 (.66)	48.35	2, 398	.0001*	L & h
	middle	1.65 (.59)				M & h
	high	2.25 (.61)				
NumPA	Low	1.82 (.40)	38.85	2, 498.338	.0001*	L & h
	middle	1.89 (.33)				M & h
	high	2.12 (.39)				

*Significance at p<.001

In conclusion, SES, gender and ethnicity all affect achievement scores. SES and student ethnicity had an effect on these children’s mathematics achievement scores. Gender also had an affect but only on one of the measures, the PAT: mathematics scores. For this reason, it is important to consider all the variables that explain differences in achievement before concluding that the relationship between achievement and factor means is in any way directly correlational or causal.

Appendix L: Multiple regression analyses of the four maths beliefs factors

One serious limitation with regression analyses in SPSS is that the program cannot deal with categorical variables. The only way to deal with this issue was to create a series of dummy variable for ethnicities. However, the results in this case are more confusing than clarifying. Another limitation in the case of this study was that the percentages of variance accounted for by the four models were much too low to be useful.

Table L.1: Summary of linear regression analysis for variables predicting Self

	B	SE B	β
model			
(constant) **Self**	-51.49	1.087	
Stages	.76	.18	.15*
gender	.99	.40	.09*
ethnicity: Pākehā	-1.09	.44	-.09*

* Significance p < .05

R square = .04; $F_{3, 689} = 9.697$, $p < .0005$ (using stepwise method). Significant variable are achievement, gender ethnicity for Pākehā students, explaining 4% of variance in this model. SES, age and other ethnicities were not significant predictors in this model (Table L.1).

Table L.2: Summary of linear regression analysis for variables predicting Ability

	b	SE b	β
model			
(constant) **Ability**	20.15	.61	
SES	-.16	.08	-.05*
ethnicity: **Māori**	1.96	.65	.12*

*Significance p < .05

R square =.03; $F_{2, 686} = 8.988$, $p < .0005$ (using stepwise method). The model was significant for SES and Māori ethnicity, which explains 3% of variance. Gender, age, achievement and other ethnicities were not significant predictors in this model (Table L.2).

Table L.3: Summary of linear regression analysis for variables predicting Learning Environment

	b	SE b	β
model			
(constant) LE	12.88	.16	
SES	-.07	.02	-.11*

*Significance p < .05

R square = .01; $F_{1, 688} = 8.854$, $p < .005$ (using stepwise method). The model was significant for SES as a predictor variable, explaining only 1% of variance. None of the other variables were significant (Table L.3).

Table L.4: Summary of linear regression analysis for variables predicting Nature

	b	SE b	β
model			
(constant) Nature	3.15	1.69	
Stages	.60	.10	.23*
ethnicity: Pākehā	.48	.23	.08*
age	.38	.19	.08*

* Significance p < .05

R square = .08; $F_{3, 686} = 19{,}766$, $p < .0005$ (using stepwise method). Achievement, Pākehā ethnicity and age were significant variables explaining 8% of variance in this model. Gender and other ethnicities were not significant predictors (Table L.4).

Appendix M: Focus Schools statistics

In this section, the means for the two schools on the mathematics beliefs factors are compared. The effects of gender, school year, ethnicity, age and achievement data for each school on these means are explored.

Table M.1: Means (SD) for four belief factors by focus school and student characteristics

			Self	Group	L.E.	Nature	Total	n
School	Kikorangi		22.55 (4.92)	20.05 (6.11)	12.82 (1.31)	10.05 (2.81)	65.48 (10.22)	56
	Whero		21.47 (5.16)	18.13 (6.15)	11.96 (1.95)	10.60 (2.39)	62.16 (9.71)	179
Gender	Kikorangi	F	21.04 (4.38)	19.32 (6.65)	12.61 (1.37)	10.43 (2.38)	66.42 (10.74)	28
		M	24.07 (5.03)	20.79 (5.55)	13.04 (1.23)	9.68 (3.18)	64.67 (9.87)	28
	Whero	F	20.93 (5.31)	17.40 (6.54)	12.21 (1.55)	10.63 (2.06)	61.20 (10.35)	82
		M	22.00 (5.03)	18.77 (5.82)	11.84 (2.11)	10.59 (2.66)	63.22 (8.89)	95
Age	Kikorangi	8	13.00	10.00	11.00	10.00	44.00	1
		9	22.67 (4.40)	21.04 (5.51)	12.71 (1.40)	10.92 (2.39)	67.33 (10.21)	24
		10	22.89 (4.85)	19.63 (6.10)	13.04 (1.09)	9.52 (2.83)	65.15 (8.95)	27
		11	23.67 (8.51)	16.33 (8.33)	12.67 (2.52)	8.33 (5.03)	61.00 (18.19)	3
	Whero	8	22.50 (7.78)	21.00 (1.41)	12.50 (2.12)	14.50 (.71)	70.50 (9.19)	2
		9	20.94 (5.20)	16.61 (6.41)	12.09 (1.82)	10.04 (2.40)	59.69 (9.98)	70
		10	21.67 (5.14)	18.91 (5.70)	11.89 (2.06)	10.77 (2.23)	63.25 (9.22)	101
		11	23.60 (5.08)	20.80 (8.76)	11.60 (.55)	12.40 (2.70)	68.40 (6.84)	5
School year	Kikorangi	Year 5	22.15 (4.60)	21.12 (6.00)	12.54 (1.36)	10.62 (1.23)	66.42 (10.74)	26
		Year 6	22.90 (5.23)	19.13 (6.20)	13.07 (1.23)	9.57 (3.05)	64.67 (9.87)	30
	Whero	Year 5	21.39 (5.03)	17.11 (6.38)	12.14 (1.86)	10.56 (2.63)	61.20 (10.35)	94
		Year 6	21.56 (5.33)	19.26 (5.71)	11.76 (1.99)	10.64 (2.10)	63.22 (8.89)	85
Ethnicity	Kikorangi	Asian	22.50 (4.80)	16.00 (4.90)	12.00 (1.41)	12.00 (2.94)	62.50 (8.43)	4
		Māori	24.00 (4.15)	23.55 (7.22)	13.09 (.70)	11.09 (2.84)	71.73 (10.88)	11
		Pākehā	22.17 (5.15)	19.51 (5.59)	12.83 (1.41)	9.59 (2.69)	64.10 (9.76)	41
	Whero	Asian	24.95 (4.84)	16.25 (6.66)	12.24 (1.79)	10.38 (2.22)	63.81 (9.08)	21
		Māori	23.00 (5.28)	21.25 (4.85)	11.75 (2.32)	10.31 (2.72)	66.31 (7.31)	16
		Pākehā	20.87 (5.00)	17.95 (6.12)	11.94 (1.92)	10.60 (2.42)	61.36 (10.03)	139
		Pasifika	17.00 (2.00)	23.00 (1.73)	12.33 (1.53)	13.33 (1.53)	65.67 (1.16)	3
NumPA	Kikorangi	Middle	22.58 (4.96)	20.31 (5.86	12.58 (1.32)	10.11 (2.80)	65.48 (10.22)	55
	Whero	Middle	21.20 (5.19)	18.16 (6.15)	12.08 (1.78)	10.40 (2.35)	61.85 (9.78)	137
		High	23.30 (4.60)	18.20 (6.20)	11.93 (1.98)	11.10 (2.62)	64.53 (9.12)	30

Kikorangi had a higher mean than Whero on each of the mathematics beliefs factors apart from the *Nature of Mathematics* (see Figure M.1). On Table M.1, the means and standard deviation are reported for the mathematics beliefs factors for each school by each of these aspects.

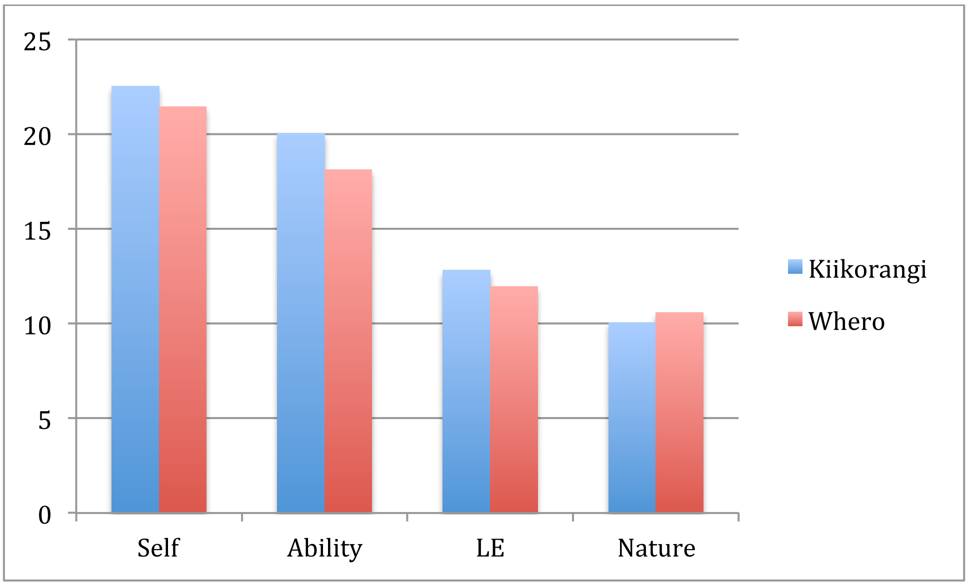


Figure M.1: Comparison of Focus Schools' beliefs means

Significant differences between Kikorangi and Whero were found on the means for *Ability* (*M* (SD)= 20.05 (6.11) & 18.13 (6.15), *t* = 2.05, p<.05), *Learning Environment* (*M* (SD)= 12.82 (1.31) & 11.98 (1.92), *t* = 3.80, p<.005) and Total Belief means (*M* (SD) = 65.48 (10.22) & 62.16 (9.71), *t* = 2.21, p<.05); however, only *Learning Environment* had an effect size above .4 (Table M.2)

Table M.2: Summary of significance on means comparison between maths beliefs factors

schools	t	p	Cohen's *d*
Self	1.38	.169	.24
Ability	2.05	.042*	.31
L.E.	3.80	.0001*	.52
Nature	-1.43	.155	.21
Total	2.21	.028*	.33

*Significance at p<.05

Gender accounted for a difference of means only on one of the mathematics beliefs factors at one of the schools (Tables M.3 and M.4). The boys at Kikorangi had significantly higher means than the girls on the *Self* factor with a moderately good effect size: M (SD) = 24.07 (5.03) & 21.04 (4.48), t =.90, $p< .05$, d = .64. In other words, the boys were more confident about their ability to do maths then were girls.

Table M.3: Kikorangi t-values, significance and effect size for belief factors differentiated by gender

Gender Kikorangi	*t*	p	Cohen's *d*
Self	-2.41	.019*	.64
Ability	-.90	.375	.24
L.E.	-1.23	.224	.33
Nature	1.00	.322	.27
Total	.64	.526	.19

*Significance at p<.05

Table M.4: Whero t-values, significance and effect size for belief factors differentiated by gender

Gender Whero	*t*	p	Cohen's *d*
Self	-1.38	.170	.19
Ability	-1.47	.143	.22
L.E.	1.32	.187	.20
Nature	.126	.900	.03
Total	-1.39	.165	.21

Most of the students attending the two focus schools were in combined mathematics classes; however, their year seemed to account for some of the differences in means. In the case of Kikorangi, Year 5 had higher means on *Ability* and *Nature* and lower means on *Learning Environment*; although none of these differences were significant, the effect size was above .4 for both *Learning Environment* and *Nature* (Tables M.1 and M.5).

Table M.5: Kikorangi t-values, significance and effect size for belief factors differentiated by year

Yr5/6 Kikorangi	*t*	p	Cohen's *d*
Self	-1.56	.576	.15
Ability	1.22	.229	.33
L.E.	-1.53	.133	.41
Nature	1.41	.165	.45
Total	.64	.526	.17

For Whero, Year 5 students had significantly lower means for the *Ability* factor than did Year 6: M (SD) = 17.11 (6.38) & 19.26 (5.71), t = 2.37, $p < .05$, but with an effect size below.4 (See Tables M.1 and M.6).

Table M.6: Whero t-values, significance and effect size for belief factors differentiated by year

Yr5/6 Whero	*t*	p	Cohen's *d*
Self	1.22	.825	.03
Ability	-2.37	.019*	.36
L.E.	1.30	.197	.20
Nature	-.20	.840	.03
Total	-1.39	.165	.21

*Significance at p<.05

Ethnicity seems to account for some marked differences of means between groups. At Kikorangi, Māori students had the highest means on the factors followed by Pākehā and then Asian students (Table M.1). Even though ANOVAs indicated no significant differences of means for Kikorangi, large effect sizes indicate that they are worth addressing. For instance, *Ability* where the p value is just over .05, the effect size between Asian and Māori students is 1.25, between Asian and Pākehā .67 and Māori and Pākehā.63; similarly, for *Learning Environment*, the effect size between Asian and Māori .98 for the total .95, and for *Nature* the difference between Asian and Pākehā has an effect size of .73 (see Table M.7). These results may be skewed by the small number of Asian students (N=4) involved; however, the effect sizes between Māori and Pākehā still remain well above .4 for three of the factors.

Table M7: ANOVA for Kikorangi belief factors by ethnicity

Ethnicity Kikorangi	*F*	df	p	Cohen's *d*
Self	.591	2, 53	.557	M & Pak .39
Ability	3.050	2, 53	.056	A & M 1.25 A & Pak .67 M & Pak .63
L.E.	1.023	2, 53	.367	A & M .98
Nature	2.402	2, 53	.100	A & Pak .73 A & M .31 M & Pak .52
Total	2.766	2, 53	.072	A & M .95 A & Pak .18 M & Pak .74

The pattern for Whero was slightly different where Māori and Pasifika students had the highest total means scores followed by Asian student and then Pākehā (see Table L.1). Because only 3 students identified themselves as Pasifika, not enough to give an accurate picture, the effect sizes that involve this group have been eliminated from Table M.7. For the *Self* factor, Asian students had the highest means (M = 24.95 as opposed to Māori = 23.00, Pākehā = 20.87 and Pasifika = 17.00) which was found to be significant (F (3, 175) = 5.400, p<.005,) between Asian and Pākehā students.

In addition, moderate to large effect sizes above .4 were found between Asian and Pākehā and Māori and Pākehā means (see Tables M.1 and M.8). A significant difference was also found for *Ability* (F (3, 175) = 2.785, p < .05, but Scheffé *post hoc* tests indicated no significant differences between groups. On the other hand, an examination of effect size indicated moderate to large differences between Asian and Māori and Māori and Pākehā. Finally, a moderate effect size was found between Māori and Pākehā means on the Total score

Table M.8: ANOVA for Whero belief factors by ethnicity

Ethnicity Whero	**F**	**df**	**p**	**Post hoc**	**Cohen's d**
Self	5.400	3, 175	.0001**	A & Pak (.008)	A & M .39 A & Pak .83 M & Pak .41
Ability	2.785	3, 175	.042*	NS	A & M .86 A & Pak .27 M & Pak .60
L.E.	.250	3, 175	.861		
Nature	1.461	3, 175	.227		
Total	1.639	3, 175	.182		A & M .30 A & Pak .26 M & Pak .56

The age range of the students taking part in this study was between 8 and 11, yet most of them fell in the 9-10 age range. Only a total of three eight –year olds and 8 eleven-year olds were included in the Focus Schools database: for this reason, only the differences in means for 9 and 10-year olds were explored. For Kikorangi, there were no significant differences of means that could be attributed to age; however, the effect sizes between age group on *Nature of Mathematics* were over .4 (Table M.9).

Table M.9: T-tests for Kikorangi belief factors differentiate between 9 and 10 year olds

Age: Kikorangi	***t***	**p**	**Cohen's *d***
Self	1.71	.865	.05
Ability	.864	.392	.25
L.E.	.941	.351	.26
Nature	1.791	.080	.53
Total	.815	.419	.23

For Whero, age accounted for significant differences in means on the *Ability* (*M*(SD) = 16.61(6.41) & 18.91(5.70), t = 2.461, p < .05), *Nature of Mathematics* (*M*(SD) = 10.04(2.40) & 10.77(2.23), *t* = 2.041, p < .05) and the Total Maths Beliefs factors (*M*(SD) = 59.69(9.98) & 63.25(9.22), *t = 2.366,* p < .05). On the other hand, effect size above .4 was only found for the *Ability* factor (Tables M.1 and M.10).

Table M.10: T-tests for Whero belief factors differentiate between 9 and 10 year olds

Age: Whero	***t***	**p**	**Cohen's *d***
Self	.909	.365	.14
Ability	2.461	.015*	.40
L.E.	.637	.525	.10
Nature	2.041	.043*	.32
Total	2.366	.017*	.37

*Significance at p<.05

The achievement data is not useful when trying to explain differences in means scores for Kikorangi because PAT: mathematics scores were not available for this group of students, and after the NumPA levels had been merged, all the students from this school ended up in the middle band. The students at Whero were in both the middle and high bands on this measure. Students in the higher band had significantly higher means scores than those in the middle on Self (*M* (SD) = 23.30 (4.60) and =21.20 (5.19), *t* =2.04, p<.05, d =.43) (Tables M.10 and M.11).

Table M.11: Whero t-values, significance and effect size for belief factors differentiated by achievement

NumPA: Whero	*t*	p	Cohen's *d*
Self	-2.04	.043*	.43
Ability	-.03	.975	.001
L.E.	.40	.689	.08
Nature	-1.45	.150	.28
Total	-1.38	.170	.28

*Significance at p<.05

To sum up, there is a significant difference between the two schools' means for the *Ability*, *Learning Environment*, and Total beliefs. Boys had significantly higher means than girls for *Self* at Kikorangi. For Whero, Year 6 students had significantly higher means than Year 5 students for the *Ability*, ethnicity accounted for differences for the *Self* and *Ability* factors, age for *Ability*, *Nature of Mathematics* and Total, and achievement for *Self*. The effect size calculations indicated a different pattern: for Kikorangi effect sizes above .4 were found by year on *Ability*, *Learning Environment*, *Nature* and Total and by age for *Ability* beliefs means. For Whero noticeable effect sizes were found by ethnicity on *Ability*, *Nature* and Total, by age on *Self*, by NumPA on *Self*, and by PAT on *Self*, *Learning Environment*, *Nature of Mathematics* and Total Belief means.

Appendix N: Statistical tables from the Mathematics Personal Mini factor (MBM)

Table N.1: Means and standard deviations on the MPM

			N	Mean	SD
Total			804	12.17	2.06
Ethnicity	Asian	Total	50	12.36	2.15
		Female	24	12.25	1.87
		Male	26	12.46	2.42
	Māori	Total	147	12.32	1.93
		Female	75	12.08	1.90
		Male	72	12.57	1.93
	Pākehā	Total	573	12.10	2.08
		Female	290	11.87	2.06
		Male	283	12.33	2.08
	Pasifika	Total	34	12.38	2.20
		Female	16	13.06	1.84
		Male	18	11.78	2.37
Gender	Female		405	11.98	2.02
	Male		399	12.36	2.09
Year	5		386	12.23	2.07
	6		416	12.11	2.05
Age	8		20	12.85	2.13
	9		338	12.14	2.08
	10		417	12.14	2.03
	11		19	12.68	1.92

	N	Mean	SD	
Decile	Low	159	11.98	2.25
	Middle	322	12.30	2.03
	High	323	12.13	1.99
NumPA	Low	62	12.03	2.21
	Middle	584	12.21	2.10
	High	42	12.07	1.80
	Total	688	12.18	2.09
PAT	High	89	12.04	2.08
	Middle	220	12.22	1.93
	Low	88	12.16	1.96
	Total	397	12.17	1.97

Table N.2: T-tests from MPM means by gender and year

Student mini factor		*t*	p	Cohen's *d*
Gender	**all**	2.643	**.008***	.18
	Asian	.344	.733	.10
	Māori	1.547	.124	.26
	Pākehā	2.699	**.007***	.22
	Pasifika	1.750	.090	.60
Year	**all**	.874	.383	.06
	Asian	.094	.926	.03
	Māori	2.285	**.024***	.38
	Pākehā	.393	.694	.03
	Pasifika	1.222	.231	.42

*Significance at p<.05

Table N.3: ANOVAs comparing student means on MPM, decile and achievement

		F	**df**	**p**	**Cohen's *d***
Age	**all**	1.183	3, 790	.315	
	Māori	4.275	3, 141	**.006***	8 & 9 = .34 8 & 10 = .34 8 & 11 = .08 9 & 10 = .00 9 & 11 = .27 10 & 11 = .27
Decile		1.328	2, 801	.266	
NumPA		.276	2, 685	.759	
PAT		.260	2, 394	.771	
Female/ethnicity		2.058	3, 401	.105	A & M = .09 A & P = .19 A & Pas = .44 M & P = .06 M & Pas = .52 P & Pas = .61
Male/ethnicity		.744	3, 395	.526	A & M = .05 A & P = .06 A & Pas = .28 M & P = .12 M & Pas = .37 P & Pas = .25

*Significance at p<.05

Appendix O: Mapping individual responses to the questionnaires

Table O.1: Focus teachers' maths beliefs

	Self	Ability	L/T	Nature	Total
Ron	18	31	9	9	67
Mr Forrest	19	24	8	10	61

Table O.2: Focus children's maths beliefs on the maths beliefs factors

	gender	age	year	stages	Self	Ability	LE	Nature	Total	group
George	m	10	6	6	16	12	11	15	54	h
Jasmine	f	9	5	5	20	19	14	15	68	m
Fred	m	9	5	6	17	29	12	7	65	h
Sammie	f	10	6	5	23	24	13	14	74	l
Harry	m	9	5	5	9	16	7	11	43	l
Jack	m	9	5	5	21	7	13	9	50	m
Caroline	f	10	5	-	19	20	13	7	59	l
Chloe	f	9	5	5	22	13	12	12	59	h
Lilly	f	9	5	6	19	18	14	9	60	h

Appendix P: Focus children's drawing data

Table P.1: Drawing belief factors

	Self	Ability/others	LE	Nature
George	✓	✓	?	✓
Jasmine			?	✓
Fred	✓	✓	✓	✓
Sammie			?	✓
Harry	✓	✓	✓	✓
Jack	✓	✓		✓
Caroline	✓?			?
Chloe	✓		?	✓
Lilly	✓		✓	✓

Table P.2: Comparison of Alien Task and drawing

	#		symbols		measurement		geom		money		algebra		affect		easy		metaphor		utility	
	AT	D	AT	D	AT	D	AT	D	AT	D	AT	D	AT	D	AT	D	AT	D	AT	D
George	✓	✓	✓	✓										✓		✓×		✓	✓	
Jasmine	✓	✓		✓		✓				✓							✓	✓		✓
Fred	✓	✓		✓					✓					✓×		✓×	✓	✓	✓	
Sammie	✓	✓	✓	✓		✓		✓		✓								✓		✓
Harry		✓		✓									×	×	×	×		✓		
Jack	0		0		0		0		0		0		0	✓	0		0	✓	0	
Caroline														✓	✓×	?		✓?		
Chloe	✓		✓											✓		?		✓		
Lilly	✓	✓		✓		✓✓							✓	✓		?		✓		

Key: ✓ = present or positive
x = negative
0 = did not answer
blank = not present

Appendix Q: Drawings from Island School, Ecuador

Figure Q.1: Ariane
I think maths is easy but when it comes to big numbers I don't like it

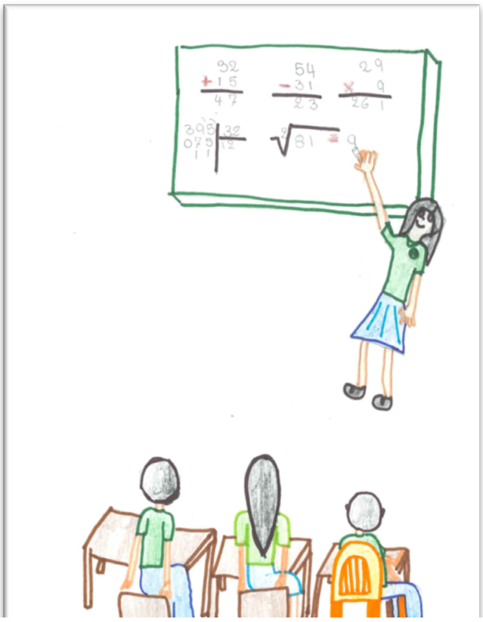


Figure Q.2: Pamela
Mathematicas has big numbers and is difficult

Metaphors used by both Ecuadorian and New Zealand children

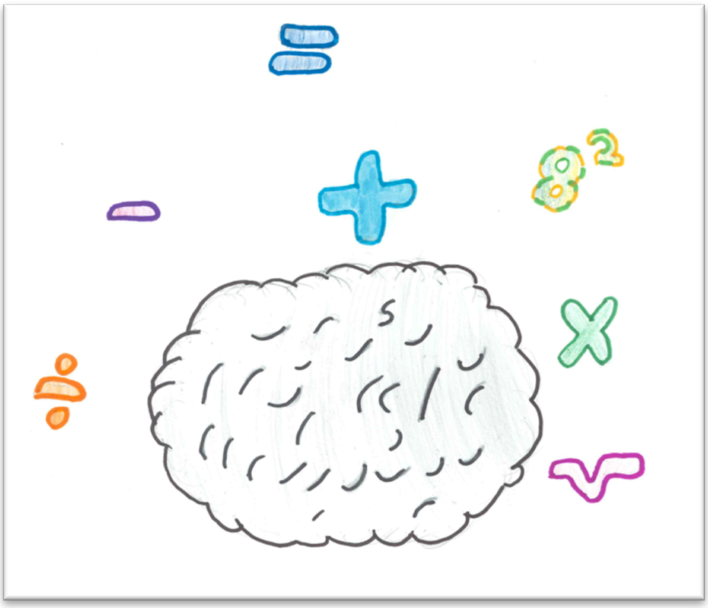


Figure Q.3: Tod
Math is difficult because I think a lot

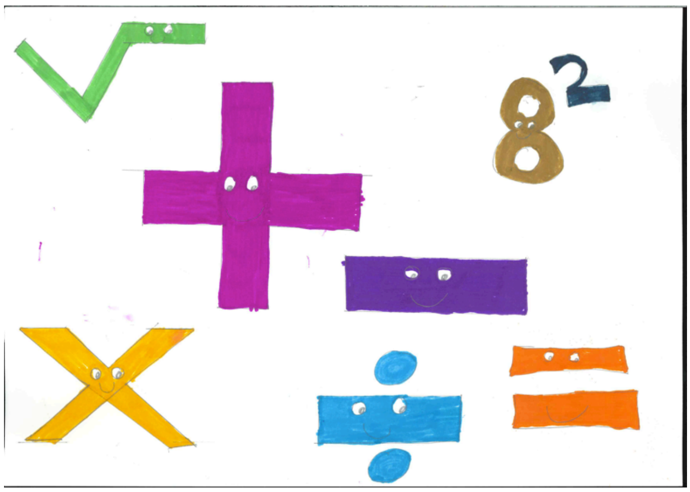


Figure Q.4: Melisa (Ecuador)
I think maths is amazing because there are very different tipes (types) of math

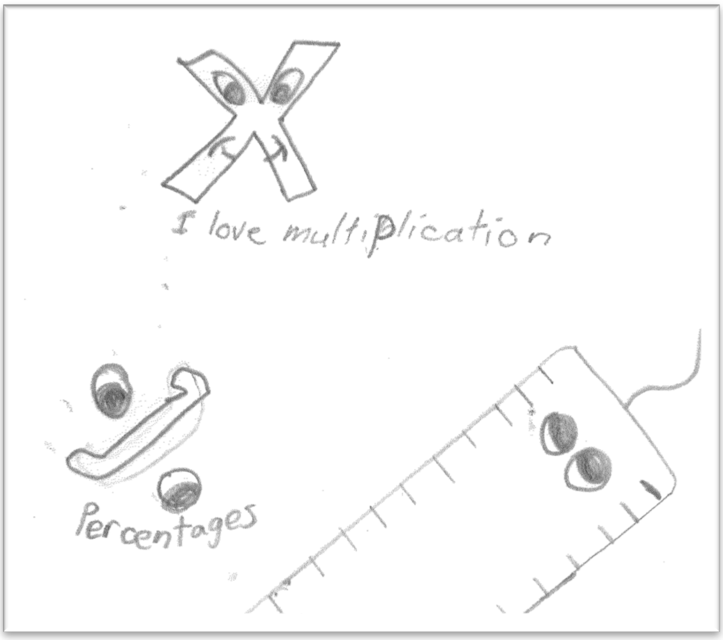


Figure Q.5: Ari (NZ), detail

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